OPTIMAL POWER DISPATCH AND CONVERSION IN SYSTEMS WITH MULTIPLE ENERGY CARRIERS

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Abstract - This paper introduces a general optimization approach for power dispatch and conversion in power systems that include multiple energy carriers such as electricity, natural gas, and district heating. The classical Economic Dispatch method is modified in order to account for certain system properties, such as the possibility of conversion between the different energy carriers, or local overproduction and power feedback to the grid. In this work both a system model as well as an optimization approach are developed which are suitable for the integration of an arbitrary number of energy carriers. Analytical results show how the optimal conversion of power affects the marginal prices related to the different energy carriers. Finally the proposed optimization procedure is demonstrated in numerical examples.

Keywords - Power system optimization, multiple energy carriers, economic dispatch, distributed generation

1 INTRODUCTION

THE increasing utilization of gas-fired and other distributed generation (co- and trigeneration) is expected to have significant influence on both the technical and economical operation of power delivery systems. The conversions between the different energy carriers establish a coupling of the corresponding power flows, which result also in economic interactions. For example, a micro turbine can be used to produce both electricity and heat from natural gas, and such a device will then affect natural gas and electricity power flows as well as the heat supply of the load.

With different energy carriers available and the possibility of demand-side conversion between them, the customer gets flexible in supply. An electric load for instance can be supplied directly from the electric grid but also from a device that converts natural gas into electricity. This flexibility raises new questions concerning optimal system operation.

Figure 1 illustrates the situation for a customer connected to a number of power delivery networks via a so-called *energy hub*, which represents an interface between networks and loads [1]. A similar concept focusing on somewhat different aspects was developed by EPRI [2]. The different energy carriers provided at the input of the energy hub can be consumed directly from the networks or indirectly by converting the power into other forms. It is up to the customer to decide a) how much of which energy carriers to consume from the networks and b) how to convert the input powers in order to meet the load demand.

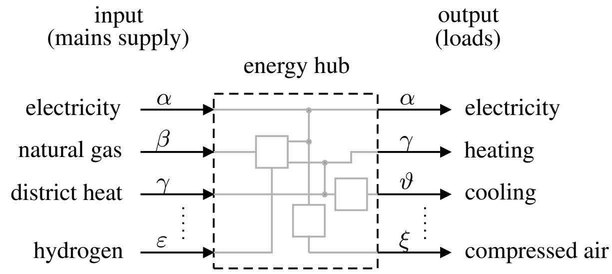

Figure 1: Power is demanded from the mains supply and converted in the energy hub in order to meet the load demand. $\alpha, \beta, \ldots, \xi$ are different energy carriers.

The related optimization problem is similar to the well-known Economic Dispatch (ED) problem [3]. The energy carriers offered at the input can be seen as generators with different cost structures. However, there are some significant differences which have to be considered in the formulation of this problem:

- *Power conversion*: The power demanded from the networks can be converted into other forms at the customer-side.
- *Local surplus*: Power conversion may exceed the local demand resulting in reverse power flow. With the sign of power flow also the sign of energy costs can reverse, i.e. the load can be paid for the power delivered back to the grids.
- *Converter by-products*: Many converter devices have multiple outputs, e.g. electricity and heat, which can normally not be controlled independently. The by-products, e.g. heat, can be used to support the corresponding load resulting in lower consumption from other sources.

A lot of research has been done on the operational optimization of power systems, in particular for electricity [3, 4], natural gas [5, 6], and district heating systems [7, 8]. More recently the combined optimization of different networks has been investigated [9, 10, 11], but—to our knowledge—no general optimization technique regarding the operation of mixed energy carrier systems has been published.

It is the objective of this paper to develop a generalized approach for optimal power dispatch and conversion in systems including an arbitrary number of energy carriers. In the following sections both a novel system model as well as an optimization approach based on this model are introduced and demonstrated in examples.

2 SYSTEM MODEL

Power flow within the hub is mathematically described by a simple steady-state model which is based on nodal power balance and only two physical values—power and energy efficiency [12].

