# AN INTEGRATED DISPATCH MODEL OF GAS SUPPLY AND THERMOELECTRIC SYSTEMS

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**Abstract – This paper presents an integrated dispatch model of a natural gas supply system and a gas power plants system. The proposed model integrates the operation of the power plant system with the operation of the natural gas pipeline network. The methodology decomposes the problem in two models. The first one corresponds to an economic dispatch model and the second one simulates the pipeline network operation. The proposed approach was applied to an example of six power plants supplied by a natural gas system.**

***Keywords: Natural Gas, Economic Dispatch, Pipelines, Power Generation***

## 1 INTRODUCTION

Since the early 1970's crude oil crisis and the development of combined-cycle power plants, the natural gas (NG) becomes a strategic and economically competitive energy resource, and extensive pipeline networks were constructed at U.S.A., Russia and Europe. During last 50 years, the world energy consumption supplied by natural gas expanded from 9% to 25%. This increase on the natural gas consumption should be maintained due the great number of unexplored natural gas reserves and its low environmental impact compared to other fossil fuel. Nowadays, only 20% of natural gas reserves are explored against 50% of crude oil reserves.

In the last decades, several papers have been published about natural gas transmission and/or power generation. Goldberg in [1] applied techniques connected with artificial intelligence and genetics to the problem of computer-based control of gas pipeline systems. Wolf and Smeers developed an algorithm to solve the problem of the optimal dimensioning of a gas transmission network [2]. In [3], Venkataramanan et al presented a technique that can be used to optimize the fuel consumption in the gas pipeline operation. A model to compute the maximum power generation of a combined-cycle power plants system is presented by Munoz, Redondo and Ruiz in [4]. In [5], the gas distribution pipeline network operation problem was formulated as a cost minimization problem subject to nonlinear flow-pressure relations, material balance equations, and pressure bounds. The solution method proposed was tested using the Belgium gas network.

The NG pipeline network operation must comply with the natural gas consumption due its low storage capacity. As the electric power plant is one of the major natural gas consumers, there is a close interaction between the NG power plant operation and the gas supply system operation. The dispatch of the NG power plants affects the NG flows in the pipelines, and, by the other side, the pipeline network operational requirements can impose limits on power plant generation. In this context (Figure 1), the models that integrate the operation of these two systems are important for an economic and secure planning and operation.

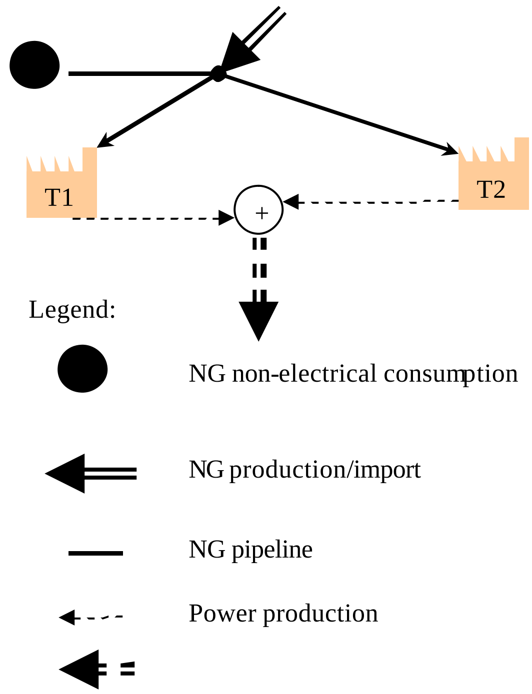


**Figure 1:** Relation between the NG transmission system and the thermoelectric system operation.

This paper proposes a model that considers a set of NG power plants supplied by a gas pipeline supply system. The objective is to minimize the costs of power generation, NG production and/or acquisition, and transmission. System requirements, such as electric load demand, power generation limits, NG flow pressure limits at pipeline network and take-or-pay contracts are represented in the formulation.

The paper is organized as follows. The formulation of the problem is presented in Section 2. Section 3 discusses the solution methodology. Section 4 introduces a test system based on the Brazilian Southeast NG supply system and NG thermoelectric generation plants. The conclusions are presented at Section 5.

