Transmission cost allocation in pool systems.

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Abstract

The transmission cost allocation problem may be divided into three different subproblems: the flow allocation to transactions, the definition of these transactions, and the cost allocation based on the flow allocation. The flow allocation subproblem is solved here using a differential, slack invariant method. The last one can be given many different solution, but each choice has different consequences. These consequences are analyzed for the IEEE-RTS96 test system, and can be easily extrapolated to real networks.
Keywords - transmission cost allocation, differential method.

1 Introduction

Transmission cost allocation methods are of increasing importance in the deregulating process that is taking place in the European Union and elsewhere. They are specially interesting in the oncoming European Internal Electricity Market, since they must be applied to quantify the compensations for Cross Border Trades (CBT) of electricity from the use that a foreign transaction makes of a national grid. Although this paper does not deal directly with this problem, the conclusions heres exposed has direct application to it.

A commonly agreed feature that a transmission cost allocation method must have is to provide locational signals and incentives in order to encourage efficient use of the transmission facilities. They also must comply with some conditions, namely to avoid cross-subsidies, to be transparent and easy to implement, to ensure cost recovery, to provide adequate economic signals and to have continuity with time [1].

The cost allocation methods proposed in literature could be classified as embedded cost methods and marginal cost methods. The latter, however, do not guarantee cost recovering in real networks [2]. Embedded cost methods, on the other hand, allocate the transmission costs according to the extent of use of generators and consumers. Several methods of this kind have been proposed and are in use in different systems. They can be divided into rolled-in methods and load flow based methods. Rolled-in methods charge a fixed amount per energy unit, and their main drawback is that they ignore actual network use and that they do not send adequate economic signals to grid users. Flow-based methods, on the other hand, charge the users in proportion to the use they make of grid facilities. Some proposed methods of this kind may be classified as proportional [3], or differential methods [4]. The proportional method has several advantages: it is simple to understand and provides several results such as loss allocation, grid use and load sharing among generators. However, although beginning from a solved load flow, it does not follow the Voltage Kirchhoff Law in the allocation process, ignores the counterflows, and the results seem to be too volatile.

Differential (or incremental) methods are, on the other hand, well known in literature and are based on the sensitivities of branch flows to power injection in nodes. These sensitivities, however, depend on the choice of the slack bus in the studied case, and, therefore, there is a part of arbitrariness in the allocation. One method for overcoming this difficulty, is given in [4]. This method make use of the well know property of the invariance of these sensitivities to a transaction when using DC load flow equations. It allocates transmission costs to transactions, obtaining participation factors in the transmission network for each one of these transactions, and assuming unnecessarily that the generator node is a slack bus in each transaction.

Usually, the proposed allocation methods seem linked to a definition of power exchanges in the system. For instance, [4] proposes the Equivalent Bilateral Exchange principle, or the sharing of the load proportionally to the power provided by each generator. Also, classical differential methods assume an exchange between each node and the slack. In [3], the allocation of costs and the definition of the exchanges are also made following the Proportional Sharing Principle (PSP).

In this paper, however, the problems of transmission grid cost allocation to transactions are divided into three subproblems, that can be addressed independently. First, the definition of transactions. Second, the allocation of power flows through branches to each transaction, and finally, the cost allocation to the already allocated flows. Different solutions can be given to all of them. This last problem is given special importance in this paper, because it has a great influence in the results. The allocation is finally solved assuming an equal share of the grid costs for generators and loads.

The definition of the transactions is straightforward for markets based on bilateral contracts, but in pool organized markets a definition of transaction must be made. This definition has a great influence on the results, and this topic is duly addressed in the paper. Two principles for defining these transactions in a pool based system that have been proposed in literature, the Proportional Sharing Principle (PSP) and the Equivalent Bilateral Exchanges (EBE), are examined in this paper and the consequences of this choice analyzed. It must be recalled that when using differential methods applied directly to nodes, a transaction is tacitly assumed between each node and the slack node.

The allocation of flows to transactions is made from a solved load flow of the studied grid. This load flow may be representative of a certain load and generation pattern. The method could be applied to all the hours of the year in order to quantify the amount of use that a transaction has made of the transmission grid. The allocation is made to transactions, looking for the invariance property of the allocation to the slack bus choice made in the initial load flow. Unlike in [4], the AC load flow equations are used. This requires some additional considerations for achieving this invariance.

