ASSESSMENT AND ENHANCEMENT OF POWER SYSTEM STATE ESTIMATION QUALITY

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Abstract –The accuracy of the power system state estimation determines the usefulness of real-time power system operation and control applications. The quality of the state estimator results is judged by computing two classes of accuracy indexes namely, the post-estimation value of the ratio between the weighted least square (WLS) objective function and its corresponding threshold, as well as the ratios between the standard deviation of the estimated and measured quantities.

The paper describes (a) an original method to compute the standard deviations of the estimated values (branch power transits and nodal power injections) even if the corresponding measured quantities are not available and (b) an original procedure for the detection of topology errors.

Finally numerical simulations based on real-time measurement data are presented.

Keywords: power system state estimation, quality evaluation, standard deviations, topology error.

1 INTRODUCTION

The power system state estimation constitutes the core of the on-line security analysis function. It acts like a filter between the raw measurements received from the system and all the application functions that require the most reliable data base for the current state of the system [1÷3]. In almost all state estimation implementation, a set of measurements obtained by a SCADA system at approximately the same time instant throughout the whole supervised network is centrally processed by a static state estimator at regular intervals or by operator request. State estimation is based on the mathematical relations between system state variables (i.e. bus voltage magnitudes and phase angles) and the measurements used to compute the real-time system state (e.g. real and reactive power measurements and bus voltage measurements). Various techniques (coupled and decoupled formulations) have been used to obtain the system state from a set of noisy measurements and system parameters [4÷15]. An excellent survey on power system state estimation can be found in [16].

The quality of results obtained from a state estimator can be judged by using three classes of magnitudes:

- The standard deviations of the estimated quantities;
- The post estimation value of the state estimator objective function;
- The Lagrange multipliers of the equality constraints corresponding to the “zero injection” buses.

The paper is divided in three main parts. The first part describes an original method to compute the standard deviations of all the estimated quantities (branch power transits and nodal power injections) even if the corresponding measured quantities are not available. The standard deviations of estimated values are expressed as a function of bus voltages, co-variances and standard deviations of these voltages, issued at the end of the estimation process. These quantities are used to evaluate the quality of the state estimation results.

The second part concerns the description of an original post-estimation procedure for the detection of topology errors. This procedure is implemented in the state estimation algorithm before the bad data processing. This approach preserves the necessary information for a substantiated judgment whether a measurement or a topology error is the likely cause of an error group.

The third part of the paper presents numerical simulations based on real-time measurement data collected in the Belgian energy control centre.

The proposed method has been implemented into the state estimation algorithm of the Belgian EMS.

2 QUALITY INDEXES USING STANDARD DEVIATIONS OF THE ESTIMATED QUANTITIES

2.1 Description

At the end of the state estimation process the standard deviations of the following estimated quantities are computed:

- Bus voltage magnitudes and angles
- Active and reactive branch power transits
- Active and reactive nodal power injections

The accuracy and quality of the estimates provided by the state estimator is judged according to the value of the ratio:

$$\boldsymbol{R_i} = \frac{\sigma_{mes,i}}{\sigma_{est,i}} \tag{1}$$

where:

$\sigma_{mes,i}$: standard deviation of the measured quantity $\boldsymbol{i}$

$\sigma_{est,i}$: standard deviation of the estimated quantity $\boldsymbol{i}$

For the estimates $\boldsymbol{i}$ whose corresponding measured quantities are not available, the quality of estimation can be judged according to the value of the ratio:

$$R_i^{'} = \frac{\sigma_{mesv,i}}{\sigma_{est,i}} \tag{1bis}$$

where:

$\sigma_{mesv,i}$: standard deviation of a virtual measurement associated to the estimated quantity i ;

$\sigma_{est,i}$: standard deviation of the estimated quantity i

The larger the ratio R_i or $R_i^{'}$ the better the estimate i

☑ *Comment*: The use of the quality indexes (1) and (1bis) is limited to the system areas having a good redundancy.

2.2 Computation of standard deviations

For those estimates i which have a measured quantity available, the value of the standard deviation $\sigma_{est,i}$ is directly accessible from the covariance matrix of estimation errors evaluated at the end of the estimation process [17,18].