2.1 Input-Output Notation

The conversion of power from one energy carrier α to another one β is characterized by a certain efficiency $\eta_{\alpha\beta}$:

$$P_\beta^{out} = \eta_{\alpha\beta} P_\alpha^{in} \tag{1}$$

where P_α^{in} is the power input, and P_β^{out} is the power output of the converter. A combination of converter devices as illustrated in figure 1 demands different forms of power at the input and delivers the power output to the loads. The conversion from the input to the output of the hub can be described with an *input-output coupling matrix* $\mathbf{C}$:

$$\underbrace{\begin{bmatrix} P_\alpha^{out} \\ P_\beta^{out} \\ \vdots \\ P_\zeta^{out} \end{bmatrix}}_{\mathbf{P}^{out}} = \underbrace{\begin{bmatrix} c_{\alpha\alpha} & c_{\beta\alpha} & \cdots & c_{\zeta\alpha} \\ c_{\alpha\beta} & c_{\beta\beta} & \cdots & c_{\zeta\beta} \\ \vdots & \vdots & \ddots & \vdots \\ c_{\alpha\zeta} & c_{\beta\zeta} & \cdots & c_{\zeta\zeta} \end{bmatrix}}_{\mathbf{C}} \underbrace{\begin{bmatrix} P_\alpha^{in} \\ P_\beta^{in} \\ \vdots \\ P_\zeta^{in} \end{bmatrix}}_{\mathbf{P}^{in}} \tag{2}$$

where $\alpha, \beta, \ldots, \zeta$ are members of a set of energy carriers $\mathcal{E}$. The entries of the coupling matrix are called *coupling factors*; these values depend on the converter efficiencies, the hub-internal topology and power dispatch. Note that converter efficiencies (and therefore also the coupling factors) are usually dependent on the power.

The input-output coupling matrix of an energy hub can be derived step by step:

1. Define input and output power vectors
2. Introduce *dispatch factors* at input junctions
3. Express converter outputs as functions of the inputs
4. State nodal power balance at output junctions
5. Formulate the results from 1.–4. in (2)

The procedure ist demonstrated in the following example.

2.2 Example

Consider the energy hub in figure 2. The input is connected to electricity, natural gas, and district heating networks. The load at the output requires electricity, compressed air, and heat. The hub contains three converter devices:

- A compressor device (C) producing compressed air from electricity. The waste heat is injected in the heating system.
- A combined heat and power plant (CHP) converting natural gas into electricity and heat.
- A furnace (F) burning gas and delivering heat.

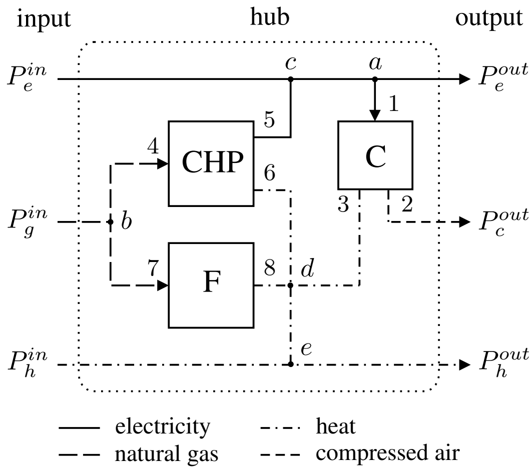

Figure 2: Example hub supplied with electricity, natural gas, and district heat at the input. The load connected to the output requires electricity, compressed air, and heat. Power conversion takes place in a combined heat and power plant (CHP), a compressor (C), and a gas furnace (F).

The coupling matrix for this converter configuration can be stated following the aforementioned procedure. Input and output power vectors can be defined as

$$\mathbf{P}^{in} = \left[\begin{array}{ccc} P_e^{in} & P_g^{in} & P_h^{in} \end{array}\right]^T \tag{3a}$$

$$\mathbf{P}^{out} = \left[\begin{array}{ccc} P_e^{out} & P_c^{out} & P_h^{out} \end{array}\right]^T \tag{3b}$$

