ASSESSING NONLINEARITY AND NON-STATIONARITY IN POWER SYSTEM BEHAVIOR USING HIGHER ORDER STATISTICAL ANALYSIS TECHNIQUES

E. Barocio
Universidad Autonoma de Nuevo Leon
Monterrey, NL,Mexico
ebarocio@gama.fime.uanl.mx

R.J. Betancourt
Universidad de Colima
Manzanillo, Col, Mexico
rjimenez@gdl.cinvestav.mx

J. Arroyo, A.R. Messina
CINVESTAV-IPN
Guadalajara, Jal, Mexico
jarroyo,aroman@gdl.cinvestav.mx

Abstract –A novel analysis framework based on nonlinear statistics, for detection and quantification of nonlinear power system behavior is presented. The technique uses output-only measurements for the extraction of nonlinear characteristics of systems subjected to large perturbations and can be used to determine the strength and distribution of nonlinear behavior of both, ambient and power system response data.

First, linear statistic analysis techniques are used to determine critical machines following the occurrence of severe perturbations. Then the method of surrogate data is utilized to detect nonlinearities and non-stationarities in the underlying process. Criteria for determining possible nonlinearities with respect to variables are proposed and statistical significance tests are used to give a quantitative measure of nonlinearity. Finally, higher order spectral analysis techniques are employed to investigate the nature and extent of nonlinear interaction involving the electromechanical modes of oscillation of the system.

A detailed case study of a 68-bus, 16-machine test system is presented to illustrate the use of the proposed techniques. Examples are given of the application of statistical criteria to detect nonlinearity in power system signals, and estimating the strength of nonlinear interaction.

Keywords: Nonlinear time series analysis, higher-order spectral analysis techniques

1 INTRODUCTION

In recent years statistical models are increasingly being used to facilitate model selection and quantifying nonlinearity and non-stationarity in time series. Quantification of nonlinearity is important because it allows identifying operating conditions under which linear analysis may not provide adequate characterization of system behavior [1-4].

The analysis of time varying characteristics is also an important problem. Non-stationarity in power system behavior may arise from changing mechanisms in the underlying phenomena giving rise to oscillations and may have important implications for the analysis of nonlinearity [5]. Detecting nonstationarities enables to decide about the length of the data required to provide better estimates of the parameters of concern, or selecting the best analysis techniques that can allow us to track changes in system behavior [6,7].

Over the past few years there has been a rapid development in the application of nolinear techniques for time series to the analysis of power system behavior. This interest arises from a variety of motivations; these include the detection and quantification of nonlinearity and non-stationarity, the analysis of modal interaction and the detection of window lengths to assess system behavior [5,8].

Recent studies reported in the literature [1,2], reveal that the dynamic response of stressed power systems subjected to critical contingencies may result in a complex dynamic pattern not accounted for by conventional linear analysis techniques. Detection of such transitions depends on the quality of the available signals and the relevance of the nonlinear statistics employed. Statistically-based methods of nonlinear time series analysis may provide additional information into various aspects of system dynamic behavior that is complementary to perturbation analysis. This approach can be applied to both ambient and power system response data and show promise in identifying nonlinear dynamics in complex systems owing to the flexibility of the model structure.

In this paper, we explore the use of higher-order statistic analysis techniques to assess the significance of nonlinear behavior arising from higher-order analytical solutions. Using artificial or *surrogate* data, nonlinearities and non-stationarities in the process are identified. In particular, nonlinear-based statistical techniques are used to determine the strength and distribution of nonlinear behavior following the inception of large perturbations. The goal is to extract the underlying dynamics of the process giving rise to the oscillatory process and assess the distribution of nonlinearity directly in physical coordinates.

Finally, higher order spectral (HOS) analysis techniques are applied to identify modal interaction involving the spectral components. The analysis methodology is demonstrated on a 68-bus, 16-machine test system.

