THE CHOICE OF SMES CONTROL FOR POWER SYSTEM TRANSIENT STABILITY IMPROVEMENT.

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Abstract – The paper presents an approach to the choice of control for electric energy storage devices in power system aiming at maximum transient stability improvement. This approach is based on Pontryagin's maximum principle. Though the solution thus obtained for nonlinear dynamic system is in the form of control trajectories and in general it can not be directly implemented, it is very useful in estimating technical possibilities of different controlled elements, and corresponding solution may serve as a standard to be obtained.

Pontryagin's maximum principle has been expanded to nonlinear dynamic system described by both differential and algebraic equations. It has been shown that for a dynamic system described by *n* differential equations, *m* algebraic ones and containing *l* controls the optimality conditions are determined by the solution of 2*n* differential equations (for the state and costate variables) and *m*(*n*+*l*+1)+*l* algebraic equations. The introduction of algebraic subsystem considerably increases the dimension of the problem, though it is not critical for modern computers.

The paper outlines general solution algorithm using quasilinearization approach which allows for eliminating algebraic equations at each integration step and provides an easy account of transversality conditions.

The above approach has been used for the choice of SMES control in a simple EPS. It has been shown that an automatic regulator, reacting to the integral of active power deviation in the transmission line adjacent to SMES gives results close to optimal ones.

Keywords: power system, transient stability, optimal control, SMES

1 INTRODUCTION

A permanent interest in electrical energy storage devices (EES) is observed during recent years in electrical power systems (EPS) of several countries. These devices are meant for daily load curves leveling, EPS stability improvement, etc. EES devices are of high efficiency as they require no energy conversion. Thyristor converters are used for the connection of accumulating part of EES to the system network, resulting in fast response. EES can control EPS power flows thus affecting transient stability. It has been shown [1, 2] that EES of SMES type may noticeable increase EPS transient stability limits. This effect depends on many factors: configurations and operating conditions of EPS, the kind of fault and post-fault conditions, SMES allocation and its control [1-4]. However, the choice of SMES control, based on the measurement and processing of local control signals has not been given due attention.

SMES control by power station synchronous generator rotor speed deviation is proposed in [1,2] for the purpose of transient stability improvement. However, implementation of a such control requires telecommunication channel to be used. In [5] it is proposed to control SMES similarly to PSS excitation control of synchronous generators. However, this mode of control is chosen for linearized EPS model, and its effectiveness under large disturbances is not clear. The main purpose of SMES control is maximum of its effect in improving transient stability and electromechanical transients.

Maximum technical possibilities of SMES or some other devices in controlling EPS transients can be determined with the help of optimal control theory [6]. On the basis of this theory a generalized Pontyagin's maximum principle has been formulated which is applicable to the system described by both differential and algebraic equations thus allowing to find optimal control of any power system element.

The report aims at: explaining algorithmic implementation of generalized Pontryagin's maximum principle, determining control trajectories for EES, choosing automatic regulator which uses local variables and gives results close to optimal ones. SMES control was chosen as a specific example.

2 MATHEMATICAL MODEL OF EPS.

The principle diagram of EPS is shown in Fig.1. It consists of hydro power station (HPS), represented by equivalent generator (G), operating through transmission line on infinite bus system (S). At the intermediate substation of the line a SMES and SVC (for reactive power compensation of SMES) are connected in parallel to medium voltage load. This diagram was reduced to three nodes circuit, namely generator (G), system (S) and SMES node (L) in order to reduce the dimension of the problem.

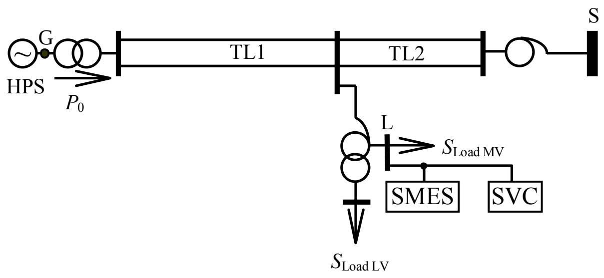

Figure 1.Test EPS.