## 2 FORMULATION OF THE PROBLEM

### 2.1 The Electric-Gas Dispatch Model

This section presents a dispatch model of a gas pipeline system and a natural gas electric power plants system. The proposed model, denominated Electric-Gas Dispatch Model (EGDM), minimizes the costs related to power generation, NG production/import and NG transmission, and it takes into account the electric load demand, power plant generating limits, NG pipeline system constraints, such as pressure and flow limits and production/import capacities.

A pipeline is composed by a set of equipments, such as pumps, compressors, control valves and separators [6]. In some points of the pipeline system, the NG is injected (suppliers nodes), and in other points the NG is delivered (consumer nodes). Each supplier has a specific NG production/import cost and capacity, and the pipelines have capacity limits represented by constraints on node pressure. The pipeline supplies NG for many end users and it must meet contractual obligations in terms of flow and pressure.

The electric-gas dispatch problem integrates the electric energy system and the NG pipeline network. In the first system, the main objective is to minimize the operational costs of power plants, attending the electric load demand and operational constraints. This is the classical economic dispatch problem [8], where only global load demand and power generation limits are considered. The second system, the pipeline network system, aims to attend the NG demands with minimum production/import and transmission costs [2][5][7].

### 2.2 Mathematical Formulation

The EGDM model described in the last subsection is formulated as an optimization problem as follow.

Minimize

$$\sum_{i\in T} c_i(g_i) + \sum_{n\in S} b_n(w_n) + \sum_{ij\in D} t_{ij}(y_{ij}) \qquad (1)$$

Subject to

$$\sum_{i\in T} g_i(q_i^e) = d; \qquad (2)$$

$$g_i^{\min} \le g_i \le g_i^{\max}; \qquad i \in T \qquad (3)$$

$$\sum_{j\in D} y_{ji}(p_j, p_i) - \sum_{j\in D} y_{ij}(p_i, p_j) = q_i^e + q_i^o; \qquad i \in C \qquad (4)$$

$$\sum_{j\in D} y_{ji}(p_j, p_i) - \sum_{j\in D} y_{ij}(p_i, p_j) = w_i; \qquad i \in S \qquad (5)$$

$$w_i^{\min} \le w_i \le w_i^{\max}; \qquad i \in S \qquad (6)$$

$$sign(y_{ij}).(y_{ij})^2 = (C_{ij})^2[(p_i)^2 - (p_j)^2]; \qquad ij \in D_P \qquad (7)$$

$$sign(y_{ij}).(y_{ij})^2 \ge (C_{ij})^2[(p_i)^2 - (p_j)^2]; \qquad ij \in D_A \qquad (8)$$

$$p_i^{\min} \le p_i \le p_i^{\max}; \qquad i \in N \qquad (9)$$

A legend of symbols used in the equations (1)-(9) is presented in the table below.

The first component of the objective function (1) is the electric generating cost; the second component represents the NG production/import cost; and the last component represents the NG transmission cost.

| Symbol | Meaning |
|---|---|
| $c_i(\,)$ | Generating cost function at $i$th power plant. |
| $b_n(\,)$ | Production/import cost function at $n$th supplier. |
| $t_{ij}(\,)$ | Transport cost function of pipeline $ij$. |
| $g_i$ | Electric power generated at $i$th power plant. |
| $w_n$ | Production/import of NG at $n$th supplier. |
| $y_{ij}$ | NG flow through pipeline that links node $i$ to node $j$. |
| $p_i$ | Pressure at node $i$. |
| $d$ | Electric load demand. |
| $q_i^e, q_i^o$ | NG flow rates delivered at $i$th node for power generation and non-electrical consumption, respectively. |
| $C_{ij}$ | Constant that depends on the length, the diameter, the absolute rugosity of pipe, and the gas composition [5]. |
| $T$ | Power plants set. |
| $S, C$ | Sets of suppliers and consumers nodes, respectively. |
| $D, D_P, D_A$ | Sets of all pipelines, passive pipelines and active pipelines, respectively. |
| $N$ | Set of all nodes. |

**Table 1:** Legend of mathematical formulation.

Constraint (2) attends the global electric load requirement, and constraints (3) represent the units' generating limits. In equation (2), the generation at the $i$th power plant depends on NG flow rate consumed at this unit. Then, this equation relates the electric power generation and its NG flow rate consumed.