2 DETECTION OF NONLINEARITY USING LINEAR STATISTICAL MEASURES

2.1 Metrics or Distance Functions

Suppose $\mathbf{x}(t) = [x_1(t), x_2(t),..., x_n(t)]^T$ and $\mathbf{x}^*(t) = [x^*{}_1(t), x^*{}_2(t),..., x^*{}_n(t)]^T$ are two real-valued time series describing the same physical process. Our goal is to evaluate the statistical significance of the evidence for nonlinear structure in each case. Following Jordan and Smith [9], the separation between them at any time t is defined by

$$\left\|\mathbf{x}(t) - \mathbf{x}^*(t)\right\| = \sqrt{\left(\sum_{i=1}^{n}\left|x_i(t) - x_i^*(t)\right|^2\right)} \tag{1}$$

where $|\cdots|$ denotes the magnitude and $\|\cdots\|$ represents the metric or distance function on the space. Corresponding to the distance function (1), the norm of vector $\mathbf{x}$ is given by $\|\mathbf{x}\| = (\sum_{i=1}^{n}|x_i|^2)^{1/2}$.

In a statistical sense, $\mathbf{x}(t)$ and $\mathbf{x}^*(t)$ may represent the set of original and surrogate data or denote the predicted and observed values, respectively. Several relationships between model selection criteria for linear (nonlinear) time series have been established in the literature. We briefly review the nature of these models from the standpoint of linear statistics analysis of physical processes.

2.2 The Variance and Standard Deviation

The standard deviation σ of a probability distribution is defined as the square root of the variance σ^2. Given a set of observed data $\mathbf{x}(t)$, we may be interested in the fluctuation of the measurements around the mean, $\mathbf{x}(t) - \bar{x}$, where $\bar{x} = {x_1 + ... + x_N}/{N}$ is the mean. In order to characterize the size of the fluctuations of the observed data around the mean, it is useful to obtain a measure of the mean value of the square of the fluctuations, $\sigma^2 = {1}/{N}\sum_{i=1}^{N}(x_i - \bar{x})^2$, called the variance. The square root of the sample variance of a set of N values is the sample standard deviation,

$$\sigma_N(t) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2} \tag{2}$$

An important attribute of the standard deviation as a measure of spread is that if the mean and standard deviation of a normal distribution are known, it is possible to compute the percentile rank associated with any given score. The Box-whisker plot (see Figure 1) is a commonly used graphical method to provide information about both the location and dispersion of the data.

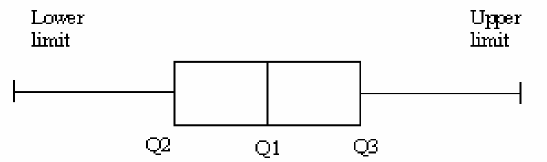


Figure 1: Box-whisker plot

The quartiles are given around the median, splitting the distribution into two parts. The first quartile (Q_1), is the median of the data values in the lower half of the data set. The third quartile (Q3), is the median of the data values in the upper half of the data set. A more accurate and precise statement of statistical significance is given elsewhere [8].

A limitation of the techniques just described is that they implicitly assume that the data are statistically independent and that the underlying mechanism is linear and stationary.

3 QUANTIFAYING NONLINEAR BEHAVIOR USING SURROGATE DATA

3.1 Testing for Nonlinearity in Time Series

A number of measures have been developed for testing for nonlinearity in time series. A common approach is to specify some linear process as a null hypothesis, and then generate artificial or surrogate data sets which are consistent with this null hypothesis [6,7]. If the computed statistics for the original data is significantly different than the values obtained for the surrogate sets, then the null hypothesis is rejected and nonlinearity is detected.

In what follows we examine practical algorithms to generate surrogate data using Fourier transform based surrogate data tests.

3.2 The Method of Surrogate Data

Let $x(t)$ denote a time series of N values obtained from measurements or simulation taken at regular intervals of time $t = t_o, t_1,..., t_{N-1} = 0, \Delta t,..., (N-1)\Delta t$.

Application of the Fourier transform results in

$$X(f) = \Im\{x(t)\} = \sum_{n=0}^{N-1} x(t_n)e^{2\pi i f n\Delta t} = A(f)e^{i\phi(f)} \tag{3}$$

where $A(f)$ is the amplitude and $\phi(f)$ is the phase.