The transients in EPS are described by differential equations and nonlinear algebraic ones. In the calculations generator and transmission system are represented in a simplified form [7]. It is also assumed that in calculating electromechanical transients the generator can be represented by constant EMF behind transient reactance $(E' = \mathrm{const})$. The generator's rotor motion is described by the following equations:

$$\left.\begin{aligned}\dot\delta &= s\\ \dot s &= \frac{\omega_{nom}}{M}\begin{bmatrix} P_t - E'^2 y_{\mathrm{GG}}\sin\alpha_{\mathrm{GG}} - \\ -E'Uy_{\mathrm{GL}}\sin(\delta-\vartheta-\alpha_{\mathrm{GL}}) - \\ -E'U_{\mathrm{S}} y_{\mathrm{GS}}\sin(\delta-\alpha_{\mathrm{GS}})\end{bmatrix}\end{aligned}\right\}, \qquad (1)$$

where: δ- rotor displacement angle with respect to S, M- inertia constant, sec, P_t- turbine power, p.u., $y_{i,j}$, $\alpha_{i,j}$- modules and complementary angles of nodal admittances, U, ϑ- module and phase angle of the SMES terminal voltage.

SMES is simulated by differential equation of stored energy variation and by equations of active and reactive power balances in EPS node [8]:

$$\left.\begin{aligned}
&P_{\mathrm{SMES}} = \frac{dW_{\mathrm{SMES}}}{dt} = \frac{12\sqrt{W_{\mathrm{SMES}}}}{\pi\sqrt{L_{\mathrm{SMES}}}}U\cos\alpha - \frac{12x_c\sqrt{W_{\mathrm{SMES}}}}{\pi L_{\mathrm{SMES}}}\\
&P_{\mathrm{SMES}} + U^2 y_{\mathrm{LL}}\sin\alpha_{\mathrm{LL}} + E'Uy_{\mathrm{GL}}\sin(\vartheta-\delta-\alpha_{\mathrm{GL}}) +\\
&+ UU_{\mathrm{S}}y_{\mathrm{LS}}\sin(\vartheta-\alpha_{\mathrm{LS}}) = 0\\
&-\frac{3U^2}{\pi x_c}[\gamma - \sin\gamma\cos(2\alpha+\gamma)] + Q_{\mathrm{SVC}} - U^2 y_{\mathrm{LL}}\cos\alpha_{\mathrm{LL}} +\\
&+ E'Uy_{\mathrm{GL}}\cos(\vartheta-\delta-\alpha_{\mathrm{GL}}) + UU_{\mathrm{S}}y_{\mathrm{LS}}\cos(\vartheta-\alpha_{\mathrm{LS}}) = 0\\
&\gamma - \cos^{-1}\left(\cos\alpha - \frac{2x_c\sqrt{W_{\mathrm{SMES}}}}{U\sqrt{L_{\mathrm{SMES}}}}\right) + \alpha = 0
\end{aligned}\right\}, \qquad (2)$$

where: α- firing angle of SMES converter, x_c- commutating reactance, W_{SMES}, L_{SMES}- stored energy and inductance of SMES, Q_{SVC}- reactive power of SVC. The rest of notation can be found elsewhere.

The sets of nonlinear differential and algebraic equations (1) and (2) describe transient processes in EPS under consideration. Similar description can be used for EPS of more complicated structure. Steady state EPS operating conditions with the account of SMES are calculated according to [4].

3 GENERALIZED PONTRYAGIN'S MAXIMUM PRINCIPLE.

Mathematical model of EPS transients can be represented as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{y},\mathbf{u},t), \qquad (3)$$

$$0 = \varphi(\mathbf{x},\mathbf{y},\mathbf{u},t), \qquad (4)$$

where: $\mathbf{x} = (x_1,\ldots,x_n)_{\mathrm{T}}$- state variables vector with $\mathbf{x}(0) = \mathbf{x}_0$, $\mathbf{f}$- vector-function, $\mathbf{u} = (u_1,\ldots,u_l)_{\mathrm{T}}$, $0 \le l \le n$- control vector; $\mathbf{y} = (y_1,\ldots,y_m)_{\mathrm{T}}$- vector of complementary variables, - symbol of transpose.