For the estimates i whose corresponding measured quantities are not available, the standard deviations $\sigma_{est,i}$ and $\sigma_{mesv,i}$ are computed as shown here after in §2.2.1 and §2.2.2

2.2.1 Computation of standard deviations ($\sigma_{est,i}$) of the estimated quantities which are not measured.

The standard deviations of estimated values are expressed as a function of estimated bus voltages, co-variances and standard deviations of these voltages. The values of standard deviations and co-variances of bus voltage magnitudes and angles are available from the inverse of the gain matrix issued at the end of the estimation process [18].

a) List of symbols:

- N : set of power system buses.
- V_i, θ_i : voltage magnitude and phase angle at bus i
- $\theta_{ij} = \theta_i - \theta_j$
- $G_{ik} + jB_{ik}$: ik th element of the complex bus admittance matrix.
- $g_{ik0} + jb_{ik0}$: admittance of the shunt branch connected at bus i
- Ωi : set of bus numbers that are directly connected to bus i
- $E(X)$: expected value of the random variable X
- $E(X^k)$: k th moment of the random variable X
- $\mathrm{cov}(X,Y)$: co-variance of the random variables X and Y

b) Assumptions:

- $$\mathrm{cov}(V_i, \theta_k) = 0 \qquad i,k \in N \tag{2}$$
- for all series branches connecting buses i and k :
$$\begin{aligned} \sin\theta_{ik} &= \theta_{ik} \\ \cos\theta_{ik} &= 1 \end{aligned} \tag{3}$$

c) Statistical relations:

- The joint moments of two random variables X and Y is given by [19]:

$$E\left(X^k Y^n\right) = E\left(X^k\right)E\left(Y^n\right) + kn \int_0^{\mathrm{cov}(X,Y)} E\left(X^{k-1}Y^{n-1}\right) d\,\mathrm{cov}(X,Y) \tag{4}$$

- The joint moments of three random variables X, Y and Z having each of them a nil expected value, is given by [20]:

$$\begin{aligned} &E\left(X^2YZ\right) = \sigma_X^2\,\mathrm{cov}(Y,Z) + 2\,\mathrm{cov}(X,Z)\mathrm{cov}(X,Y) \\ &E(XYZ) = 0 \end{aligned} \tag{5}$$

The expressions of the real and reactive power flow from bus i to bus k are:

$$P_{ik} = -V_i^2\left(G_{ik} - g_{iko}\right) + V_iV_k\left[G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}\right] \tag{6}$$

$$Q_{ik} = V_i^2\left(B_{ik} - b_{iko}\right) - V_iV_k\left[B_{ik}\cos\theta_{ik} - G_{ik}\sin\theta_{ik}\right] \tag{7}$$

The expressions of the real and reactive power injection at bus i are:

$$P_i = G_{ii}V_i^2 + \sum_{k\in i;k\neq i} V_iV_k\left[G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}\right] \tag{8}$$

$$Q_i = -B_{ii}V_i^2 - \sum_{k\in i;k\neq i} V_iV_k\left[B_{ik}\cos\theta_{ik} - G_{ik}\sin\theta_{ik}\right] \tag{9}$$

Considering the assumptions (2) and (3) and the relations (4)÷(7), the standard deviations of the estimated real and reactive power flow from bus i to bus k are given by:

$$\sigma_{est,P_{ik}} = \sqrt{E\left(P_{ik}^2\right) - E^2\left(P_{ik}\right)} =$$

$$\sqrt{\begin{aligned} &2\left(G_{ik} - g_{iko}\right)^2\left(2V_i^2\sigma_{V_i}^2 + \sigma_{V_i}^4\right) + \left[G_{ik} + B_{ik}\theta_{ik}\right]^2\begin{bmatrix} 2V_iV_k\,\mathrm{cov}(V_i,V_k) + \mathrm{cov}^2(V_i,V_k) + \\ +V_i^2\sigma_{V_k}^2 + V_k^2\sigma_{V_i}^2 + \sigma_{V_i}^2\sigma_{V_k}^2 \end{bmatrix} + \\ &+ B_{ik}^2\left[\sigma_{\theta_i}^2 + \sigma_{\theta_k}^2 - 2\,\mathrm{cov}(\theta_i,\theta_k)\right]\begin{bmatrix} V_i^2V_k^2 + 4V_iV_k\,\mathrm{cov}(V_i,V_k) + 2\,\mathrm{cov}^2(V_i,V_k) + \\ +V_i^2\sigma_{V_k}^2 + V_k^2\sigma_{V_i}^2 + \sigma_{V_i}^2\sigma_{V_k}^2 \end{bmatrix} - \\ &- 4\left(G_{ik} - g_{iko}\right)\left[G_{ik} + B_{ik}\theta_{ik}\right]\left[V_i^2\,\mathrm{cov}(V_i,V_k) + V_iV_k\sigma_{V_i}^2 + \sigma_{V_i}^2\,\mathrm{cov}(V_i,V_k)\right] \end{aligned}} \tag{10}$$

$$\sigma_{est,Q_{ik}} = \sqrt{E\left(Q_{ik}^2\right) - E^2\left(Q_{ik}\right)} =$$