A *phase-randomized* Fourier transform $\tilde{X}(f)$ is made by rotating the phase ϕ at each frequency f by an independent random variable φ which is chosen uniformly in the range [0,2π], namely [7]

$$\tilde{X}(f) = A(f)e^{i[\phi(f)+\varphi(f)]} \tag{4}$$

The surrogate time series is then given by the inverse Fourier transform $x^*(t) = \Im^{-1}\{\tilde{X}(f)\}$. By construction $x^*(t)$ will have the same power spectrum as the original data set, and the same autocorrelation function. Since the surrogate data is assumed to have no dynamical nonlinearities, a number of discriminating statistics can be used to test for nonlinearity and non-stationarity.

3.3 Takens Best Estimator forHypothesisTesting

Among the several discriminating statistics, we use the Takens best estimator of correlation dimension motivated by previous work on nonlinear dynamics statistic [7]:

$$Q = D_{Takens} = \frac{C(r_o)}{\int_o^{r_o} [C(r)/r]dr'} \tag{5}$$

where r_o is an upper cutoff, and $C(r)$ is the correlation integral

$$C(r) = \frac{2}{N^2} \sum_{k=W}^{N-1} \sum_{j=0}^{N-1-k} \Theta(r - \left\| x^*(t_{j+k}) - x^*(t_j) \right\|) \tag{6}$$

Here, Θ is the Heaviside function, $\|.\|$ is the maximum norm, and W is a constant accounting for the order of a few autocorrelation times, which is used to remove autocorrelative effects; x^* can be either a multivariate signal or a time delay embedding; $x^*(t) = [x^*(t), x^*(t-\tau), ..., x^*(t-(m-1)\tau)]$. The correlation dimension can be interpreted as a measure of the average complexity of a signal and gives insight into nonlinearity [7]. In practical terms, an estimated correlation dimension provides a discriminating statistics in tests for nonlinear structure.

A simple measure of significance can then be defined as

$$S = \frac{\left| Q - \langle Q_{surr} \rangle \right|}{\sigma_{surr}} \tag{7}$$

where Q is the Takens estimator for the original data set, $\langle Q_{surr} \rangle$ is the mean value statistics for the surrogates, and σ_{surr} is the standard deviation of the statistic for the surrogates. Analogous reasoning can be applied to linear measures of statistical significance.

3.4 Surrogate Data for Non-Stationary Signals

Most realistic signals have non-stationary characteristics (i.e. their frequency content changes with time). This may render the above analysis unreliable or uninformative.

To circumvent this problem we split the time series into segments that can be considered nearly stationary and perform individual tests. A three step procedure is developed for testing for non-stationarity:

(1) Divide the data into M records (segments) of K length , (2) Generate a surrogate for each segment using the procedure in section 3.2, and (3) Obtain the statistical test by averaging the individual ensemble statistics and determine measures of significance.

4 HIGHER-ORDER STATISTICAL ANALYSIS

Higher order spectral analysis techniques such as the bispectrum, constitute an important tool for the analysis of nonlinear behavior.

4.1 Conventional Spectral Analysis

Let $x(t)$ be a time series that is stationary up to order k with kth order cumulant function and let $X(\omega)$ be its Fourier transform as a function of frequency, where $X(\omega)$ is defined as

$$X(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\omega t} x(t) dt \tag{8}$$

The power spectrum, $P(\omega)$, can then be defined in terms of the signal´s DFT as

$$P(\omega) = \int_{-\infty}^{\infty} R_{2x}(\tau) e^{-i2\pi f\tau} dt = X^*(\omega) X(\omega) \tag{9}$$

where $R_{2x}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x(t+\tau)dt$ is the autocorrelation function or a cumulant of the second order.

4.2 Bispectral Analysis

The bispectrum is a double Fourier transform of the second-order autocorrelation function

$$B(\omega_1, \omega_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{3x}(\tau, \lambda) e^{-i2\pi(\omega_1 \tau + \omega_2 \lambda)} d\lambda d\tau \tag{10}$$

where $R_{3x}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x(t+\tau)x(t+\lambda)dt$ is the third order cumulant. Given estimates of the spectrum and the bispectrum, the bicoherence is defined as [2,8]

$$\left| b(\omega_1, \omega_2) \right|^2 = \frac{\left| B(\omega_1, \omega_2) \right|^2}{P(\omega_1) P(\omega_2) P(\omega_1, \omega_2)} \tag{11}$$