Regarded to pipeline system, equations (4) and (5) represent the mass conservation principle in each node. In these equations, the first component is given by the summation of all NG inflows at the $i$th node, and second component represents the summation of all NG outflows at the $i$th node. The right-hand side of the equation (4) is the NG delivered at the consumer nodes, and the right-hand side of equation (5) is the NG production/import at the supplier nodes. Equation (7) defines the relation between the flow rate through a passive pipeline (lines without compressors) and its terminal nodes pressures. The sign function determines the NG flow direction. This term introduces a binary variable in the model, and it turns this formulation in a combinatorial problem. In the active pipeline (with compressor) the flow rate can be greater than the pressure drops along the pipeline (8). The limits on the suppliers' capacities and node's pressure are given by constraints (6) and (9) respectively. The take-or-pay contracts can be represented changing the limits in (6). The problem (1)-(9) is mixed and nonlinear.

This is a basic formulation. Other constraints, both the pipeline system and electric transmission system constraints can be included in the model.

## 3 SOLUTION PROCEDURE

EGDM problem (1)-(9) is solved by an iterative resolution of a Dispatch Model (DM) and a Simulation Model (SM). In the first one, the pressure-flow nonlinear constraints are relaxed and a linear optimization problem is obtained. This model determines the dispatch of each thermal plant, the NG production/import level at each NG sources, and the natural gas flow rate in pipelines. In the second model a simulator of NG pipeline system that consider the non-linear flow equations (7) and (8), and pressure limits (9), is used to simulate the dispatch obtained by DM model. If the Simulator identifies some pipeline pressure limits violations, these indicate that the solution determined by the linear optimization problem is not feasible for the pipeline system operation. Thus, the flow limits of linear optimization problem are updated in order to meet the violated constraints. Then, a new dispatch is obtained, and the process is repeated until no more violations are detected.

In the DM model was introduced a slack variable associated to each power plant. These variables can be interpreted as NG supply deficit, penalized with high cost. If at least one of these slack variables is non-zero in the optimal solution, this indicates that it is not possible to attend the power demand requirements with present NG availability.

### *3.1 Dispatch Model (DM)*

The DM presented as follow determines the power generation output at each power plant ($g_i$), the NG production level at each supply node ($w_i$), and the flow at each pipeline ($y_{ij}$). This model considers the electric load demand requirement, the material balance equation at each node in the NG pipeline system, and the limits in the NG production/import and in the flow rate.

In this formulation, all generation units are NG power plants, but other kind of power plants can be included to meet the electric energy demand. Hydroelectric power plants also can be considered [9]. In the problem (10)-(16) is included the NG flow limits (16) that are not considered in the original problem (1)-(9). In that problem there are pressure limits.

The NG is transported by pipeline network that can be represented by a graph, in which equations (13)-(14) represent the material balance at each node. In the present implementation each branch of the pipeline was modeled by two arcs with opposite directions. So, the flow direction at each branch is determined by an optimization problem without using binary variables. In the nodes with NG power plants were included slack variables with high costs that represent artificial NG supplies. If at least one of these artificial is not zero at optimal solution, then this indicates a not feasible problem.

Minimize

$$\sum_{i\in T} c_i(g_i) + \sum_{n\in S} b_n(w_n) + \sum_{ij\in D} t_{ij}(y_{ij}) \tag{10}$$

Subject to

$$\sum_{i\in T} g_i(q_i^e) = d; \tag{11}$$

$$g_i^{\min} \le g_i \le g_i^{\max}; \qquad i \in T \tag{12}$$

$$\sum_{j\in D} y_{ji} - \sum_{j\in D} y_{ij} = q_i^e + q_i^o; \qquad i \in C \tag{13}$$

$$\sum_{j\in D} y_{ji} - \sum_{j\in D} y_{ij} = w_i; \qquad i \in S \tag{14}$$

$$w_i^{\min} \le w_i \le w_i^{\max}; \qquad i \in S \tag{15}$$

$$y_{ij}^{\min} \le y_{ij} \le y_{ij}^{\max} \qquad ij \in D \tag{16}$$

If objective function (10) and equation (11) are represented by smooth non-linear functions, then problem (10)-(16) can be treated by a Newton Technique [10]. In the present paper, all cost function in (10) and the relation between power generation and NG consumption were modeled by linear functions. Thus, the resulting DM problem was solved by Linear Programming software. The solution obtained by DM can violate the pressure limits, because the pressure aspects were not represented in DM problem.

The pressure violations are identified by the Simulation Model (SM) presented as follow.