$$\sqrt{\begin{aligned}&2\left(B_{ik} - b_{iko}\right)^2\left(2V_i^2\sigma_{V_i}^2 + \sigma_{V_i}^4\right) + \left[B_{ik} - G_{ik}\theta_{ik}\right]^2\begin{bmatrix}2V_iV_k\operatorname{cov}(V_i,V_k) + \operatorname{cov}^2(V_i,V_k) + \\ + V_i^2\sigma_{V_k}^2 + V_k^2\sigma_{V_i}^2 + \sigma_{V_i}^2\sigma_{V_k}^2\end{bmatrix} + \\ &+ G_{ik}^2\left[\sigma_{\theta_i}^2 + \sigma_{\theta_k}^2 - 2\operatorname{cov}(\theta_i,\theta_k)\right]\begin{bmatrix}V_i^2V_k^2 + 4V_iV_k\operatorname{cov}(V_i,V_k) + 2\operatorname{cov}^2(V_i,V_k) + \\ + V_i^2\sigma_{V_k}^2 + V_k^2\sigma_{V_i}^2 + \sigma_{V_i}^2\sigma_{V_k}^2\end{bmatrix} - \\ &- 4\left(B_{ik} - b_{iko}\right)\left[G_{ik} + B_{ik}\theta_{ik}\right]\left[V_i^2\operatorname{cov}(V_i,V_k) + V_iV_k\sigma_{V_i}^2 + \sigma_{V_i}^2\operatorname{cov}(V_i,V_k)\right]\end{aligned}} \tag{11}$$

Considering the assumptions (2) and (3) and the relations (4),(5),(8),(9), the standard deviations of the estimated real and reactive power injection at bus i are given by:

$$\sigma_{est,P_i} = \sqrt{E\left(P_i^2\right) - E^2\left(P_i\right)} =$$

$$\sqrt{\begin{aligned}&2G_{ii}^2\sigma_{V_i}^2\left(\sigma_{V_i}^2 + 2V_i^2\right) + 4G_{ii}\sum_{k\in\Omega i;k\neq i}\left[G_{ik} + B_{ik}\theta_{ik}\right]\left[V_i^2\operatorname{cov}(V_i,V_k) + V_iV_k\sigma_{V_i}^2 + \sigma_{V_i}^2\operatorname{cov}(V_i,V_k)\right] + \\ &+ \sum_{k\in\Omega i;k\neq i} B_{ik}^2\left[\sigma_{\theta_i}^2 + \sigma_{\theta_k}^2 - 2\operatorname{cov}(\theta_i,\theta_k)\right]\begin{bmatrix}V_i^2V_k^2 + 4V_iV_k\operatorname{cov}(V_i,V_k) + \\ + 2\operatorname{cov}^2(V_i,V_k) + V_i^2\sigma_{V_k}^2 + V_k^2\sigma_{V_i}^2 + \sigma_{V_i}^2\sigma_{V_k}^2\end{bmatrix} + \\ &+ \sum_{k\in\Omega i;k\neq i}\left[G_{ik} + B_{ik}\theta_{ik}\right]^2\left[2V_iV_k\operatorname{cov}(V_i,V_k) + \operatorname{cov}^2(V_i,V_k) + V_i^2\sigma_{V_k}^2 + V_k^2\sigma_{V_i}^2 + \sigma_{V_i}^2\sigma_{V_k}^2\right] + \\ &+ 2\sum_{k\in\Omega i;k\neq i}\ \sum_{\substack{l\in\Omega i;l\neq i\\ l>k}}\begin{bmatrix}V_i^2V_kV_l + V_i^2\operatorname{cov}(V_k,V_l) + 2V_iV_k\operatorname{cov}(V_i,V_l) + 2V_iV_l\operatorname{cov}(V_i,V_k) + \\ + V_kV_l\sigma_{V_i}^2 + \operatorname{cov}(V_i,V_k)\operatorname{cov}(V_i,V_l) + \sigma_{V_i}^2\operatorname{cov}(V_k,V_l)\end{bmatrix} \\ &\qquad * B_{ik}B_{il}\left[\sigma_{\theta_i}^2 - \operatorname{cov}(\theta_i,\theta_l) - \operatorname{cov}(\theta_i,\theta_k) + \operatorname{cov}(\theta_k,\theta_l)\right] \\ &+ 2\sum_{k\in\Omega i;k\neq i}\ \sum_{\substack{l\in\Omega i;l\neq i\\ l>k}}\begin{bmatrix}V_i^2\operatorname{cov}(V_k,V_l) + V_iV_k\operatorname{cov}(V_i,V_l) + V_iV_l\operatorname{cov}(V_i,V_k) + V_kV_l\sigma_{V_i}^2 + \\ + \operatorname{cov}(V_i,V_k)\operatorname{cov}(V_i,V_l) + \sigma_{V_i}^2\operatorname{cov}(V_k,V_l)\end{bmatrix} * \\ &\qquad * \left[G_{ik} + B_{ik}\theta_{ik}\right]\left[G_{il} + B_{il}\theta_{il}\right]\end{aligned}} \tag{12}$$