It is required to find such controls, belonging to a specified limited area Ω $(u \in \Omega)$, which minimized cost function

$$J = \int_0^T [\mathbf{x}_{\mathrm{T}}\mathbf{Q}\mathbf{x} + \mathbf{u}_{\mathrm{T}}\mathbf{R}\mathbf{u}]dt, \qquad (5)$$

where $\mathbf{Q}$ and $\mathbf{R}$- are diagonal matrices of weighting factors, [0, T]- time interval of optimal control problem solution.

The main problem in Pontryagin's maximum principle application to the system, represented by (3) and(4) is subset of algebraic equations (4). This problem has been solved by implicit elimination of $\mathbf{y}$ variables and by corresponding transformation of optimality conditions, resulting in:

$$\left.\begin{aligned}
&\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{y},\mathbf{u},t)\\
&\dot{\mathbf{p}} = -\left(\frac{\partial\mathbf{f}}{\partial\mathbf{x}} + \frac{\partial\mathbf{f}}{\partial\mathbf{y}}\mathbf{G}\right)_{\mathrm{T}}\cdot\mathbf{p} + \frac{\partial F}{\partial\mathbf{x}}\\
&0 = \varphi(\mathbf{x},\mathbf{y},\mathbf{u})\\
&0 = \left(\frac{\partial\mathbf{f}}{\partial\mathbf{u}} + \frac{\partial\mathbf{f}}{\partial\mathbf{y}}\mathbf{I}\right)_{\mathrm{T}}\cdot\mathbf{p} - \frac{\partial F}{\partial\mathbf{u}}\\
&0 = \frac{\partial\varphi}{\partial\mathbf{x}} + \frac{\partial\varphi}{\partial\mathbf{y}}\mathbf{G}\\
&0 = \frac{\partial\varphi}{\partial\mathbf{u}} + \frac{\partial\varphi}{\partial\mathbf{y}}\mathbf{I}
\end{aligned}\right\}, \qquad (6)$$

where $\mathbf{G} = \frac{\partial\mathbf{y}}{\partial\mathbf{x}}$, $\mathbf{I} = \frac{\partial\mathbf{y}}{\partial\mathbf{u}}$, $\mathbf{p}$- is vector of costate variables. The dimension of the problem is $n(m+2) + l(m+1) + m$.

4 SOLUTION OF GENERALIZED PONTRYAGIN'S MAXIMUM PRINCIPLE PROBLEM BY QUASILINEARIZATION METHOD.

This method has been developed for the solution of optimal control problem for dynamic system described by the set of differential equations. In our case it should be expanded to the problem described by the set of equations (6).

This set can be represented in the form of two subsets:

$$\dot{\mathbf{z}} = \mathbf{F}\begin{pmatrix}\mathbf{z}\\ \mathbf{z}'\end{pmatrix}, \qquad (7)$$

$$0 = \mathbf{F}'\begin{pmatrix}\mathbf{z}\\ \mathbf{z}'\end{pmatrix}, \tag{8}$$

where $\mathbf{F}\begin{pmatrix}\mathbf{z}\\ \mathbf{z}'\end{pmatrix}$ and $\mathbf{F}'\begin{pmatrix}\mathbf{z}\\ \mathbf{z}'\end{pmatrix}$ - are vector-functions of differential and algebraic subsets, correspondingly, $\mathbf{z} = \begin{pmatrix}\mathbf{x}\\ \mathbf{p}\end{pmatrix}$, $\mathbf{z}' = \left(\mathbf{y}_{\mathrm{T}}, \mathbf{u}_{\mathrm{T}}, \{\mathbf{G}\}_{j,i}, \{\mathbf{I}\}_{j,i}\right)_{\mathrm{T}}$.