### *3.2 Simulation Model (SM)*

The main objective of SM is to verify if it is possible to transport the NG productions and flows calculated by DM model. This test is executed calculating the upper ($pu_i$) and lower ($pl_i$) pressure bounds at each node considering the equipments capacities and NG dispatch. These upper and lower limits do not necessarily coincide with the minimum and maximum node pressure limits ($p_i^{\min}$ and $p_i^{\max}$).

The simulation process starts at a supply node, and considers its node pressure ranges. At each node and for each one of the branches connected to this node, the SM determines the maximum and minimum pressure bounds considering the NG flow calculated by DM. Initially, the upper and lower bounds ($pu_i$ and $pl_i$) are initiated at the maximum and minimum node pressure limits.

#### *3.2.1 Node Pressure Upper Bound*

The square of the pressure at the end node $j$ of the branch $ij$, when the pressure at the node $i$ is at its maximum and the flow through the branch is $y_{ij}$ (calculated by DM), is given by

$$\hat{p}_j^2 = (pu_i)^2 - (y_{ij}^2 / C_{ij}^2). \qquad (17)$$

If $\hat{p}_j^2 < 0$ this indicates that this NG flow is not possible and the maximal flow through $ij$ must be limited to

$$y_{ij}^{\max} = \sqrt{[(pu_i)^2 . C_{ij}^2} . \qquad (18)$$

If $\hat{p}_j^2 > 0$, then it has three possibilities.

First, the pressure upper bound ($pu_j$) at the $j$ node is given by:

$$pu_j = \hat{p}_j , \quad \text{if } p_j^{\min} \leq \hat{p}_j \leq p_j^{\max} . \qquad (19)$$

Second, if $\hat{p}_j > p_j^{\max}$, this indicates that the pressure upper bound at node $i$ should be reduced to

$$pu_i = \sqrt{(pu_j)^2 + (y_{ij}^2 / C_{ij}^2)} . \qquad (20)$$

If $pu_i < pl_i$, then the minimal flow through this branch must be

$$y_{ij}^{\min} = \sqrt{[(pl_i)^2 - (pu_j)^2].C_{ij}^2} . \qquad (21)$$

Third, if $\hat{p}_j < p_j^{\min}$, this indicates that the flow through this branch exceed its capacity, and it must be reduced to

$$y_{ij}^{\max} = \sqrt{[(pu_i)^2 - (pl_j)^2].C_{ij}^2} . \qquad (22)$$

### 3.2.2 Node Pressure Lower Bound

When the pressure at node $i$ is given by its minimum limit and the flow through the branch is $y_{ij}$, then the square of the minimum pressure at node $j$ is given by:

$$\hat{p}_j^2 = (pl_i)^2 - (y_{ij}^2 / C_{ij}^2). \qquad (23)$$

If $\hat{p}_j^2 < 0$, it has two cases. First, $\sqrt{y_{ij}^2 / C_{ij}^2} > pl_i$, and then

$$pl_i = \sqrt{y_{ij}^2 / C_{ij}^2} . \qquad (24)$$

Otherwise,

$$y_{ij}^{\min} = \sqrt{[(pu_i)^2 . C_{ij}^2} . \qquad (25)$$

If $\hat{p}_j^2 \geq 0$ and $\hat{p}_j > p_j^{\min}$, then

$$pl_j = \hat{p}_j . \qquad (26)$$

Otherwise, if $\hat{p}_j^2 \geq 0$ and $\hat{p}_j < p_j^{\min}$, this indicates that the pressure lower bound at node $i$ should be increased to

$$pl_i = \sqrt{(pl_j)^2 + y_{ij}^2 / C_{ij}^2} . \qquad (27)$$

### 3.2.3 Simulation Process

The simulation process initializes the pressure bounds at their respective node pressure limits. Pressure bounds actualization begins as in 3.2.1 and 3.2.2 at a supply node. The node pressure bounds actualization follow the NG flows from that supply node. When a pressure limit is changed the Simulation Process must be restarted with update bounds.

When a flow capacity is actualized by equation (18), (21), (22) or (25), this indicates that the present NG dispatch is not feasible, and the simulation process is stopped. A new NG dispatch is obtained by solving the DM with actualized NG flow capacities (16), and the simulation process is repeated again.

If the simulation does not actualize any NG flow capacity, this indicates that it is possible to transport through the pipeline system the present NG dispatch.