The cost allocation problem has also been addressed, and different solutions have been considered. A further decision must be taken about the percentage of costs allocated to generation and load. In non slack invariant methods, this choice is taken by choosing a slack node close to the generation or load centers. However, the fact that the costs are allocated to transactions, and from this to users, allows more flexibility.

The main contributions of this paper are intended to be the following

- The split of the whole problem into three subproblems: transaction definition, use of grid allocation, and cost allocation, with different possible solutions for each of them. This division allows to make choices for each subproblem being aware of their consequences.
- The proposal of a flow allocation method to transactions that is differential and slack invariant (DSI method). This method makes use of the AC load flow equations.
- A study about the consequences of the definition of transactions in pool systems.
- A study about the consequences of the cost allocation choice.

The paper is organized as follows. Section 2 exposes the differential, slack-invariant (DSI) method of flow and losses allocation to transactions. Section 3 deals with the subject of transaction definition choice, while in Section 4 the problem of cost allocation to transactions is addressed. In Section 5 the numerical results of the application of the method to the IEEE 24 nodes Reliability Test System [6] system under different hypothesis are given and commented.

2 Flows allocation

The method begins from the results of a solved AC load flow in a system with a given load and generation. This solution is used as a start point.

If P_{k-m} and P_{m-k} are the active powers in branch r between nodes k and m, injected, respectively, in nodes k, and m, the average power flow and losses in branch r are given by equation (1).

$$F_r = \frac{P_{k-m} - P_{m-k}}{2} \tag{1}$$

Let us define T_{ij} as the transaction t of a power P_{ij} between the generation node j and the demand node i. Differential methods allocate these power flows to transactions from the value of sensitivities of average flow and losses to a differential increase in each transaction. These sensitivities of average power in branch r to a differential variation of the transaction t, T_{ij}, are given by (2).

$$\phi_{rt} = \frac{\partial F_r}{\partial T_{ij}} \simeq \frac{\partial F_r}{\partial P_j} - \frac{\partial F_r}{\partial P_i} \tag{2}$$

The values of these sensitivities may be obtained by means of incremental load flows. The conditions under which this sensitivities are slack invariant are given in the Appendix.

Hence, the increment of average flow of branch r due to the differential transaction t, dF_{rt}, could be found as (3).

$$dF_{rt} = \phi_{rt} dT_{ij} \tag{3}$$

These equations are only valid for differential variations in transactions. In order to obtain the allocation of the flow to a transaction, it would be necessary to integrate them somehow. The integration process requires an initial point, to define an integration path P, and to know the value of the sensitivities along this path, $\phi_{rt}(T_{ij})$. To take a decision for both initial point and integration path is difficult and sometimes unrealistic, and the required computation time is much higher, since this process must be numerical.

For this reason a reconciliation process is followed here. This method is equivalent to the integration, when considering only one integration step, being the integration made with the implicit Euler rule and assuming that the integration path begins from zero value of all the transactions, and then, that all are raised at the same rate to their actual value. Under these assumptions, the allocation of flow to each transaction is shown in (4).

$$\overline{F}_{rt} = \phi_{rt}(T_{ij}) T_{ij} \tag{4}$$

These coefficients $\overline{F}_{rt}$ do not lead to the correct flow of each line,

$$F_r \neq \sum_{t=1}^{N_T} \overline{F}_{rt}$$

and a reconciliation process is necessary, finding the corrected flow allocations by (5).

$$F^c_{rt} = \frac{F_r}{\sum_{t=1}^{N_T} \overline{F}_{rt}} \overline{F}_{rt} \tag{5}$$

Some points must be highlighted:

- The fact that the sensitivities are found with respect to transactions makes them practically independent of the slack bus given. A demonstration of this fact is given in the Appendix.

- Once the allocation to a transaction is made, it may be necessary to share it between generation and load. This may be made in several ways, by agreement between agents, in the case of a bilateral contract, or by dividing it between them somehow. In this way, a great flexibility is possible for the allocation among users, because it might be even different for each transaction. The direct allocation to nodes, and the choice of an adequate slack bus closer to the generation or the demand, in order to allocate to them a part of the power flow, is also possible and has been proposed in literature. However, this last solution is less flexible than the proposal made here. Besides, in systems with no clear spatial separation between generation and load is not straightforward to choose the adequate slack bus for this purpose.