$$\sigma_{est,Q_i} = \sqrt{E\left(Q_i^2\right) - E^2\left(Q_i\right)} =$$

$$\sqrt{\begin{aligned}&2B_{ii}^2\sigma_{V_i}^2\left(\sigma_{V_i}^2 + 2V_i^2\right) + 4B_{ii}\sum_{k\in\Omega i;k\neq i}\left[B_{ik} - G_{ik}\theta_{ik}\right]\left[V_i^2\operatorname{cov}(V_i,V_k) + V_iV_k\sigma_{V_i}^2 + \sigma_{V_i}^2\operatorname{cov}(V_i,V_k)\right] + \\ &+ \sum_{k\in\Omega i;k\neq i} G_{ik}^2\left[\sigma_{\theta_i}^2 + \sigma_{\theta_k}^2 - 2\operatorname{cov}(\theta_i,\theta_k)\right]\begin{bmatrix}V_i^2V_k^2 + 4V_iV_k\operatorname{cov}(V_i,V_k) + \\ + 2\operatorname{cov}^2(V_i,V_k) + V_i^2\sigma_{V_k}^2 + V_k^2\sigma_{V_i}^2 + \sigma_{V_i}^2\sigma_{V_k}^2\end{bmatrix} + \\ &+ \sum_{k\in\Omega i;k\neq i}\left[B_{ik} - G_{ik}\theta_{ik}\right]^2\left[2V_iV_k\operatorname{cov}(V_i,V_k) + \operatorname{cov}^2(V_i,V_k) + V_i^2\sigma_{V_k}^2 + V_k^2\sigma_{V_i}^2 + \sigma_{V_i}^2\sigma_{V_k}^2\right] + \\ &+ 2\sum_{k\in\Omega i;k\neq i}\ \sum_{\substack{l\in\Omega i;l\neq i\\ l>k}}\begin{bmatrix}V_i^2V_kV_l + V_i^2\operatorname{cov}(V_k,V_l) + 2V_iV_k\operatorname{cov}(V_i,V_l) + 2V_iV_l\operatorname{cov}(V_i,V_k) + \\ + V_kV_l\sigma_{V_i}^2 + \operatorname{cov}(V_i,V_k)\operatorname{cov}(V_i,V_l) + \sigma_{V_i}^2\operatorname{cov}(V_k,V_l)\end{bmatrix} \\ &\qquad * B_{ik}B_{il}\left[\sigma_{\theta_i}^2 - \operatorname{cov}(\theta_i,\theta_l) - \operatorname{cov}(\theta_i,\theta_k) + \operatorname{cov}(\theta_k,\theta_l)\right] \\ &+ 2\sum_{k\in\Omega i;k\neq i}\ \sum_{\substack{l\in\Omega i;l\neq i\\ l>k}}\begin{bmatrix}V_i^2\operatorname{cov}(V_k,V_l) + V_iV_k\operatorname{cov}(V_i,V_l) + V_iV_l\operatorname{cov}(V_i,V_k) + V_kV_l\sigma_{V_i}^2 + \\ + \operatorname{cov}(V_i,V_k)\operatorname{cov}(V_i,V_l) + \sigma_{V_i}^2\operatorname{cov}(V_k,V_l)\end{bmatrix} * \\ &\qquad * \left[B_{ik} - G_{ik}\theta_{ik}\right]\left[B_{il} - G_{il}\theta_{il}\right]\end{aligned}} \tag{13}$$

2.2.2 Computation of standard deviations ($\sigma_{mesv,i}$) of the virtual measurements

a) *Standard deviations of the virtual measurements concerning the real and reactive power flow from bus i to bus k*

The value of the, real or reactive, power flow virtual measurement ($T_{mesv,ik}$) from bus i to bus k can be defined as follows:

$$T_{mesv,ik} \overset{def}{=} I_{mes,i} - \sum_{\substack{l\in\Omega i\\ l\neq k}} T_{mes,il} - \sum_{\substack{l\in\Omega i\\ l\neq k}} T'_{est,il} \tag{14}$$

where:

$I_{mes,i}$: power injection measurement at bus i

$T_{mes,il}$: power flow measurement from bus i to bus l

$T'_{est,il}$: power flow estimate from bus i to bus l on the branch $i-l$ whose measured quantity are not available