### 3.3 Methodology Overview

The diagram in the Figure 2 shows the main steps of the methodology.

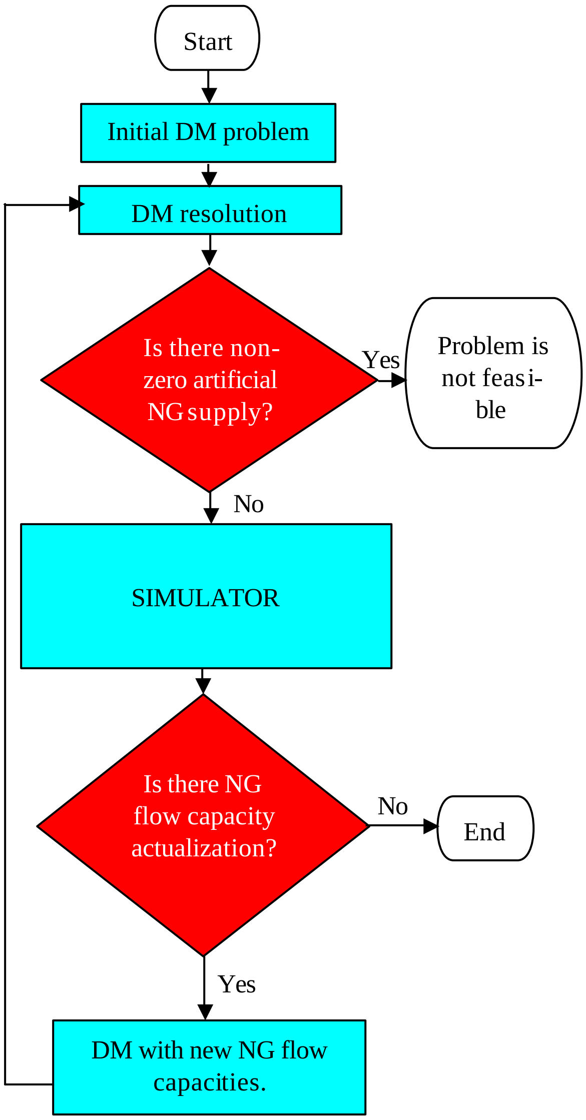


**Figure 2:** Methodology overview.

## 4 APPLICATION TO A TEST NETWORK

The Brazilian NG reserves are currently estimated in 657 billions of cubic meters, where the major reserves are at Brazilian Southeast. The extension of the Brazilian NG pipeline system is about 7600 km. GASBOL is the biggest NG pipeline system; it starts in Bolivia and supplies the NG consumers at five states in the Brazilian South/Southeast region.

The methodology presented in the Section 3 was applied to a test network based in the NG transmission network and thermoelectric generation plants in the Brazilian Southeast, composed by six NG thermoelectric power plants, two NG production plants and one NG import node.

Table 2 presents the data of the test system showed in the network graph (Figure 3). The nodes types are described in the second column. In the third column are the capacities of the nodes. Node number 1 represents the NG imported from Bolivia. In the present example a minimum NG import flow is considered due a contractual obligation, and fixed in 5 $m^3/s$.

| Number | Type | Capacity of production/import/generation |
|---|---|---|
| 1 | Import | Between 5$m^3/s$ and 347.2 $m^3/s$ |
| 2 | NG Producer | 11.5 $m^3/s$ |
| 3 | NG Producer | 104.2 $m^3/s$ |
| 4 | Thermal Plant | 880 MW |
| 5 | Connection Node | — |
| 6 | Thermal Plant | 320 MW |
| 7 | Thermal Plant | 1036 MW |
| 8 | Thermal Plant | 143 MW |
| 9 | Thermal Plant | 720 MW |
| 10 | Thermal Plant | 870 MW |

**Table 2:** Nodes of the test network.

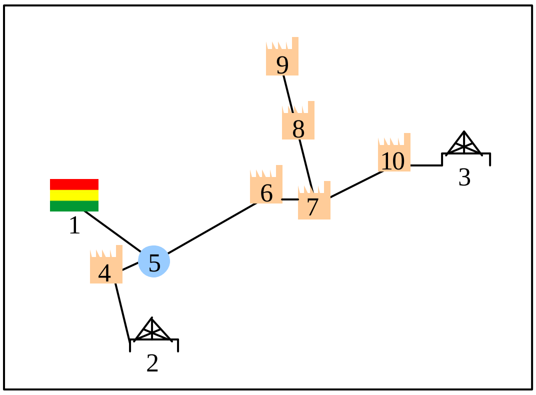


**Figure 3:** The gas-electric system.

The electric load demand was 1984.5 MW. NG production/import costs were \$0.10/$m^3$/s, \$0.07/$m^3$/s, and \$0.07/$m^3$/s to nodes 1, 2 and 3 respectively.