In the next paragraphs, the transaction definition and the cost allocation procedures are examined in detail.

3 Transactions

When the market is based on bilateral contracts, the transactions are clearly defined. However, in pool operated systems, it is necessary to define them in a nondiscriminatory way. This definition of transactions is a key issue because if affects decisively the results. Two principles are examined here, the Equivalent Bilateral Exchange (EBE) principle and the Proportional Sharing Principle (PSP). The first one assumes that all the loads are supplied by all the generators proportionally to the power provided by them. This principle has been proposed in [4] and some aspects have been studied in [5]. The second one assumes that the outgoing power of each branch from a node comes from the incoming flows proportionally to the amount of these incoming flows. This principle has been proposed in [3], where it is also used for flow and losses allocation. In the proposed method the PSP principle will be only used for the definition of the transactions. The application of this principle to the system shown in figure 1, where the arrows show the direction of the active power through the branches, would say that load 3 is supplied by generators 1 and 2, while loads 2 and 4 are supplied only by generator 4. Unlike in [3], the principle is only used to define the transactions, but the flows in all the lines are sensitive to a variation of every transaction. Therefore, each transaction participates in the flow of every line.

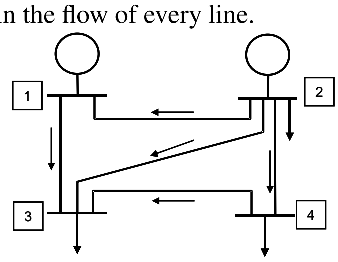

Figure 1: Oriented graph.

The EBE principle has the property of smaller variability with space and time [4]. Besides, as all the generation is supposed to supply to all the loads, the peripheral generation and load make more use of the grid than those located in the center of the system. They are, consequently, charged more. Another characteristic of this principle is that if it were applied to very large systems, it would lead to unrealistic results, since the relation between very distant nodes would ignore grid congestions or other constraints and nonlinearities.

The Proportional Sharing Principle has advantages and disadvantages complementary to the EBE principle. First, it has more variability with time and space. Although this may be a disadvantage, it must be taken into account that in some cases, such as the compensations for CBT, the charges for grid use are computed throughout a long period- for instance a year- and charged annually. Under these conditions, the volatility with time is less important.

Under this principle, the generation and demand located in the center of the system would likely supply or receive power to or from more nodes, and they would have greater relative charges than if EBE principle is followed. On the other hand, peripheral generation and demand are more confined and their charges tend to be smaller than under the EBE principle.

If this principle is applied to very large systems, it would split them into smaller balanced ones, so there are less interference with system nonlinearities and congestions.

4 Cost allocation

4.1 Different methods.

Once the flow has been allocated to the different transactions, it still remains the problem of allocating the cost to them. Throughout this section, it will be assumed that the cost recovery principle is applied, and therefore, the costs are allocated to each transaction according to the use that it makes of the grid. It must be also remarked that the flow allocation to a transaction may be positive or negative. If it is positive, it means that this transaction contribute to the dominant flow of the branch.

The cost of each branch can be allocated in different ways to the branch users. If the negative terms are allowed, those flows opposed to the dominant flow should be paid. The allocation of the cost a branch r to a transaction t, S_{rt} would be given by (6).

$$S_{rt} = C_r \frac{F^c_{rt}}{F_r} \tag{6}$$

Where C_r is the branch cost, F^c_{rt} is the flow allocation of branch r to transaction t, and F_r the flow through branch r.

The total grid cost allocation to a transaction t would be given by (7).

$$S_t = \sum_{r=1}^{N_R} S_{rt} \tag{7}$$

Another method is to allocate the cost according to the absolute value of the flow allocation to a transaction. This is given by equation (8).

$$U_{rt} = C_r \frac{|F^c_{rt}|}{\sum_{t=1}^{N_T} |F^c_{rt}|} \tag{8}$$

where U_{rt} is the cost allocation of branch r to the transaction t and C_r is the cost of branch r. With this allocation, there are no incentives to the counterflows, but the payments for the use of one branch is proportional to the use that the transaction makes of it in a given situation.

The total flow allocation to a transaction is given by equation (9).

$$U_t = \sum_{r=1}^{N_R} U_{rt} \tag{9}$$

In the next paragraph these choices are discussed.