There are two junctions at the input side of converters, in the electrical and in the natural gas system (nodes a and b). Both require the introduction of a dispatch factor which specifies the dispatch of power to the branches:

$$P_1 = \nu_1 \left(P_e^{in} + P_5\right) \tag{4a}$$

$$P_4 = \nu_4 P_g^{in} \tag{4b}$$

where ν_1 and ν_4 are the dispatch factors ($0 \le \nu_i \le 1 \; \forall \, i$). Now the converter outputs are expressed as the product of input and efficiency:

$$P_2 = \eta_{12} P_1 = \nu_1 \eta_{12} \left(P_e^{in} + P_5\right) \tag{5a}$$

$$P_3 = \eta_{13} P_1 = \nu_1 \eta_{13} \left(P_e^{in} + P_5\right) \tag{5b}$$

$$P_5 = \eta_{45} P_4 = \nu_4 \eta_{45} P_g^{in} \tag{5c}$$

$$P_6 = \eta_{46} P_4 = \nu_4 \eta_{46} P_g^{in} \tag{5d}$$

$$P_8 = \eta_{78} P_7 = (1-\nu_4) \eta_{78} P_g^{in} \tag{5e}$$

The efficiencies of converter devices are normally dependent on the converted power. This dependency can be regarded by replacing the constant efficiencies with functions $\eta_{\alpha\beta} = \eta_{\alpha\beta}\left(P_\alpha^{in}\right)$. In this example, efficiencies are assumed to be constant, but the power dependency is straightforward to include.

The next step is to formulate nodal power balance for every node at the output side of the converters (c, d, e). Conservation of power yields

$$P_e^{out} = (1-\nu_1)\left(P_e^{in} + \nu_4 \eta_{45} P_g^{in}\right) \tag{6a}$$

$$P_c^{out} = P_2 = \nu_1 \eta_{12} \left(P_e^{in} + \nu_4 \eta_{45} P_g^{in}\right) \tag{6b}$$

$$\begin{aligned} P_h^{out} &= P_h^{in} + \nu_1 \eta_{13} \left(P_e^{in} + \nu_4 \eta_{45} P_g^{in}\right) \\ &\quad + P_g^{in} \left(\nu_4 \eta_{46} + (1-\nu_4)\,\eta_{78}\right) \end{aligned} \tag{6c}$$

Finally, the power coupling from the input to the output can be written according to (2):

$$\begin{bmatrix} P_e^{out} \\ P_c^{out} \\ P_h^{out} \end{bmatrix} = \underbrace{\begin{bmatrix} 1-\nu_1 & (1-\nu_1)\,\nu_4\eta_{45} & 0 \\ \nu_1\eta_{12} & \nu_1\nu_4\eta_{12}\eta_{45} & 0 \\ \nu_1\eta_{13} & \begin{matrix}\nu_4\eta_{46} \\ +(1-\nu_4)\,\eta_{78} \\ +\nu_1\nu_4\eta_{13}\eta_{45}\end{matrix} & 1 \end{bmatrix}}_{\mathbf{C}} \begin{bmatrix} P_e^{in} \\ P_g^{in} \\ P_h^{in} \end{bmatrix} \quad (7)$$

With given loads P_e^{out}, P_c^{out}, and P_h^{out}, both the input powers P_e^{in}, P_g^{in}, P_h^{in} as well as the dispatch factors ν_1 and ν_4 are subject to optimization. After the optimization is performed, the converter inputs can be calculated from the result.

3 OPTIMIZATION APPROACH

3.1 Assumptions

Some basic assumptions and simplifications are made at this point in order to keep the optimization problem formulation simple:

- Polynomial cost curves for each energy carrier. This is a common assumption in electricity and natural gas networks [3, 9]. However, it is possible to introduce other functions for the optimization.
- The cost of the energy carriers are independent of each other
- Inelastic loads
- Within the hub, losses occur only in converter devices
- Constant converter efficiencies

For general investigations in the system behavior, a model based on these assumptions is believed to provide sufficient accuracy. If the simplified model cannot yield the desired features, then the assumptions above can be relaxed resulting in more accurate and complex equations.

3.2 Problem Formulation

The optimization problem is basically stated by three relations. An objective function accounts for the cost of power demanded from the networks. Physical laws result in an equality constraint, and technical limitations require the introduction of inequality constraints.