According to quasilinearization method the above equations are solved by iterative process, and in each iteration they are numerically integrated being linearized at each integrated step of the previous iteration, namely:

$$\dot{\mathbf{z}}^{(k)} = \mathbf{F}\begin{pmatrix}\mathbf{z}^{(k-1)}\\ \mathbf{z}'^{(k-1)}\end{pmatrix} + \mathbf{J}\begin{pmatrix}\mathbf{z}^{(k-1)}\\ \mathbf{z}'^{(k-1)}\end{pmatrix}\cdot\begin{pmatrix}\Delta\mathbf{z}^{(k)}\\ \Delta\mathbf{z}'^{(k)}\end{pmatrix}, \tag{9}$$

$$0 = \mathbf{F}'\begin{pmatrix}\mathbf{z}^{(k-1)}\\ \mathbf{z}'^{(k-1)}\end{pmatrix} + \mathbf{J}'\begin{pmatrix}\mathbf{z}^{(k-1)}\\ \mathbf{z}'^{(k-1)}\end{pmatrix}\cdot\begin{pmatrix}\Delta\mathbf{z}^{(k)}\\ \Delta\mathbf{z}'^{(k)}\end{pmatrix}, \tag{10}$$

where $\mathbf{J}(\mathbf{z}) = \partial\mathbf{F}\begin{pmatrix}\mathbf{z}\\ \mathbf{z}'\end{pmatrix}\Big/\partial\begin{pmatrix}\mathbf{z}\\ \mathbf{z}'\end{pmatrix}$, $\mathbf{J}'(\mathbf{z}) = \partial\mathbf{F}'\begin{pmatrix}\mathbf{z}\\ \mathbf{z}'\end{pmatrix}\Big/\partial\begin{pmatrix}\mathbf{z}\\ \mathbf{z}'\end{pmatrix}$ - are Jacoby matrices, $\mathbf{z}$, $\mathbf{z}'$ - time trajectories of variables, $\Delta\mathbf{z}^{(k)} = \mathbf{z}^{(k)} - \mathbf{z}^{(k-1)}$, $\Delta\mathbf{z}'^{(k)} = \mathbf{z}'^{(k)} - \mathbf{z}'^{(k-1)}$, k- iteration number.

Linearization of the equations makes it possible:

1) to eliminate algebraic equations at each integration step thus reducing (6) to (9) with $\mathbf{z}$ variables only,
2) to ensure at each iteration transversality condition $\mathbf{p}(T) = 0$ fulfillment by supplementing (9) by a homogenous set

$$\dot{\mathbf{z}}_{h(i)}^{(k)} = \mathbf{J}\left(\mathbf{z}^{(k-1)}\right)\cdot\mathbf{z}_{h(i)}^{(k)}, i = 1,\ldots,n, \tag{11}$$

where $x_{h(i)}(0) = 0,\ i = 1,\ldots,n$, and $p_{h(i)}(0),\ i = 1,\ldots,n$ - form nonsingular matrix, for instance unit matrix.

By linear combination of (9) and (11) we get $\mathbf{p}(T) = 0$ and, correspondingly $\mathbf{p}(0)$ which satisfy this condition thus reducing two point boundary problem to the problem of Cauchy at each iteration.

Equations (9) and (10) can be written as:

$$\dot{\mathbf{z}}^{(k)} = \mathbf{F}\begin{pmatrix}\mathbf{z}^{(k-1)}\\ \mathbf{z}'^{(k-1)}\end{pmatrix} + \left.\frac{\partial\mathbf{F}}{\partial\mathbf{z}}\right|_{\begin{pmatrix}\mathbf{z}^{(k-1)}\\ \mathbf{z}'^{(k-1)}\end{pmatrix}}\Delta\mathbf{z}^{(k)} + \left.\frac{\partial\mathbf{F}}{\partial\mathbf{z}'}\right|_{\begin{pmatrix}\mathbf{z}^{(k-1)}\\ \mathbf{z}'^{(k-1)}\end{pmatrix}}\Delta\mathbf{z}'^{(k)}, \tag{12}$$

$$0 = \mathbf{F}'\begin{pmatrix}\mathbf{z}^{(k-1)}\\ \mathbf{z}'^{(k-1)}\end{pmatrix} + \left.\frac{\partial\mathbf{F}'}{\partial\mathbf{z}}\right|_{\begin{pmatrix}\mathbf{z}^{(k-1)}\\ \mathbf{z}'^{(k-1)}\end{pmatrix}}\Delta\mathbf{z}^{(k)} + \left.\frac{\partial\mathbf{F}'}{\partial\mathbf{z}'}\right|_{\begin{pmatrix}\mathbf{z}^{(k-1)}\\ \mathbf{z}'^{(k-1)}\end{pmatrix}}\Delta\mathbf{z}'^{(k)}. \tag{13}$$