4.2 *Consequences of the cost allocation method choice.*

Let us consider the small system in figure 2, with the different load and generation patterns shown in table 1.

Two situations are considered in order to illustrate also the importance of the transaction definition. The first one assumes that all the nodes belong to a single system (*Domestic system*). The second one supposes that the nodes 101 and 102 are one system (System 1) surrounded by other two that exchange the power through the line *L1* of System 1 (*Multiarea system*). For simplicity, all the lines have the same cost and the grid costs are shared equally by generation and load.

Results are given for the two assumptions,*Domestic* (table 2) and *Multiarea* (table 3, and for the two cost choices, *Flow proportional* ('Flow') and *Absolute value proportional* ('Abs'). Values in both tables represents the total amount of the grid cost in p.u. paid by each node.

The following conclusions may be drawn from the results:

- If the costs are allocated proportionally to the branch flow, an incentive for discharging the lines is provided, as shown for situations *I* and *II*.
- When the flow in line *L1* is very small, the allocation changes abruptly with small differences in flows. In situation *III-B*, nodes 101 and 102 are paid more than 12 times[1] the cost of the whole grid, while in situation *III-C*, very similar to the previous one, it is them who must pay and the other receive. The only condition is that the amount paid by the four nodes is 1. The limit is situation *III-A*, where the amount to pay or being paid is infinite, as may be seen in equation 6.

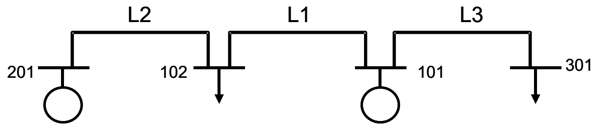

Figure 2: Four nodes system.

Case	Power in nodes (MW) 101	102	201	301
I	50	50	150	150
II	100	100	150	150
III-A	150	150	150	150
III-B	150	150	151	151
III-C	151	151	150	150

Table 1: Load and generation patterns in the 4 nodes system (MW).

Case	Choice	Nodes 101	102	201	301
I	Flow	0.0208	0.0208	0.4792	0.4792
	Abs	0.0583	0.0583	0.4417	0.4417
II	Flow	-0.0667	-0.0667	0.5667	0.5667
	Abs	0.1179	0.1179	0.3821	0.3821
III-A	Flow	$\mp\infty$	$\mp\infty$	$\pm\infty$	$\pm\infty$
	Abs	0.1667	0.1667	0.3333	0.3333
III-B	Flow	-12.37	-12.37	12.87	12.87
	Abs	0.1658	0.1658	0.3342	0.3342
III-C	Flow	12.71	12.71	-12.21	-12.21
	Abs	0.1675	0.1675	0.3325	0.3325

Table 2: Allocation for Domestic system.

Case	Choice	Nodes 101	102	201	301
I	Flow	-0.0833	-0.0833	0.5833	0.5833
	Abs	0.0417	0.0417	0.4583	0.4583
II	Flow	-0.3333	-0.3333	0.8333	0.8333
	Abs	0.0667	0.0667	0.4333	0.4333
III-A	Flow	$\mp\infty$	$\mp\infty$	$\pm\infty$	$\pm\infty$
	Abs	0.0833	0.0833	0.4167	0.4167
III-B	Flow	-25	-25	25.5	25.5
	Abs	0.0831	0.0831	0.4169	0.4169
III-C	Flow	25.17	25.17	-24.67	-24.67
	Abs	0.0836	0.0836	0.4164	0.4164

Table 3: Allocation for Multiarea system.

- The allocation proportional to the absolute value results in a negative incentive for nodes 101 and 102, because they pay more although they have reduced the flow in *L1*. However, the behavior is much more steady in low flow situations in line *L1*.

[1]Net flow in L2 is 1 MW due to flows of 75,75 MW from 201 to 301 and 74,75 MW from 101 to 102. Nodes 201 and 301 would have to pay 37,875 times the cost of L2. Cost of L2 is a third part of system total cost.

- Transaction definition has a key role in the final allocation. In the Domestic example, the generators are supposed to supply every load proportionally to the generated and demanded power. There are, therefore, four different transactions. In the Multiarea situation, however, there are only two exchanges, between nodes 101 and 102, and between nodes 201 and 301. In this cases, the differences between the allocation proportional to flow allocation, and to the absolute values, are even greater.