The cost function reflects the cost of power[1] input, which are to be minimized. The costs for demand of a certain energy carrier α are modelled as polynomials of the corresponding power. For negative flows, the hub is paid for the delivered energy. In order to account for that, the variable cost for power depend on the direction of power flow whereas the fixed cost are constant (see example cost curve in figure 3):

$$C_\alpha\left(P_\alpha^{in}\right) = a_{\alpha,0} + \begin{cases} \sum\limits_{m=1}^{M_\alpha} a_{\alpha,m}\left(P_\alpha^{in}\right)^m & \text{if } P_\alpha^{in} \geq 0 \\ \sum\limits_{n=1}^{N_\alpha} b_{\alpha,n}\left|P_\alpha^{in}\right|^n & \text{else} \end{cases} \quad (8)$$

where

- C_α are the cost related to the energy carrier α in monetary units (mu)
- P_α^{in} is the power flowing into the hub input α in power units (pu)
- $a_{\alpha,0}$ represents fixed costs related to α in mu/pu
- $a_{\alpha,m}$ are cost coefficients for *demand* of α in mu/pu^m
- $b_{\alpha,n}$ are cost coefficients for *delivery* of α in mu/pu^n
- M_α, N_α are the orders of the cost polynomials for demand and delivery, respectively

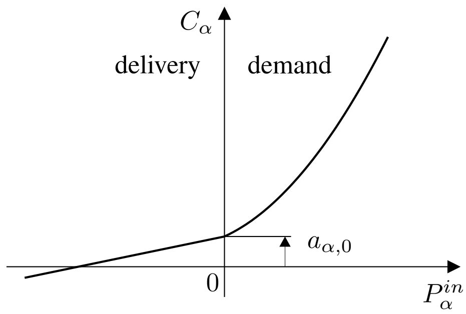

Figure 3: Example of a cost function (8) with $M_\alpha = 2$, $N_\alpha = 1$. The variable costs are dependent on the direction of power flow.

Conversion of power yields an equality constraint which can be formulated as (2). This relation accounts for the conversion of power between the input and the output of the hub. As mentioned, the inequality constraints are given by the power limitations of the converters and the connections to the grid. Both lower limits (flame out, stand-by) and upper limits (rating) have to be considered.

Finally, the optimization problem can be stated as:

$$\begin{aligned} &\text{minimize} \quad C\left(\mathbf{P}^{in}\right) = \sum_{\alpha\in\mathcal{E}} C_\alpha\left(P_\alpha^{in}\right) \\ &\text{subject to} \quad \begin{cases} \mathbf{P}^{out} = \mathbf{C}\,\mathbf{P}^{in} \\ \widetilde{\mathbf{P}}_{min}^{in} \leq \widetilde{\mathbf{P}}^{in} \leq \widetilde{\mathbf{P}}_{max}^{in} \end{cases} \end{aligned} \quad (9)$$

where

- $\mathcal{E}$ is the set of energy carriers taken into account
- C are the total costs for energy demand
- C_α are the individual costs according to (8)
- $\mathbf{P}^{in}, \mathbf{P}^{out}$ are the vectors of power inputs and outputs, respectively

[1] Actually the cost for energy per unit of time, e.g. MWh/h are considered.

- $\mathbf{C}$ is the coupling matrix according to (2)
- $\widetilde{\mathbf{P}}^{in}_{min}, \widetilde{\mathbf{P}}^{in}_{max}$ contain the minimal and maximal power values for each network connection and converter input
- $\widetilde{\mathbf{P}}^{in}$ contains the actual power values of each network connection and converter input

3.3 Central versus Decentral Dispatching

The operation of power systems can be controlled centrally or locally. So-called self dispatchers can optimize their consumption without regarding the rest of the system, resulting in a certain demand which is optimal for this particular hub. In a centralized dispatch procedure, the converters can be operated in order to meet an objective related to the overall system performance, resulting in a different optimum.

The way power is dispatched comes finally down to a question of market organization. The optimization procedure outlined in this paper is applicable to different scenarios. For self-dispatching hubs, the objective function in (9) is appropriate; if the hubs are centrally dispatched, a different objective function accounting for the total power consumption in the system can be applied.