The, real or reactive, power flow estimated value ($T_{est,ik}$) from bus i to bus k is given by the relation:

$$T_{est,ik} = I_{est,i} - \sum_{\substack{l\in\Omega i\\ l\neq k}} T_{est,il} - \sum_{\substack{l\in\Omega i\\ l\neq k}} T'_{est,il} \tag{15}$$

where:

$I_{est,i}$: power injection estimate at bus i

$T_{est,il}$: power flow estimate from bus i to bus l on the branch $i-l$ whose measured quantity are available

Subtracting the relation (15) from the relation (14) gives:

$$T_{mesv,ik} - T_{est,ik} = (I_{mes,i} - I_{est,i}) - \sum_{\substack{l\in\Omega i\\ l\neq k}} (T_{mes,il} - T_{est,il}) \tag{16}$$

or:

$$RT_{mesv,ik} = RI_{mes,i} - \sum_{\substack{l\in\Omega i\\ l\neq k}} RT_{mes,il} \tag{17}$$

where:

$RT_{mesv,ik}$: power flow virtual measurement residual from bus i to bus k

$RI_{mes,i}$: power injection measurement residual at bus i

$RT_{mes,il}$: power flow measurement residual from bus i to bus l

Using the relation (17), the standard deviation of the residual $RT_{mesv,ik}$ can be evaluated as a function of standard deviations and co-variances of the residuals $RI_{mes,i}$ and $RT_{mes,il}$ ($l \in \Omega i\,;\, l \neq k$):

$$\sigma\left(RT_{mesv,ik}\right) = \sqrt{\sigma^2\left(RI_{mes,i} - \sum_{\substack{l\in\Omega i\\ l\neq k}} RT_{mes,il}\right)} =$$

$$\sqrt{\sigma^2\left(RI_{mes,i}\right) + \sum_{\substack{l\in\Omega i\\ l\neq k}}\sigma^2\left(RT_{mes,il}\right) - 2\sum_{\substack{l\in\Omega i\\ l\neq k}}\operatorname{cov}\left(RI_{mes,i}, RT_{mes,il}\right) + 2\sum_{\substack{l\in\Omega i\\ l\neq k}}\ \sum_{\substack{j\in\Omega i\\ j\neq k;\, j>l}}\operatorname{cov}\left(RT_{mes,il}, RT_{mes,ij}\right)} \tag{18}$$

The values of variances and co-variances in the relation (18) are directly available from the co-variance residual matrix evaluated at the end of the estimation process.

The standard deviation of the power flow virtual measurement $T_{mesv,ik}$ from bus i to bus k is computed with the relation:

$$\sigma\left(T_{mesv,ik}\right) = \sqrt{\sigma^2\left(T_{est,ik}\right) + \sigma^2\left(RT_{mesv,ik}\right)} \tag{19}$$

where $\sigma(T_{est,ik})$ is computed using the relation (10) or (11).

☑ *Comments*:

- If the power injection measurement at bus $\boldsymbol{i}$ is not available, the quantity $\boldsymbol{I}_{mes,i}$ is replaced by the quantity $\boldsymbol{I}_{est,i}$. The relation (18) becomes:

$$\sigma(\boldsymbol{RT}_{mesv,ik}) = \sqrt{\sigma^2\left(\sum_{\substack{l\in\Omega i\\ l\neq k}} \boldsymbol{RT}_{mes,il}\right)} = \sqrt{\sum_{\substack{l\in\Omega i\\ l\neq k}} \sigma^2(\boldsymbol{RT}_{mes,il}) + 2\sum_{\substack{l\in\Omega i\\ l\neq k}}\sum_{\substack{j\in\Omega i\\ j\neq k;\, j>l}} \operatorname{cov}(\boldsymbol{RT}_{mes,il}, \boldsymbol{RT}_{mes,ij})} \tag{18'}$$

- If the power flow measurement on the branch $\boldsymbol{i}-\boldsymbol{l}$ ($\boldsymbol{l}\in\Omega\boldsymbol{i}$; $\boldsymbol{l}\neq\boldsymbol{k}$) is available only from bus $\boldsymbol{l}$ to bus $\boldsymbol{i}$, the quantities $\boldsymbol{T}_{mes,il}$ and $\boldsymbol{T}_{est,il}$ are replaced by the quantities: $\boldsymbol{T}_{mes,li}$, respectively $\boldsymbol{T}_{est,li}$.

b) *Standard deviations of the virtual measurements concerning the real and reactive power injection at bus* $\boldsymbol{i}$