The bispectrum is a complex, two-dimensional doubly periodic quantity that measures the magnitude and the phase of the correlation of a signal at different Fourier frequencies in the $\omega_1 - \omega_2$ plane. The amplitude of the bicoherence can be interpreted as the contribution of energy of nonlinear interaction to the wave energy with a frequency $(\omega_1 + \omega_2)$ .A value of $\left| b(\omega_1, \omega_2) \right|$ close to unity indicates a nonlinear production mechanism. In contrast to this, the system is linear if $B(\omega_1, \omega_2) = 0$ for all ω_1, ω_2. It follows that, $b(\omega_1, \omega_2)$ has constant modulus if $x(t)$ is linear. Therefore, testing the linearity of $x(t)$ reduces to testing the constancy of the squared modulus of the normalized bispectrum. Bispectral estimators employ statistical averaging, and implicitly assume there is randomization between segments; this approach can be used as a reliable detector of modal interaction only if the data to which it is applied satisfy a number of criteria, namely stationarity.

5 APLICATION STUDY

5.1 Operation Conditions Study

The developed procedures are tested on the 16-machine, 68-bus model of the NPCC system. A pictorial representation of this system illustrating the location of coherent areas is provided in Figure 2. The system operating conditions and data are taken from Ref. [10].

To examine the onset of nonlinear behavior a three-phase fault is applied at bus 28 cleared in 0.03 s by opening the tie-line to bus 26. This contingency excites several local and inter-area modes and leads to mode coupling under certain critical operating conditions.

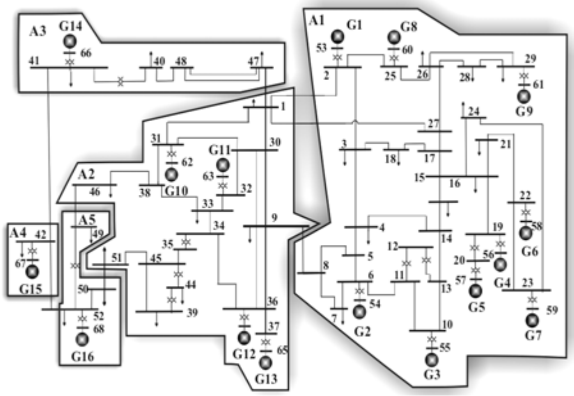


Figure 2: Pictorial of the study system.

Two operating scenarios were considered for analysis: (a) Operating scenario I. A scenario corresponding to the nominal operating condition, and (b) Operating scenario II. A scenario obtained by rescheduling generation. By re-dispatching generation both, the strength and distribution of nonlinearity are modified. Figure 3 shows the speed rotor deviations of the system generators for the above operating scenarios.

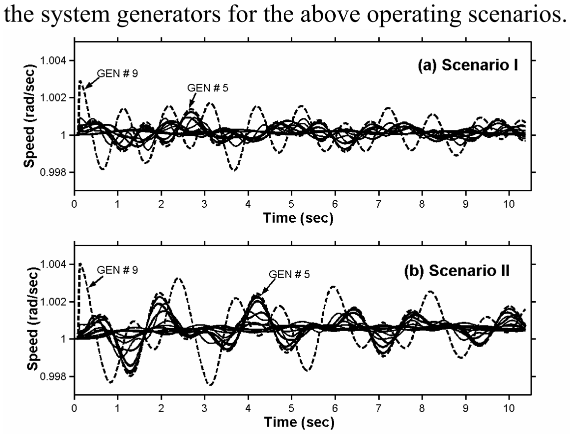


Figure 3: Speed rotor deviations following a short circuit at bus 28. a) Scenario I, b) Scenario II.

As expected from physical considerations, generator # 9, in the neighborhood of the fault exhibits the largest speed rotor deviation. For operating scenario I, visual inspection of the speed rotor deviation of this generator in Figure 3(a), suggests the presence of essentially a simple harmonic behavior. In contrast, examination of the rotor speed deviations in Figure 3(b) for operating scenario II suggests the onset of nonlinear behavior involving more than significant mode of oscillation. Although no presented here, spectral analyses enable us to confirm these findings.