3.4 Type of Problem and Solution

The statement in (9) represents a non-linear, multi-variable, and inequality-constrained optimization problem. Note that the cost functions (8) are continuous,[2] which is an important property for the application of numerical optimization methods. The convexity of (8) depends on the parameters of the cost function $a_{\alpha,m}$ and $b_{\alpha,n}$.

This class of problem can be solved using Nonlinear Programming algorithms [13]. However, it is not the aim of this paper to investigate mathematical optimization methods, therefore commercially available optimization software is used (in particular the Matlab function `fmincon` [14]).

4 GENERAL RESULTS

Some general results which can be obtained from analytical considerations are discussed in the following part.

4.1 Marginal Prices

The Lagrange function for an equality-constrained optimization problem defined by (9) is[3]

$$L = C\left(\mathbf{P}^{in}\right) + \lambda\left[\mathbf{P}^{out} - \mathbf{C}\,\mathbf{P}^{in}\right] \tag{10}$$

The dimension of λ corresponds to the number of energy carriers available at the output. The first-order optimality conditions due to Karush-Kuhn-Tucker (KKT) require that the partial derivatives of the Lagrange function are zero [4, 13]:

$$\frac{\partial L}{\partial \mathbf{P}^{in}} = \frac{\partial C}{\partial \mathbf{P}^{in}} + \lambda \frac{\partial}{\partial \mathbf{P}^{in}}\left[\mathbf{P}^{out} - \mathbf{C}\,\mathbf{P}^{in}\right] = \mathbf{0} \tag{11}$$

The sensitivity of the cost function in the optimum with respect to the input powers can be derived from (11). This sensitivity is commonly called ”incremental cost”, ”shadow price”, or ”marginal price”; the vector containing the marginal prices of all energy carriers is defined as

$$\frac{\partial C}{\partial \mathbf{P}^{in}} \triangleq \lambda' = \lambda \mathbf{C} \tag{12}$$

where

- $\lambda' = \left[\begin{array}{cccc} \lambda'_\alpha & \lambda'_\beta & \ldots & \lambda'_\zeta \end{array}\right]$ represents the marginal prices of the energy carriers offered at the input side of the hub in mu/pu. These marginal prices are the same for all hubs connected to the grid. The entries of λ' are therefore called *System Marginal Prices (SMP).*
- $\lambda = \left[\begin{array}{cccc} \lambda_\alpha & \lambda_\beta & \ldots & \lambda_\zeta \end{array}\right]$ represents the marginal prices at the load side of the hub named *Hub Marginal Prices (HMP)* in mu/pu.

The first row in the matrix equation (12) is

$$\lambda'_\alpha = \lambda_\alpha c_{\alpha\alpha} + \lambda_\beta c_{\alpha\beta} + \ldots + \lambda_\zeta c_{\alpha\zeta} \tag{13}$$

It can be observed that the SMP of an input α depends on how it is converted into other forms. Although SMP and HMP may differ, the total energy cost are equal on both sides of the hub (input and output), since power flows are different.

Equation (12) is a necessary requirement for optimal dispatch and conversion and therefore only valid if the hub is operated optimally.

4.2 Power Flow and Marginal Price Coupling

A comparison of (2) and (12) shows that the coupling matrix $\mathbf{C}$ describes both the coupling of power as well as the coupling of marginal prices from the input to the output of the hub. This analogy gives the idea of power flow coupling and related marginal price coupling due to power conversion between different power systems.

If no power is converted to another form, then the input-output coupling is described by a unity matrix ($\mathbf{C} = \mathbf{I}$). In this case power as well as the marginal costs are equal on all sides of the hub. If conversion takes place in the hub, then the coupling matrix differs from a unity matrix ($\mathbf{C} \neq \mathbf{I}$), and power as well as marginal prices are related in a similar way:

$$\begin{array}{lll} \text{no conversion:} & \mathbf{P}^{out} = \mathbf{P}^{in} & \Leftrightarrow \quad \lambda' = \lambda \\ \text{conversion:} & \mathbf{P}^{out} = \mathbf{C}\,\mathbf{P}^{in} & \Leftrightarrow \quad \lambda' = \lambda\mathbf{C} \end{array}$$

Note that the power input changes if the conversion (i.e. $\mathbf{C}$) is changed, whereas the output stays constant. A change in input (network consumption) changes the marginal prices (SMP and HMP) as well.