Substituting $\Delta\mathbf{z}'$ from (13) into (12) we obtain differential equation for state and costate variables:

$$\dot{\mathbf{z}}^{(k)} = \left[\mathbf{F} - \frac{\partial\mathbf{F}}{\partial\mathbf{z}'}\left(\frac{\partial\mathbf{F}'}{\partial\mathbf{z}'}\right)^{-1}\mathbf{F}'\right]\Bigg|_{\begin{pmatrix}\mathbf{z}^{(k-1)}\\ \mathbf{z}'^{(k-1)}\end{pmatrix}} + \left[\frac{\partial\mathbf{F}}{\partial\mathbf{z}} - \frac{\partial\mathbf{F}}{\partial\mathbf{z}'}\left(\frac{\partial\mathbf{F}'}{\partial\mathbf{z}'}\right)^{-1}\frac{\partial\mathbf{F}'}{\partial\mathbf{z}}\right]\Bigg|_{\begin{pmatrix}\mathbf{z}^{(k-1)}\\ \mathbf{z}'^{(k-1)}\end{pmatrix}}\Delta\mathbf{z}^{(k)}. \tag{14}$$

In this way algebraic system (10) is excluded and the problem is reduced to that of integrating differential equations (14). Numerically (14) is obtained from (12) and (13) with the help of $m(n+1) + l(m+1)$ steps of Gauss elimination.

The solution of two point boundary problem requires an integration both of (14) with initial conditions $\mathbf{x}(0) = \mathbf{x}_0$, $\mathbf{p}(0) = 0$, and n sets of (11) with initial conditions $\mathbf{x}_h(0) = 0$, $\mathbf{p}_{h(i)} = \mathbf{e}_i$, $i = 1,\ldots,n$, where $\mathbf{e}_i$ form unit matrix of order n.

Solution of (14) $\mathbf{z}_p^{(k)}(t)$ and those of (11) are then combined, the latter being multiplied by $a_j^{(k)}$, chosen in such a way that $\mathbf{p}(T) = 0$, i.e.

$$z_i^{(k)}(T) = z_{pi}^{(k)}(T) + \sum_{j=n+1}^{2n} z_{hij}^{(k)}(T)a_j^{(k)} = 0,\ i = n+1,\ldots,2n. \tag{15}$$

Solution of (15) with respect to $a_j^{(k)}$ provides initial values for $\mathbf{p}$: $p_j^{(k)}(0) = a_j^{(k)},\ j = 1,\ldots,n$. After that (14) is integrated for specified initial conditions $\mathbf{x}(0)$ and $\mathbf{p}(0)$. In the process of integration values of **y**, **u**, **G**, **I** are calculated for each step by solving algebraic subset of (6) using Newton's method.

The optimal control problem is considered to be solved if

$$\left|\mathbf{z}^{(k)} - \mathbf{z}^{(k-1)}\right| \le \varepsilon, \tag{16}$$

where ε- is specified mismatch.

This algorithm has no limitations on system complexity as far as it is described by (3) and (4).