5.2 Small Signal Analysis

The analysis of the system reveals four inter-area modes of concern with damping ratios below 4%. The damping ratios and frequencies are displayed in Tables 1 and 2 for the above operating scenarios, whilst Table 3 shows the linear participation factors for scenario II.

Mode	Eigenvalue	Frecuency (hz)	Damping (%)
22,23	-0.066±2.5884i	0.4119	2.57
24,25	-0.068±3.5006i	0.5571	1.96
26,27	-0.049±4.6155i	0.7346	1.06
28,29	-0.092±6.1744i	0.9827	1.49
30,31	-0.077±5.0623i	0.8057	1.54

Table 1: System eigenvalues- operating scenario I

Mode	Eigenvalue	Frequency (Hz)	Damping (%)
22,23	-0.069±2.0118i	0.3202	3.44
24,25	-0.075±3.2036i	0.5099	2.35
26,27	-0.065±3.688i	0.5870	1.77
28,29	-0.088±5.4885i	0.8735	1.61
30,31	-0.079±5.0034i	0.7963	1.59

Table 2: System eigenvalues- operating scenario II

Mode	Oscillation pattern	Participation factors
22,23	A_2 vs A_3, A_4 A_5	15(0.168),14(0.164), 13(0.105)
24,25	A_1 vs A_2, A_3 A_4,A_5	**9(0.155)**, 6(0.058), 7(0.0433)
26,27	A_3 vs A_2	16(0.303), 14(0.182), 13(0.011)
28,29	A_1	**9(0.312),** 6(0.039), 5(0.036)
30,31	A_4 vs A_3 A_5	15(0.296), 14(0.131), 16(0.067)

Table 3: Linear Participation factors-scenario II

For scenario II, the analysis of linear participations in Table 3 shows that generator # 9 has a dominant participation in the 0.51 Hz inter-area mode 24, and the local plant mode 28 at 0.87 Hz. It is also interesting to note that generators #13-16 have a significant participation in all inter-area modes.

5.3 Linear Statistics Analysis

Testing for nonlinearity using basic metrics provides insights towards identifying the extent and distribution of nonlinearity. A first estimate of nonlinear behavior is obtained from the global standard deviation for the cases of concern. Figure 4 shows the global standard deviations whilst Figure 5 presents the variance of the speed metrics. Examination of the global standard deviation results for scenarios I and II in Figure 4 suggests the onset of both nonlinear and non-stationary behavior. This effect is more pronounced for scenario II as shown by the fluctuations in the standard deviation.

Statistical measures provide information about the strength and distribution of nonlinearity. Thus, for instance, these results demonstrate that generator # 9 and to a lesser extent generator # 5 make a significant contribution to the variability of the oscillations and therefore play an important role in the dynamics of the system. For scenario I, the analysis of the variances in Figure 5 clearly shows that generator #9 has a relevant participation in system behavior. As the system is stressed, the participation of generators in Area 1 relative to generator # 9 increases as shown in Fig. 5b).

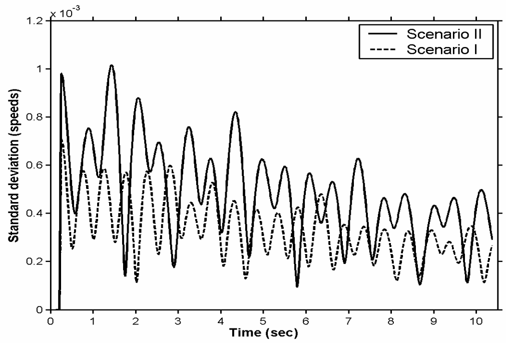


Figure 4: Comparison of global standard deviations for scenarios I and II

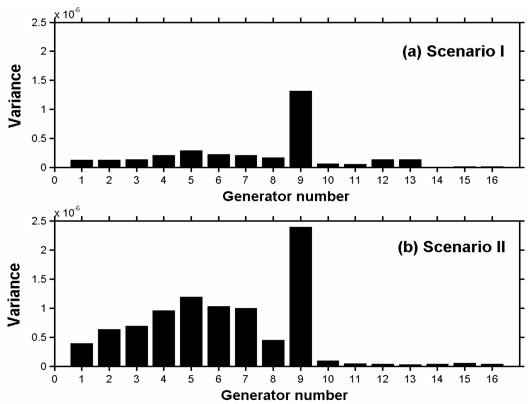


Figure 5: Variance of the speed metrics for scenarios I, II

To help in the interpretation of nonlinear behavior, we computed the Box-Whisker plot associated with the speeds solutions. Figure 6 displays the plots of the rotor speeds for both cases under investigation.