[2] See example in figure 3: $\lim_{P^{in}_\alpha \to 0^+} C_\alpha = \lim_{P^{in}_\alpha \to 0^-} C_\alpha = C_\alpha(P^{in}_\alpha = 0) = a_{\alpha,0}$

[3] The inequality constraint is not considered here.

4.3 General Dispatch Rule

It is well known that generators in an electrical network can be optimally dispatched by operating them on equal incremental cost [3].[4] Considering multiple energy carriers and conversion between them, a comparable rule for the marginal prices is given by equation (12).

As shown in figure 4, the different energy carriers supplying a load can be interpreted as generators, but it has to be regarded that they feed the load through converters characterized by certain efficiencies. If the load P_α^{out} is supplied via a number of converters with different inputs $P_\alpha^{in}, P_\beta^{in}, \ldots, P_\zeta^{in}$ and related coupling factors[5] $c_{\alpha\alpha}, c_{\beta\alpha}, \ldots, c_{\zeta\alpha}$, then the hub and system marginal prices (λ and λ' respectively) are related to each other as follows (see figure 4):

$$\lambda_\alpha = \frac{\lambda'_\alpha}{c_{\alpha\alpha}} = \frac{\lambda'_\beta}{c_{\beta\alpha}} = \ldots = \frac{\lambda'_\zeta}{c_{\zeta\alpha}} \tag{14}$$

This is an optimality condition for a system with one output, as pictured in figure 4. It means that power can be dispatched optimally by operating all inputs on *equal ratio of system marginal price to coupling factor*.

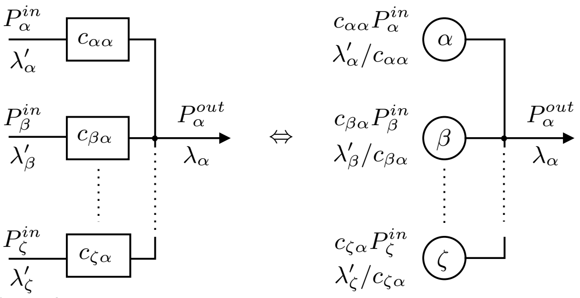

Figure 4: Left: A load is supplied by conversion of multiple energy carriers. Right: Model with equivalent generator powers and marginal prices.

Consider again the converter configuration in figure 4. Due to (14) the marginal price of an input has to be low when the related efficiency is low; low marginal price corresponds to low power. This is a reasonable result: Inputs which are converted with low efficiency are less utilized than those with higher efficiency (assuming equal cost coefficients).

5 EXAMPLES

5.1 Micro Turbine

Consider the simple hub in figure 5 that contains electrical and thermal connections as well as a micro turbine (MT) which converts gas (e.g. natural gas or hydrogen-based gas) into electricity and heat, with efficiencies of 35% and 40%, respectively. The hub has to supply an electrical load of 50 kW and a thermal load of 150 kW. Power can either be taken directly from the electricity and district heating networks or indirectly by utilizing the MT. The prices for the energy carriers offered at the hub input are given in table 1.

[4]With network losses neglected.

[5]In this case the coupling factors are equal to the corresponding efficiencies.

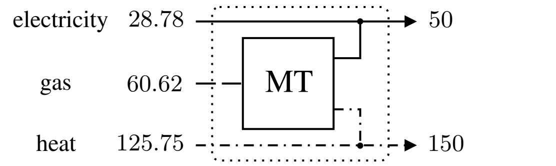

Figure 5: Simple hub with micro turbine (MT) in optimal operation mode (values in kW).