4 CALCULATION OF SMES OPTIMAL CONTROL.

Equivalent 3-node EPS circuit is shown in Fig. 2. In this system SMES is the controlled element meant for improving transient stability. The transients in the system are caused by three phase short circuit on transmission line (TL) 2 with the duration of t_{sc}=0.12 s. After the short circuit clearing SMES is controlled in such a way

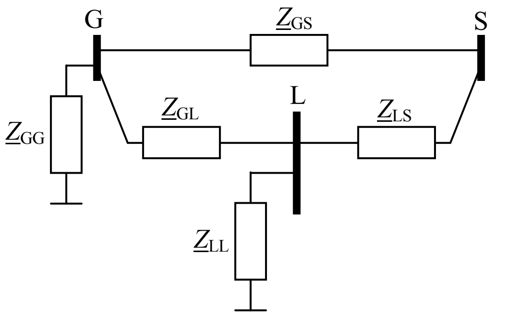

Figure 2. Equivalent 3-node EPS

so as to reach post-fault condition with minimum deviation of the variables from steady-state values according to cost function (5).

Parameters of EPS element together with the optimality conditions are presented in Appendix.

One of the main difficulties in the proposed approach is the choice of weighting factors in (5). These factors influence convergence of iterative process to the solution and transients damping. By trial and error approach the following values of weighting factors gave good results: k_δ=0.015, k_ω=0.015 and k_α=0.01. Trajectories of state variables and control for optimal problem solution are shown in Fig. 3, where $\alpha = \alpha_F$ corresponds to $P_{\text{SMES}} = 0$.

From this trajectories it can be seen that at the first stage of transient process after fault clearing the control is close to bang- bang one, and that the instances of firing angle changes practically coincide with the points of s=0. From the curves in Fig. 3 it can be seen that optimal control of SMES corresponds to the change of α proportionally to the rotor speed deviation of power plant equivalent generator:

$$\alpha = \alpha_F - k_s s\,, \tag{17}$$

where k_s- is a control gain.

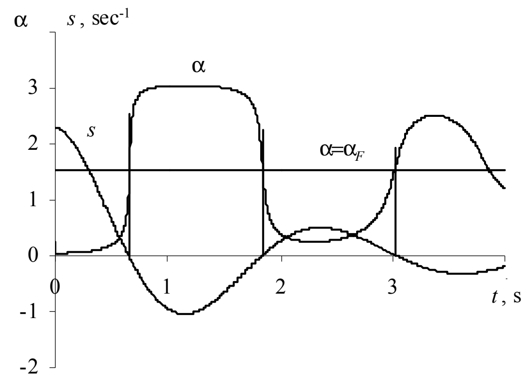

Figure 3. SMES optimal control.

5 SMES CONTROL IMPLEMENTATION.

Control (17) requires telecommunication channel, which makes this control unpractical. The better choice is the control by local variables at the point of SMES connection which gives results close to optimal ones.

As it is known [7] the acceleration of equivalent generator is proportional to the power deviation in TL1 from its post-fault value (P_{TL}-$P_{\text{TL F}}$). Speed deviation is equal to the integral of this power difference. From this it follows that

$$I_{P_{\text{TL}}}(t) = -\int_0^t \left(P_{\text{TL}} - P_{\text{TL F}}\right) dt\,, \tag{18}$$

will be close in shape to $s(t)$, the difference being due to power losses in TL. Then SMES control can be performed as

$$\alpha = \alpha_F - k_I I_{P_{\text{TL}}}\,, \tag{19}$$

where k_I- is the control gain.

Control signal $I_{P_{\text{TL}}}$ can be measured at the point of SMES connection only for $t \ge t_{\text{sc}}$. Thus if we present:

$$I_{P_{\mathrm{TL}}} = -\int_0^{t_{\mathrm{sc}}} \left(P_{\mathrm{TL}} - P_{\mathrm{TL}\,F}\right)dt - \int_{t_{\mathrm{sc}}}^{t} \left(P_{\mathrm{TL}} - P_{\mathrm{TL}\,F}\right)dt =$$

$$= I_{P_{\mathrm{TL\,sc}}} - \int_{t_{\mathrm{sc}}}^{t} \left(P_{\mathrm{TL}} - P_{\mathrm{TL}\,F}\right)dt, \qquad (20)$$

then $I_{P_{\mathrm{TL\,sc}}}$ can be found only approximately on the basis of transient process calculation. For the system under consideration we got

$$I_{P_{\mathrm{TL\,sc}}} \approx -0.9 P_{\mathrm{TL}\,F} t_{\mathrm{sc}}\,. \qquad (21)$$

The results of transients calculations with SMES control according to (20) and (21) are shown in Fig. 4. These trajectories practically coincided with optimal ones, which confirms the effectiveness of simple quazioptimal control.