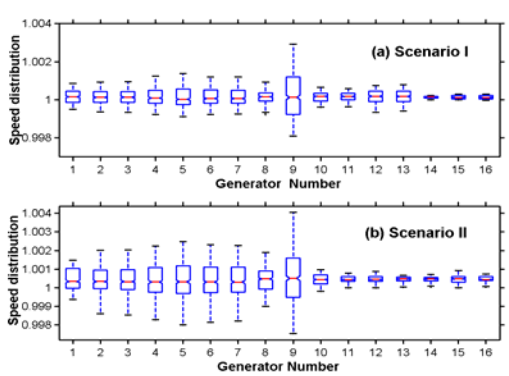


Figure 6: Box-Whisker plot associated with speeds in the scenarios I and II.

By analyzing the Box-Whisker plot of each data set it is possible to determine more precisely which states contribute to nonlinear behavior as well as to reveal the amount of variation. From this basic analysis, we select machines # 4 through 9 for further study.

6 TESTING FOR NON-STATIONARITY AND NONLINEARITY

6.1 Surrogate Data Analysis

For each operating condition, surrogate data was generated and several discriminating statistics were used to detect nonlinearity and non-stationarity. Figure 7 compares the original and surrogate data sets for generator #9. Similar results are obtained for generator #5.

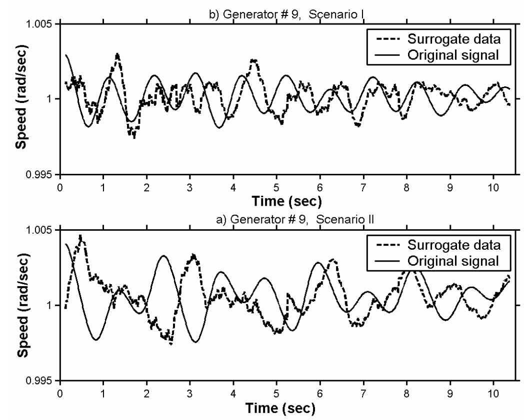


Figure 7: Comparison between original signal and the surrogate data for scenarios I and II. Generator # 9.

6.2 Testing for Non-Stationarity

Simulation results suggest that the observed time series are nonlinear and non-stationary. To identify time varying characteristics the data was divided into several records and statistical techniques were applied to determine time-varying characteristics. Figure 8 shows the speed deviation of generators #5 and #9 showing selected segments for the analysis.

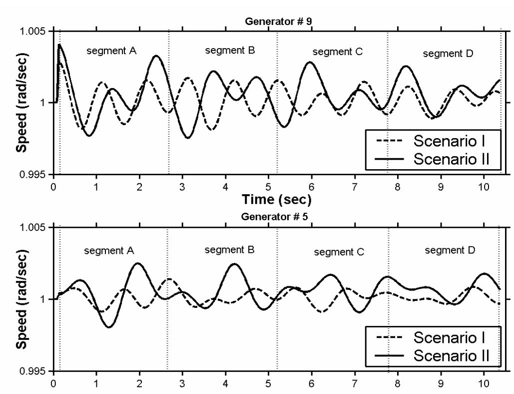


Figure 8: Rotor speed deviations of generators # 5 and 9.

For this analysis, the records were divided into 4 segments each one consisting of 256 samples. For each segment, the surrogate data was computed and correlation measures were obtained. As discussed above [5], the degree of correlation between two adjacent time windows (segments) provides a measure of nonlinearity and non-stationarity. This is an important consideration in the test for statistical significance

In order to evaluate the non-stationary of the original signal, simple dispersion measures were computed. Individual variances were generated using the selected segments and the statistics of these simulations were calculated to create an overall stationarity measure of the data. The variance of the separation between the original and surrogate data is obtained from

$$\sigma_k{}^2 = \frac{1}{N}\sum_{i=1}^{N}\left(e_i(t)-\bar{e}\right)^2 \quad \mathrm{k}=1,\ldots n \tag{12}$$

where $e_i(t)=\omega_i(t)-\omega^*{}_i(t)$ denotes the difference between the exact solution and the approximate solutions provided by surrogate method and k is a number section selected to the analysis signal. The results of this analysis are shown in Figure 9 for generators # 5 and 9.