The value of the, real or reactive, power injection virtual measurement ($\boldsymbol{I}_{mesv,i}$) at bus $\boldsymbol{i}$ can be defined as follows:

$$\boldsymbol{I}_{mesv,i} \overset{def}{=} \sum_{l\in\Omega i} \boldsymbol{T}_{mes,il} + \sum_{l\in\Omega i} \boldsymbol{T}'_{est,il} \tag{20}$$

The, real or reactive, power injection estimated value ($\boldsymbol{I}_{est,i}$) at bus $\boldsymbol{i}$ is given by the relation:

$$\boldsymbol{I}_{est,i} = \sum_{l\in\Omega i} \boldsymbol{T}_{est,il} + \sum_{l\in\Omega i} \boldsymbol{T}'_{est,il} \tag{21}$$

Subtracting the relation (21) from the relation (20) gives:

$$\boldsymbol{I}_{mesv,i} - \boldsymbol{I}_{est,i} = \sum_{l\in\Omega i} (\boldsymbol{T}_{mes,il} - \boldsymbol{T}_{est,il}) \tag{22}$$

or:

$$\boldsymbol{RI}_{mesv,i} = \sum_{l\in\Omega i} \boldsymbol{RT}_{mes,il} \tag{23}$$

where:

$\boldsymbol{RI}_{mesv,i}$: power injection virtual measurement residual at bus $\boldsymbol{i}$

Using the relation (23), the standard deviation of the residual $\boldsymbol{RI}_{mesv,i}$ can be evaluated as a function of standard deviations and co-variances of the residuals $\boldsymbol{RT}_{mes,il}$ ($\boldsymbol{l}\in\Omega\boldsymbol{i}$):

$$\sigma(\boldsymbol{RI}_{mesv,i}) = \sqrt{\sigma^2\left(\sum_{l\in\Omega i} \boldsymbol{RT}_{mes,il}\right)} = \sqrt{\sum_{l\in\Omega i} \sigma^2(\boldsymbol{RT}_{mes,il}) + 2\sum_{l\in\Omega i}\sum_{\substack{j\in\Omega i\\ j>l}} \operatorname{cov}(\boldsymbol{RT}_{mes,il}, \boldsymbol{RT}_{mes,ij})} \tag{24}$$

The values of variances and co-variances in the relation (24) are directly available from the co-variance residual matrix evaluated at the end of the estimation process.

The standard deviation of the power injection virtual measurement $\boldsymbol{I}_{mesv,i}$ at bus $\boldsymbol{i}$ is computed with the relation:

$$\sigma(\boldsymbol{I}_{mesv,i}) = \sqrt{\sigma^2(\boldsymbol{I}_{est,i}) + \sigma^2(\boldsymbol{RI}_{mesv,i})} \tag{25}$$

where $\sigma(\boldsymbol{I}_{est,i})$ is computed using the relation (12) or (13).

☑ *Comment*: If the power flow measurement on the branch $\boldsymbol{i}-\boldsymbol{l}$ ($\boldsymbol{l}\in\Omega\boldsymbol{i}$) is available only from bus $\boldsymbol{l}$ to bus $\boldsymbol{i}$, the quantities $\boldsymbol{T}_{mes,il}$ and $\boldsymbol{T}_{est,il}$ are replaced by the quantities: $\boldsymbol{T}_{mes,li}$, respectively $\boldsymbol{T}_{est,li}$.

3 IDENTIFICATION OF THE TOPOLOGY ERRORS

3.1 Principle

The identification of the topology errors is based on the concept of Normalized Lagrange Multipliers. This concept is related to the "zero injection" buses, in so far as those buses are treated as equality constraints in the weighted least square (WLS) formulation:

$$\boldsymbol{L} = 1/2(\boldsymbol{z}-\boldsymbol{h}(\boldsymbol{x}))^{\boldsymbol{T}}\boldsymbol{W}(\boldsymbol{z}-\boldsymbol{h}(\boldsymbol{x})) - \lambda^{\boldsymbol{T}}\boldsymbol{c}(\boldsymbol{x}) \tag{26}$$

where:

$\boldsymbol{L}$: Lagrangian

$\boldsymbol{z}$: measurement vector of length $\boldsymbol{m}$

$\boldsymbol{x}$: system state vector of length $\boldsymbol{n}$

$\boldsymbol{h}(\boldsymbol{x})$: vector function of m measurements

$\boldsymbol{W}$: weight matrix of size $\boldsymbol{m}$ x $\boldsymbol{m}$

λ : vector of the Lagrange multipliers

$\boldsymbol{c}(\boldsymbol{x})$: vector function of the equality constraints

The bad data (measurement) detection is based on the concept of Normalized Residuals:

$$\boldsymbol{r}^{\boldsymbol{N}} = (\boldsymbol{diag}\,\boldsymbol{R}_{\boldsymbol{r}})^{-1/2}|\boldsymbol{r}| \tag{27}$$

where:

$\boldsymbol{r}$: vector of the residuals (z-h(x))

$\boldsymbol{R}_{\boldsymbol{r}}$: co-variance matrix of the residuals

The state estimation theory shows that bad data are emphasized by large Normalized Residuals, at least if there is only one bad data and if the measurement redundancy is high enough.