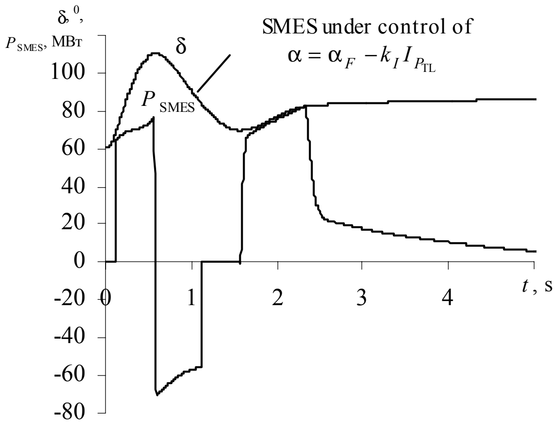

Figure 4. SMES quazioptimal control by local parameters.

It should be noted that post fault condition of the system differs from initial one as a circuit of faulted transmission line is disconnected.

6 CONCLUSION.

1. Optimal control of SMES under large disturbances has been found with the help of generalized Pontryagin's maximum principle.
2. SMES regulator responding to local variables has been designed and proved to give results close to optimal ones.

APPENDIX.

EPS model.

In order to simplify the presentation of the problem the energy stored in SMES is considered constant (which practically did not affect the results) and its reactive power is compensated by SVC, then we have (1) and (2) in the form

$$\left.\begin{aligned} &\dot{\delta} = s \\ &\dot{s} = a_1 - a_2 U \sin(\delta - \vartheta - \alpha_{\mathrm{GL}}) - a_3 \sin(\delta - \alpha_{\mathrm{GS}}) \end{aligned}\right\}, \quad (\mathrm{A1})$$

$$\left.\begin{aligned} &0 = -a_4 U \cos\alpha + a_5 - a_6 U^2 - \\ &- a_7 U \sin(\vartheta - \delta - \alpha_{\mathrm{GL}}) - \\ &- a_8 U \sin(\vartheta - \alpha_{\mathrm{LS}}) \\ &0 = -a_9 U^2 + a_7 U \cos(\vartheta - \delta - \alpha_{\mathrm{GL}}) + \\ &+ a_8 U \cos(\vartheta - \alpha_{\mathrm{LS}}) \end{aligned}\right\}, \quad (\mathrm{A2})$$

where $a_1 = \dfrac{\omega_{\mathrm{nom}}}{M}\left(P_{\mathrm{T}} - E' y_{\mathrm{GG}} \sin\alpha_{\mathrm{GG}}\right)$,

$a_2 = \dfrac{\omega_{\mathrm{nom}}}{M} E' y_{\mathrm{GL}}$, $a_3 = \dfrac{\omega_{\mathrm{nom}}}{M} E' U_{\mathrm{S}} y_{\mathrm{GS}}$,

$a_4 = \dfrac{12\sqrt{W_{\mathrm{SMES}}}}{\pi\sqrt{L_{\mathrm{SMES}}}}$, $a_5 = \dfrac{12 x_{\mathrm{c}} W_{\mathrm{SMES}}}{\pi L_{\mathrm{SMES}}}$,

$a_6 = y_{\mathrm{LL}} \sin\alpha_{\mathrm{LL}}$, $a_7 = E' y_{\mathrm{GL}}$, $a_8 = U_{\mathrm{S}} y_{\mathrm{LS}}$,

$a_9 = y_{\mathrm{LL}} \cos\alpha_{\mathrm{LL}}$.