Therefore, it seems that the benefit of the incentive for a more efficient use of the line is counterbalanced by the unsteadiness in situation of low flows due to opposite exchanges. This is not an unusual situation. It has been claimed that a 20% of the flows in the lines in 14 countries of the UCTE changes their direction because of the foreign transits. It seems, hence, more adequate to allocate proportionally to the absolute value, if the cost recovery principle is assumed.

5 Application examples.

This method has been applied to the IEEE-RTS96 system [6]. Load flows have been run using the Matlab Power System Toolbox [7]. The IEEE-RTS96 1-area system is shown in figure 3. In this system, most of the generation is situated in the upper part of the system, while the load is mostly concentrated in the lower part. Therefore, the main power flow comes downwards. The cost of the grid elements are proportional to the reactance of the branch.

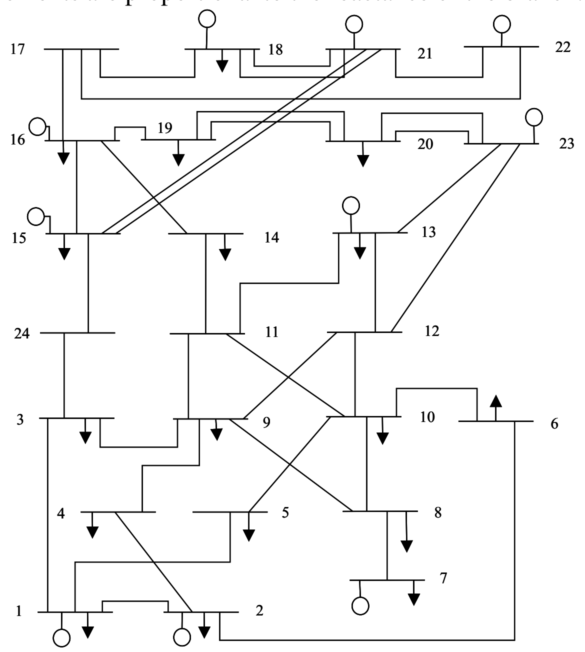

Figure 3: IEEE-RTS96 system.

Several results are shown. First of all, different studies for a single area system have been run under different definition of transactions and cost allocation methods. Then, the IEEE-RTS96 multiarea system is studied in order to draw conclusions about the transaction definition in these multiarea systems, with possible application to CBT compensation evaluation.

5.1 Single area system.

The IEEE-RTS96 system cost is allocated among its users. Results are given in figures 4 to 7. In all of them, the amount of grid cost allocated to each node, under different hypothesis, is shown. These hypothesis are the transaction definition, (EBE and PSP principle are considered), and the cost allocation (proportional to the flow allocation, or to its absolute value).

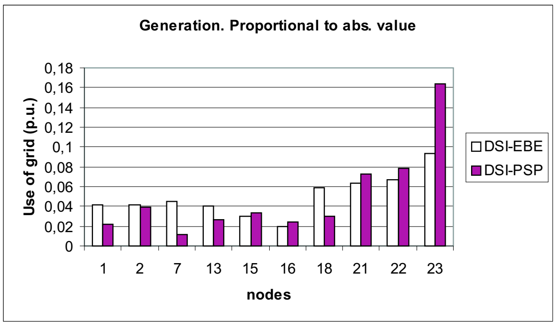

Figure 4: Charge for use of grid among generators. Charge proportional to absolute value.

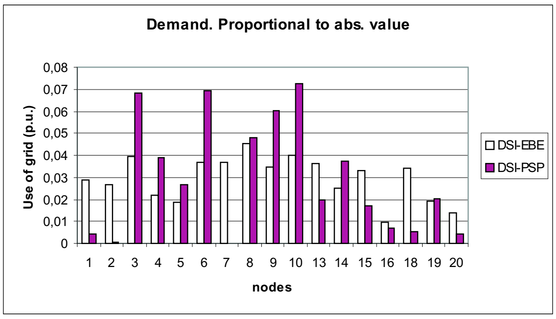

Figure 5: Charge for use of grid among demand. Charge proportional to absolute value.