Similarly, one can define the Normalized Lagrange Multipliers:

$$\lambda^{\boldsymbol{N}} = (\boldsymbol{diag}\,\boldsymbol{R}_{\lambda})^{-1/2}|\lambda| \tag{28}$$

where:

λ^N : vector of normalized Lagrange multipliers

R_λ : co-variance matrix of the Lagrange multipliers

A. Gjelsvik in [21] discussed the behavior of the Lagrange multipliers: "the Lagrange multiplier for a constraint shows the sensitivity of the objective function to a change in the constraint. If topology errors or bad data are present, it becomes more costly to fulfill the constraints, and that will give bias in the multipliers". The author also emphasized that when a topology error is present, both $\boldsymbol{r}^N$ and λ^N may have large components. Nevertheless, the computational experience of the author indicates that with a single-line topological error adjacent to a zero injection bus $\boldsymbol{i}$ and no bad data, λ_i^N will frequently be larger than any $\boldsymbol{r}_j^N$ for $\boldsymbol{j} \neq \boldsymbol{i}$.

The problem is the possible interaction between bad data and topology errors: both bad data and topology errors can cause large $\boldsymbol{r}^N$ and large λ^N. The practical implementation that is detailed in the next paragraph relies upon the assumption that both $\boldsymbol{r}^N$ and λ^N behave similarly with respect to the possible sources of error, in other words that a bad data causes mainly one large $\boldsymbol{r}^N$ and that a topology error causes mainly one large λ^N.

3.2 Practical implementation

The first step in the implementation is to gather all the buses where either the $\boldsymbol{r}^N$ or the λ^N are supposed to be abnormal. This has to be done at the very first convergence of the state estimation process, i.e. with the complete measurement set before starting any bad data detection. Since both the $\boldsymbol{r}^N$ and the λ^N are supposed to behave similarly, the same detection threshold can be considered (3, for instance). A bus is flagged during that step:

- if it is a zero injection bus with a λ^N greater than 3,
- if it is a bus with a measured injection and a $\boldsymbol{r}^N$ greater than 3,
- if it is a bus with a measured voltage and a $\boldsymbol{r}^N$ greater than 3,
- if a flow measurement on one of its adjacent branches has a $\boldsymbol{r}^N$ greater than 3.

Both active and reactive measurements are taken into account.

The next step consists in analyzing how the flagged buses are spread in the network: a breadth first procedure is used to group all the adjacent flagged buses. The result of this procedure is a set of error groups containing adjacent flagged buses. In each error group, the $\boldsymbol{r}^N$ and the λ^N are sorted.

In the last step each error group is analyzed as follows:

- if the largest $\boldsymbol{r}^N$ or λ^N in the group is a normalized residuals, then the group is assumed to be caused by a bad data measurement
- if the largest $\boldsymbol{r}^N$ or λ^N in the group is a normalized Lagrange multiplier, then the group is assumed to be caused by a topology error.

This analysis relies upon the following assumptions:

- Each error group is caused only by one error, whatever this error is a bad data or a topology error. This error is the fundamental error of the group and it is pointed out by the largest $\boldsymbol{r}^N$ or λ^N. All the other $\boldsymbol{r}^N$ and λ^N are just consequences of this fundamental error.
- Each error group is independent with respect to the other groups. In other words, error groups are independent and do not affect each other.

3.3 Field experience

The described methodology for identifying the topological errors has been put into service in the on-line state estimator of the Belgian control center. Analysis of the error groups is performed by the people who are in charge of tracking the possible sources of data errors.

This field experience showed that the methodology supplies a significant help in the understanding the causes of errors. In fact, one of the greatest benefits of the method is that the number of places with potential errors is drastically reduced from the total number of abnormal $\boldsymbol{r}^N$ and λ^N to the number of fundamental errors, which is obviously smaller. People are generally starting their investigation from the error group with the largest fundamental error. When this error is identified and corrected, they perform a new state estimation. Most of the time the error group disappears, which demonstrates that the assumptions are practically acceptable. If the error group does not disappear, this means that it contains more than one real error.