The hub input powers are optimized with respect to minimum cost of network demand. The resulting power flow and marginal price couplings are stated according to (2) and (12):

$$\underbrace{\begin{bmatrix} 50 \\ 0 \\ 150 \end{bmatrix}}_{\text{power output in kW}} = \underbrace{\begin{bmatrix} 1 & 0.35 & 0 \\ 0 & 0 & 0 \\ 0 & 0.40 & 1 \end{bmatrix}}_{\mathbf{C}} \underbrace{\begin{bmatrix} 28.78 \\ 60.62 \\ 125.75 \end{bmatrix}}_{\text{power input in kW}} \tag{15}$$

$$\underbrace{\begin{bmatrix} 0.1576 & 0.1718 & 0.2915 \end{bmatrix}}_{\text{SMP in €/kW}} = \\ = \underbrace{\begin{bmatrix} 0.1576 & 0 & 0.2915 \end{bmatrix}}_{\text{HMP in €/kW}} \underbrace{\begin{bmatrix} 1 & 0.35 & 0 \\ 0 & 0 & 0 \\ 0 & 0.40 & 1 \end{bmatrix}}_{\mathbf{C}} \tag{16}$$

In the electrical and thermal system, SMP and LMP are equal since there are direct, lossless connections to the networks. The natural gas consumption is adjusted so that the related SMP fulfills the optimality condition (in €/kW):

$$0.1718 = 0.1576 \times 0.35 + 0.2915 \times 0.40 \tag{17}$$

In this particular case, the optimally operated MT reduces the variable energy cost by 13% compared with normal network demand.

5.2 Industrial Supply

In the following the proposed optimization technique is applied to the hub shown in figure 2, which has already been discussed in section 2.2. This hub should supply an industrial load which demands electricity, compressed air, and heat. The hub loads are characterized by a certain daily load profile which is pictured in figure 6 (average power during one hour). The optimization is performed for every hour of the day. The assumed energy price coefficients and converter data are given in tables 1 and 2, respectively. Converter devices are limited with their ratings, and the district heat supply from the network should not exceed 250 kW. The operation of the energy hub is now optimized in terms of energy cost.

Table 1: Coefficients for the cost functions (8), with $M_\alpha = 2$, $N_\alpha = 1$.

energy carrier	$a_{\alpha,0}$ in €	$a_{\alpha,1}$ in €/kW	$a_{\alpha,2}$ in €/kW^2	$b_{\alpha,1}$ in €/kW
electricity	100	0.10	0.001	-0.07
natural gas	100	0.05	0.001	0
district heat	100	0.04	0.001	0

Table 2: Converter efficiencies and ratings.

converter device	efficiency in %	output rating in kW
C	25/65 (pressure/heat)	80 (pressure)
CHP	35/35 (electrical/thermal)	130 (electrical)
F	50	150

Table 3: Change in hub demand due to doubled price factor.

doubled parameter	change in daily consumption in % electricity	natural gas	district heat
$a_{e,1}$	-2.8	3.2	-0.9
$a_{g,1}$	0.7	-1.7	0.7

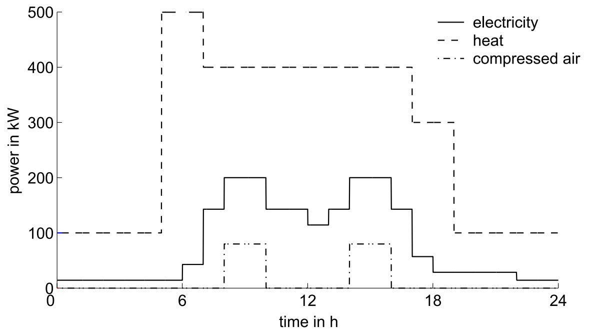

Figure 6: Daily load profile (hourly mean values).

The results for optimal operation are shown in figure 7. The thermal load peak is visible in the natural gas demand of the hub. In this period (5–7 h), the CHP and the furnace are utilized in order to produce heat. The resulting surplus electricity is delivered back to the grid. The compressor cannot be used in this case, as there is no possibility to utilize compressed air. Whenever the compressor is operating (8–10 h, 14–16 h), the heat demand from the network reduces since 65% of the compressor input are converted to heat. Although the electric consumption could be covered from the network only, the CHP is used at these times to reduce the peak demand of electricity which is caused by the compressor.