Variables in optimal control problem include

$$\mathbf{x} = \begin{pmatrix} \delta \\ s \end{pmatrix}, \quad \mathbf{p} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} \vartheta \\ U \end{pmatrix}, \quad \mathbf{u} = \alpha,$$

$$\mathbf{G} = \frac{\partial \mathbf{y}}{\partial \mathbf{x}} = \begin{pmatrix} \dfrac{\partial\vartheta}{\partial\delta} & \dfrac{\partial\vartheta}{\partial s} \\ \dfrac{\partial U}{\partial\delta} & \dfrac{\partial U}{\partial s} \end{pmatrix}, \quad \mathbf{I} = \frac{\partial \mathbf{y}}{\partial \mathbf{u}} = \begin{pmatrix} \dfrac{\partial\vartheta}{\partial\alpha} \\ \dfrac{\partial U}{\partial\alpha} \end{pmatrix}.$$

If we denote them by z_i, then we get:

$$\mathbf{z}_{\mathrm{T}} = (z_1 \; z_2 \; z_3 \; z_4 \; z_5 \; z_6 \; z_7 \; z_8 \; z_9 \; z_{10} \; z_{11} \; z_{12} \; z_{13}) = .$$

$$= \left(\delta \;\; s \;\; p_1 \;\; p_2 \;\; \vartheta \;\; U \;\; \alpha \;\; \frac{\partial\vartheta}{\partial\delta} \;\; \frac{\partial\vartheta}{\partial s} \;\; \frac{\partial U}{\partial\delta} \;\; \frac{\partial U}{\partial s} \;\; \frac{\partial\vartheta}{\partial\alpha} \;\; \frac{\partial U}{\partial\alpha}\right)$$

Vector-functions of the problem are:

$$\mathbf{f} = \begin{pmatrix} z_2 \\ a_1 - a_2 z_6 \sin(z_1 - z_5 - \alpha_{\mathrm{GL}}) - a_3 \sin(z_1 - \alpha_{\mathrm{GS}}) \end{pmatrix}, \quad (\mathrm{A3})$$

$$\varphi = \begin{pmatrix} -a_4 z_6 \cos z_7 + a_5 - a_6 z_6^2 - \\ -a_7 z_6 \sin(z_5 - z_1 - \alpha_{GL}) - \\ -a_8 z_6 \sin(z_5 - \alpha_{LS}) \\ -a_9 z_6^2 + a_7 z_6 \cos(z_5 - z_1 - \alpha_{GL}) + \\ + a_8 z_6 \cos(z_5 - \alpha_{LS}) \end{pmatrix}. \qquad (A4)$$

Hamilton function:

$$H = p_1 f_1 + p_2 f_2 - F = z_3 \cdot z_2 + \\ + z_4 \begin{bmatrix} a_1 - a_2 z_6 \sin(z_1 - z_5 - \alpha_{гн}) - \\ - a_3 \sin(z_1 - \alpha_{гс}) \end{bmatrix} - \\ - k_\delta (z_1 - z_{1F})^2 - k_\omega (z_2 - z_{2F})^2 - k_\alpha (z_7 - z_{7F})^2, \quad (A5)$$

where

$$F = k_\delta (\delta - \delta_F)^2 + k_\omega (s - s_F)^2 + \\ + k_u (\alpha - \alpha_F)^2 = k_\delta (z_1 - z_{1F})^2 + , \\ + k_\omega (z_2 - z_{2F})^2 + k_\alpha (z_7 - z_{7F})^2$$

k_δ, k_ω, k_α– are weighting factors.

EPS parameters.

Power station: M=8.806 s, x'_d =0.29 p.u.,

P_t=P_0=0.64 p.u., U_S=1.0 p.u.

Parameters of equivalent Π-circuit of transmission lines: $\underline{Z}_{\Pi\,TL1} = 7.5 + j77.082$ ohm ,

$Y_{\Pi\,TL1} = j1.771 \cdot 10^{-3}$ mho ,

$\underline{Z}_{\Pi\,TL2} = 6.13 + j48.4$ ohm ,

$Y_{\Pi\,TL2} = j1.032 \cdot 10^{-3}$ mho .

SMES: W_{SMES}=270 MW·s, P_{SMES}=100 MW.

Load: $\dot{S}_{Load\ MV} = 50 + j19.76$ MVA ,

$\dot{S}_{Load\ LV} = 100 + j48.43$ MVA .

The rest of the parameters are determined from load flow calculation.

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