Nevertheless, the following limitations have been identified:

- Inside one error group, the largest $\boldsymbol{r}^N$ and the largest λ^N have sometimes nearly the same value. In this case, it is practically impossible to settle between bad data and topology error. However, most of the time, the largest $\boldsymbol{r}^N$ and the largest λ^N are related to the same bus bar or substation, so that the investigation can be limited.
- As for the bad data detection, bad topology detection is restricted to the areas with a sufficient redundancy. Furthermore, bad topology detection is effective only if the error occurs in the neighborhood of zero injection buses. If the error occurs in the middle of buses with injections, it will cause either normalized residuals if the injections are measured or abnormal injection estimations if they are not. In that case, the diagnostic of the methodology will be wrong.

- The result of the methodology is just an analysis of the potential errors; the topological errors are not automatically corrected (as the bad data detection process which is able to reject the bad measurements). The reason is that the normalized Lagrange multipliers are related to the buses and not to the breakers! It is thus basically impossible to evaluate automatically which breaker that is connected to the faulty bus is in a wrong position. This problem would require some further analysis, which is out of the scope of this paper.

The additional burden for computing the normalized Lagrange multipliers is not significant since:

- The values of the multipliers are part of the solution vector,
- The covariance matrix of the Lagrange multipliers is part of the inverted gain matrix.

Building and analyzing the error groups is also fast since it uses simple procedures like breadth first and sorting algorithms.

4 NUMERICAL SIMULATIONS

The following part of the paper illustrates how the quality indexes computed according to the procedure described in Section 2 behave when a telemetry failure occurs. In fact, we are not displaying the $\boldsymbol{R}_i$ or $\boldsymbol{R}_i^{'}$ indexes (rel. (1) and (1bis)) as described above, but their inverse values. The reason is quite simple: we want to emphasize the potential problems, i.e. areas where the state estimation quality is poor, and not the situations where everything is perfect!

The first situation illustrated in Figure 1, is supposed to be normal. All the circles that represent the inverted ratios are rather small. Plain circles are related to injections and empty circles are related to flows. This situation is basically acceptable, which means that the estimated state is accurate and reliable.

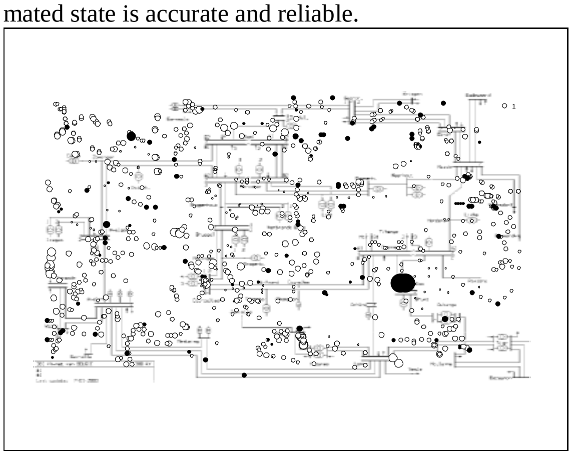

Figure 1: Acceptable state estimator results

The second situation (Figure 2) is basically the same, except that a telemetry failure is simulated in the northern part of the network: in consequence the circles that represent the inverted ratios are now larger in that part of the network.

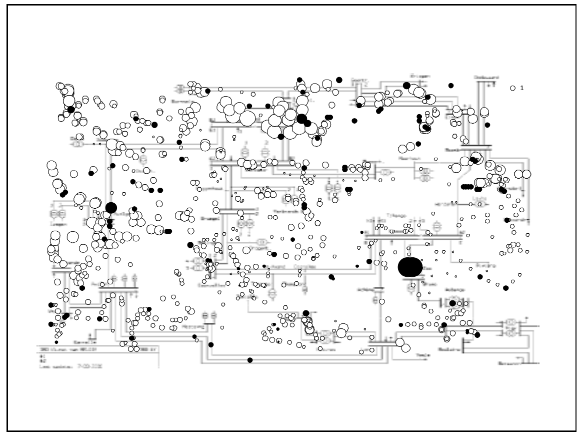

Figure 2: Poor state estimation quality

This simple example illustrates dramatically how the ratios react when a telemetry failure occurs. The graphical display immediately gives a very intelligible form to the highly theoretical ratios.

5 CONCLUSIONS

An original method is presented to compute the standard deviations:

- of the estimated quantities (e.g. branch power transits and nodal power injections) whose corresponding measured quantities are not available.
- of the virtual measurements associated to these estimated quantities.

These standard deviations are used in the expression of the quality indexes so as to assess the accuracy and the quality of the state estimation results.

An original post-estimation procedure for the detection of topology errors is presented. This procedure has to be implemented in the state estimation algorithm before the bad data processing. This approach preserves the necessary information for a substantiated judgment whether a measurement or a topology error is the likely cause of an error group.

The numerical simulations presented are based on real-time measurement data collected in the Belgian energy control centre.

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