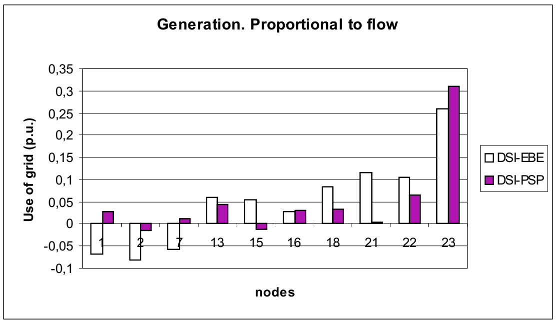

Figure 6: Charge for use of grid among generators. Charge proportional to flow.

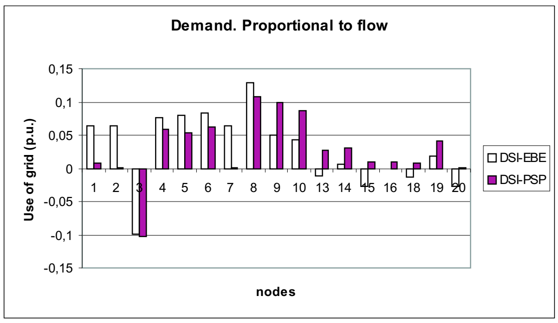

Figure 7: Charge for use of grid among demand. Charge proportional to flow.

From these results, the following conclusions could be made:

- In general, peripheral generation and load are charged more under EBE assumption than under PSP principle when allocation is made following the absolute value criteria. This is because with Equivalent Bilateral Exchanges the exchanges between remote nodes are considered. Under PSP assumptions, however, a more local allocation of load to generators is made, resulting in greater charges to centrally located nodes, because they are included in more transactions.

- When the allocation is proportional to the flow allocation, generators 1, 2 and 3 are allocated a negative amount with Equivalent Bilateral Exchanges due to the fact that there are exchanges that are opposite to the main power flow from the upper to the lower part. This effect, however, does not appear when applying the PSP principle, because the exchanges are limited to closer nodes. Therefore, this counter flows does not exist.

- It could also be added that all the generators with local loads are allocated a smaller amount under PSP assumption that under EBE principle, because the PSP principle makes these generators to supply mostly their local loads, and consequently they make less use of the transmission grid.

- Node 3 is allocated a negative amount in both cases in figure 7 because this load is supplied, according to the PSP principle, by generators 15, 21 and 22. An increase in these transactions would lead to a decrease in the flows of lines (3-1) and (3-9), also supplied by these generators, and that have a very high cost. A similar thing happens under EBE assumptions.

It can be easily concluded that great differences can be obtained with different assumptions. For the sake of stability it could be recommended to allocate costs according to the absolute value of the flow allocation, but of course this could be a subject of endless discussions.

5.2 *Multiarea system.*

As a final example, let us consider the complete (3 areas) IEEE-RTS96 system. In this example the three systems are almost exactly the same, and they are all balanced in generation and load. Only exchange loop flows take place between them. These exchanges (active power in p.u.) may be seen in figure 8, where the number of the nodes in the systems has been preceded by another number indicating its area.

In this situation, if the EBE principle is used for defining the transactions, the part of the cost of each grid and losses that would have to pay the users of each area (rows) to the other areas (columns) is given in table 4. Table 5 shows the same concept, but when the PSP is used for defining the transactions. It can be observed that PSP splits the whole grid into its balanced subsets, while the EBE principle divides the overall cost among all users. It is dubious whether the allocation following EBE principles is adequate for large systems, like the real power networks. This is specially pertinent if the whole UCTE grid were studied. In this case, the PSP principle would split it into balanced sets of generation and load.

On the other hand, it does not seem adequate to use the PSP principle in small systems, because there could be absurd situations like a generator not supplying a close load due to the sense of the power flows in the system. What is the right size for the application of each method is still an open question.

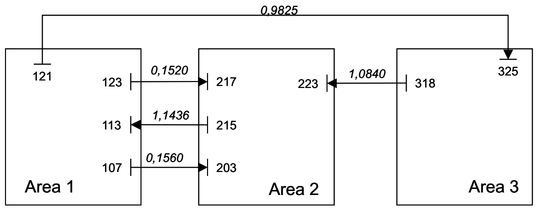

Figure 8: Active power exchanges in the IEEE-RTS96 3-area system. Active power flows in p.u.