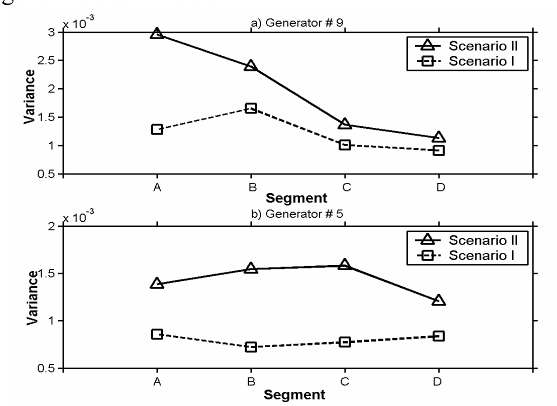


Figure 9: Variance index computed in each segment under study for scenarios I and II and generators # 5 and 9.

For scenario I, examination of the variances of generators # 9 and 5 in Figures 9a) and 9b) shows rather stationary behavior. In turn, the analysis of scenario II, suggests both, nonlinear and non-stationary behavior; nonlinearity appears to be more pronounced for segments A and B as indicated by the relative magnitude of the variances. The systematic application of this approach enables to identify the nonlinear and non-stationary structure of the data of interest for the selection and application of more advanced analytical tools.

6.3 Testing for Nonlinearity

The Takens estimator was computed for each of the signals of concern. Table 4 shows the correlation dimension for scenarios I and II. The analysis enables to confirm the presence of nonlinearity in the system as the statistic of the original data is significantly less than that for the surrogates. Also of interest, the analysis of correlation numbers and the significance of statistical measures from (7) reveals a global increase in nonlinear behavior in agreement with previous findings.

Operating condition	Correlation number
Scenario I. Original data	1.677
Scenario I. Surrogate data	2.481
Scenario II. Original data	2.323
Scenario II. Surrogate data	2.904

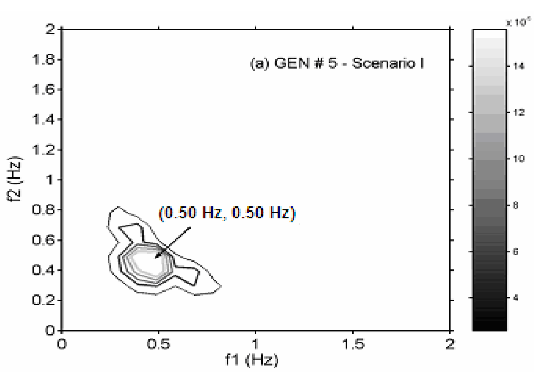
Table 4: Correlation dimension for scenarios I and II

Having determined the onset of nonlinear behavior, we next examine the potential for nonlinear behavior arising from interaction between the fundamental modes of the system using bispectral analysis.

7 HIGHER-ORDER SPECTRAL ANALYSIS

To further investigate possible the significance of the nonlinear terms nonlinear interaction between spectral components, the magnitude-squared bispectrum was obtained. Practical details of the application of the method are given in [2].

Figures 10 and 11 show the contour plot of the bispectrum of the speed rotors of generators # 5 and 9 [1]. In these plots, the contour lines indicate the bispectrum components with maximum activity; light shades represent low bispectrum components whilst dark shades indicate high bispectrum magnitudes. For generator #5, the bispectrum analysis of the scenario I and the scenario II essentially indicate self interaction of the inter-area mode at 0.50 Hz. The analysis of the bispectrum for generator # 9, on the other hand, reveals a more complex behavior suggesting the presence of nonlinear interaction between the inter-area mode and the local mode. The presence of small peaks in the bispectrum at (0.87Hz, 0.50 Hz) suggests the presence of the sum frequency of the fundamental modes and suggest phase coupling between the spectral components.



[1] As pointed out in [10], the plot is symmetrical around the equal-frequency diagonal running from the lower left to the upper right.