Table 3 gives results of a sensitivity study of price variations. If the linear price factor for electricity $a_{e,1}$ is doubled, then the network consumption of electricity *and* district heat decreases, since the CHP, which compensates for expensive electricity from the grid, generates heat as well. The demand of natural gas increases by 3.2%. If the corresponding parameter for natural gas $a_{g,1}$ is changed instead, we can observe an inverse variation of the network demand. In this case, less electricity and heat are generated within the hub, resulting in a reduction of the natural gas input of 1.7%. Note that in this study the price change in the electrical system had more influence on the optimal consumption of natural gas than an equivalent price manipulation of the natural gas price itself. This is caused by the electrical efficiency of the CHP. A part of the electrical power is generated from gas with an efficiency of 35%. If the electrical consumption from the network is decreased by 1.00 pu, then 2.86 pu more gas is needed for compensating with the CHP device.

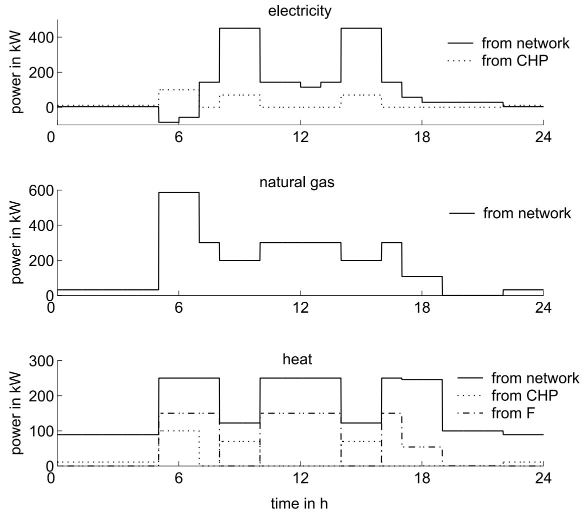

Figure 7: Results for optimal supply of the hub (hourly mean values).

6 FUTURE WORK

Current and future work is dedicated to the optimization of hub networks. Optimal power flow problems can be stated by including network losses. For calculating losses and penalty factors, a combined power flow model is developed that includes the different energy carriers.

Integrating storage in the models is another task for the future, as the combination of storage with renewable sources could be beneficial in terms of system operation. The possibility to use the gas network as a storage system is of particular interest.

In this paper, the objective function was stated for energy cost. Possible other criteria can be defined due to environmental impact (as outlined in [15, 16]) or technical characteristics such as peak power demand. The inclusion of reliability/security aspects (either as an objective or as a constraint) is also needed for a complete system assessment.

7 DISCUSSION AND CONCLUSION

This paper describes a general approach for the optimization of power demand and conversion in energy systems including different energy carriers. In a first step, a novel system model is introduced that enables the description of conversion of power as an input-output coupling. In the second step, the optimization problem is formulated similar to the classical ED approach. The objective function accounts for both power demand and delivery (demand-side surplus production). Equality and inequality constraints are stated regarding the possibility of power conversion between the different energy carriers. Analysis of the KKT optimality conditions yields a relationship for the marginal prices in the system which gives fundamental insight to economic impacts of power conversion. The analogy of power flow coupling and marginal price coupling is discussed and the well-known dispatch rule of equal marginal cost is re-stated in order to account for the use of multiple energy carriers and their conversion. Finally, the developed technique is demonstrated in examples.

The proposed dispatch method can be applied to small distributed resource scheduling as well as for optimizing bulk power systems such as transnational pipeline and electricity networks (as done in [11]).

Numerical results of the optimization clarify the mechanisms taking place in multiple energy carrier systems. Sensitivity analysis can be performed in order to determine the value of certain system components. An approach for the economic evaluation of converter devices could be based on marginal price analysis.

The optimization approach does not necessarily require an equality constraint in the form (2). Alternatively, power flow from the input to the output of the hub can be formulated in a different way, resulting in different equations. The proposed model enables to state the input-output coupling in the same way for any hub configuration. Another advantage of the model (2) is that the marginal price relation between inputs and outputs can be clearly formulated as (12). This equation enables investigations in marginal price interactions.

An assumption to be discussed is that the cost for the energy carriers are separable. This can be justified as long as for example the electricity price is not too much dependent on the natural gas price, what might be true for example in Switzerland, where almost no electricity is produced from gas. Actually the presented model can be extended to include mutual price influences by modeling the whole chain of power conversion, from the primary energy source to the end user.

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