Area	Area 1	Area 2	Area 3
1	0,5292	0,2355	0,2355
2	0,2436	0,5513	0,1983
3	0,2272	0,2132	0,5996

Table 4: EBE transactions. Payments from areas in rows to the areas in columns.

Area	Area 1	Area 2	Area 3
1	0,8220	0,0984	0,0541
2	0,1242	0,8482	0,0333
3	0,0538	0,0534	0,9126

Table 5: PSP transactions. Payments from areas in rows to the areas in columns.

6 Conclusion

From the presented results, it may be concluded that:

- Differential methods are adequate for solving the grid cost allocation problem. They follow the network physical rules and are easy to understand and implement.

- The problem of grid cost allocation may be divided in three subproblems: flow allocation to transactions, the definition of these transactions and the cost allocation based on the flow allocation. These subproblems may be dealt with independently.

- The flow allocation problem to transactions may be easily solved using a differential technique running differential load flows. The invariance of the results to the transactions is total if DC load flow equations are used. With AC load flow equations, this invariance is only approximate, and some additional measures must be taken to ensure this invariance.

- The transaction definition has a key role on the allocation results. Two principles for this definition have been examined, the Equivalent Bilateral Exchange principle, and the Proportional Sharing Principle. If the EBE principle is followed, the allocation is more uniform throughout the system, and is more like a flat rate method. It seems less adequate for large networks. The application of PSP, on the contrary, leads to a more irregular allocation pattern, while it does not seem good for small grids. What is a 'large' and a 'small' grid is a subject under discussion.

- The cost allocation criteria proportional to the flow allocation, and allowing negative allocation, may lead to unsteady situations in lines slightly charged due to great counterflows. This effect may check the incentive that provides for a more extensive use of the transmission facilities, when the cost recovery principle is assumed.

Appendix. Branch flow sensitivities to transactions with different slack buses

In figure 9 a schematic electric system is shown. The differential transaction dT_{ij} consists in an active power dP_{ij} injected in node j and delivered in node i.

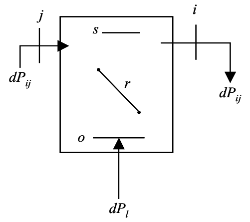

Figure 9: Electric system

When the slack bus is node o, the additional losses produced by this transaction, dP_l, are provided by this node o. The sensitivity of the flow of branch r to this differential increase of the transaction, $\left.\frac{\partial F_r}{\partial T_{ij}}\right|_o$ is given by (10):

$$\left.\frac{\partial F_r}{\partial T_{ij}}\right|_o = \left.\frac{\partial F_r}{\partial P_j}\right|_o - \left.\frac{\partial F_r}{\partial P_i}\right|_o \qquad (10)$$

If, however, the slack bus is the node s, the variation of flow dF_r when the same transaction dT_{ij} takes place, will have the same value only if the losses produced by this transaction are also supplied by the bus o. In this case, this sensitivity could be written as (11)

$$\left.\frac{\partial F_r}{\partial T_{ij}}\right|_s = \left.\frac{\partial F_r}{\partial P_j}\right|_s - \left.\frac{\partial F_r}{\partial P_i}\right|_s + \left.\frac{\partial F_r}{\partial P_s}\right|_s \frac{dP_l}{dP_{ij}} \qquad (11)$$

Equations (10) and (11) give the same results, and the difference between them is given by the term $\left.\frac{\partial F_r}{\partial P_s}\right|_s \frac{dP_l}{dP_{ij}}$. There are different actions that could be taken to consider it, namely:

1. To neglect this term, since it is a second order differential term.

2. To include the losses in the generating node j of transaction T_{ij}. This would be similar to the method proposed in [4] for the lossless case. This possibility, however, must be coherent with the definition of transaction in lossy systems, since the losses produced by each transaction should be included in it. It must be recalled, however, that for lossless systems this term does not exist, and therefore, in these systems, the differences of sensitivities are slack invariant.

3. Although the sensitivity terms could be calculated inverting the given jacobian matrix, they can also be obtained numerically running incremental load flows, where the sensitivities, together with the terms dP_l, could be found. A complete invariance could be achieved by running these incremental load flows with a distributed slack bus, where the losses would be assumed by all the nodes, except the generating node in the transaction.

The procedure used in this paper is the here numbered as 2, although it has been checked that the differences with method 3 are negligibly small.

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