The power generation costs were \$1.00/MW to nodes 4, 6, 9 and 10; \$0.90/MW to node 7, and \$1.20/MW to node 8.

The NG transport costs by branch are presented in Table 3.

The node pressure bound are presented in Table 4. The third column of Table 4 presents the pressures found by SM.

| Branches | Transport Costs (\$/$m^3$/s) |
|---|---|
| 1 — 5 | 0.000 |
| 2 — 4 | 0.000 |
| 3 — 10 | 0.000 |
| 4 — 5 | 0.320 |
| 5 — 6 | 1.585 |
| 6 — 7 | 0.255 |
| 7 — 8 | 0.985 |
| 8 — 9 | 0.745 |
| 10 — 7 | 1.390 |

**Table 3:** Branches of the test network.

| Node | Lower Bound | Upper Bound | Final Pressure |
|---|---|---|---|
| 1 | 0.0 | 88.3 | 61.0 |
| 2 | 0.0 | 88.3 | 23.3 |
| 3 | 0.0 | 88.3 | 34.2 |
| 4 | 0.0 | 68.6 | 21.9 |
| 5 | 0.0 | 58.8 | 28.7 |
| 6 | 0.0 | 29.4 | 22.1 |
| 7 | 0.0 | 39.2 | 16.7 |
| 8 | 0.0 | 73.5 | 16.2 |
| 9 | 0.0 | 58.8 | 15.6 |
| 10 | 0.0 | 78.5 | 25.2 |

**Table 4:** Node pressure bounds and calculated pressures (in bars).

| Branch | Initial DM ($m^3/s$) | New lower bound of the flow in branch 5→6 |
|---|---|---|
| 1→5 | 5.0 | 5.0 |
| 2→4 | 0.0 | 0.7 |
| 3→10 | 4.9 | 4.2 |
| 4→5 | -3.1 | -1.7 |
| 5→6 | 1.9 | 3.3 |
| 6→7 | 1.4 | 2.8 |
| 7→8 | 0.6 | 0.6 |
| 8→9 | 0.6 | 0.6 |
| 10→7 | 4.4 | 3.0 |

**Table 5:** NG productions/import and flows.

Table 5 shows the NG flows obtained by initial DM solution. The Simulation Process identified a pressure violation at the node 6 when the branch 5-6 was analised. It was necessary to change the lower bound of the NG flow of the branch 5-6 by means of equation (21). The DM was actualized with this new NG minimal flow, and a new NG dispatch was calculated. The new and final NG productions, import and flows obtained by DM are showed in the third column in Table 5.

The power generation dispatch in the first and in the second dispatch are showed in Table 6.

| Power Generation (MW) | | | | |
|---|---|---|---|---|
| Node | $g_i^{min}$ | $g_i^{max}$ | Initial Solution | Second Dispatch |
| 4 | 100 | 880 | 628.5 | 485.3 |
| 5 | — | — | — | — |
| 6 | 100 | 320 | 100 | 100 |
| 7 | 100 | 1036 | 1036 | 1036 |
| 8 | 0 | 143 | 0 | 0 |
| 9 | 120 | 720 | 120 | 120 |
| 10 | 100 | 870 | 100 | 243.2 |

**Table 6:** Power Generation.

## 5 CONCLUSIONS

This paper presented an integrated gas and electric power systems dispatch model. In these systems, the decisions about the power plants affect the operation of the gas system, and vice-versa. Thus, in this context, the models that integrate the operation of these two systems are important for a economic and secure planning and operation of electrical and gas supply system.

The decomposition approach adopted to treat the formulated problem was very efficient, and the graph representation of the pipeline network was adequated.

The tests executed showed the interaction between power generation decision and the gas system operational requirements. Thus, through an integrated model a better coordination of the operation of power plants and NG supply and transmission can be obtained.


## ACKNOWLEDGMENT

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