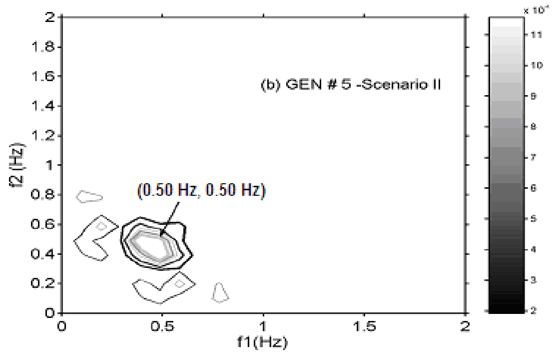


Figure 10: Contour plot of the bispectrum. Generator # 5.

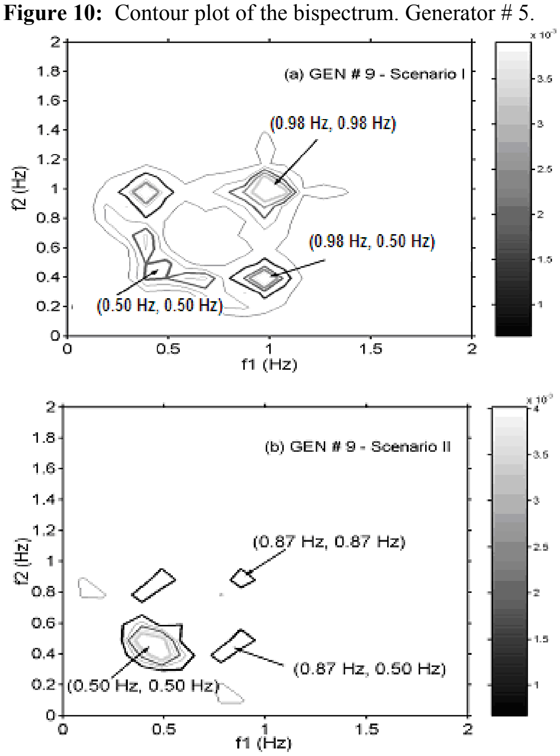


Figure 11: Contour plot of the bispectrum. Generator # 9.

The strength of the nonlinear interaction originating from these components is illustrated in Figure 12 which displays the diagonal slice of the bispectrum for the speed deviation of generator # 9. Also, the modulus of the bispectral density function shows two prominent peaks at 0.50Hz and 0.87Hz .

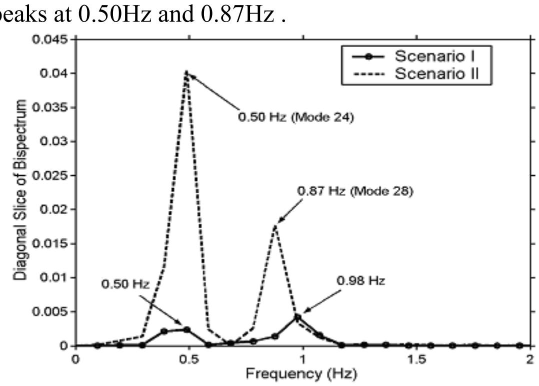


Figure 12: Diagonal slice of bispectrum of the speed Generator #9.

As predicted from statistical analysis, the magnitude of the diagonal slice of the bispectrum shows that nonlinear interaction increases with stress. The estimated parametric bicoherence spectrum (not shown), enables to confirm the presence of quadratic phase coupling between modes 25 and 28.

8 CONCLUSIONS

Nonlinear characterization of observed power system data is a difficult problem. In this paper, a statistical approach is proposed to determine the nonlinear and non-stationary structure of observed system data. The techniques can be applied to both ambient and power system response data and show promise in identifying nonlinear dynamics in complex systems owing to the simplicity and flexibility of the model structure. The proposed technique is being implemented into a production software for detection and quantification of nonlinearity and non-stationarity in the underlying process. Simulation results show that the proposed technique can be efficiently applied to identify generators having a significant participation in system behavior as well as to determine the strength and distribution of nonlinearity. Other insights from the application of the proposed techniques include the detection of modal interaction arising from interaction between spectral components and the significance of non-stationary behavior.

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