A METHOD FOR EXTRACTING RELIABILITY IMPORTANCE INDICES FROM RELIABILITY SIMULATIONS OF ELECTRICAL NETWORKS

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Abstract – This paper proposes a reliability importance index that is possible to extract from existing reliability simulations at a low additional computational cost. The index utilizes the concept of reliability worth as a measure of system reliability in order to establish the importance of components in systems with several load points. Since the method is based on simulations, it is suitable for large networks with a high level of detail. The index can be used as decision support for asset management, for example where maintenance actions will become most beneficial. The index is evaluated against the background of a number of analytically calculated indices. Furthermore, the index is applied to a network in the Stockholm area. The conclusion of the paper is that the proposed simulation based importance index provides a means of improving analysis of electrical network reliability.

Keywords: reliability importance index, Monte Carlo simulation, electrical network, customer interruption cost, reliability worth

1 INTRODUCTION

In the search for the best possible asset management strategy for electrical networks it is essential to know the importance of components involved. The importance of components can be obtained through the use of reliability importance indices. The indices can be used for prioritizing components; one example is to determine where maintenance actions will have the greatest effect. Taken further, these indices can be used in the optimization of maintenance from a system reliability perspective, which is one of the major goals for asset management of electrical networks.

This paper begins with a brief comparison/evaluation of a number of reliability importance indices, that is Fussell-Vesely's and Birnbaum's component reliability importance, the criticality index and two newly developed indices (I^H and I^{MP}) [1]. These newly developed indices define the reliability importance of individual components with respect to the reliability worth [2], i.e. total interruption cost. Except for I^H and I^{MP}, the above-mentioned indices are aimed at systems that can be modeled as networks with components in serial and/or parallel couplings between two nodes. This is a restriction that is not generally applicable for an electrical network. This motivates the indices I^H and I^{MP}, which can be used to evaluate several nodes (supply and load points). However, all of the above mentioned importance indices are based on analytical techniques. For more complicated systems, for example with repairable components and time varying failure rates, these indices become very complicated to solve with analytical techniques. A suitable alternative to the analytical approach is simulation techniques.

The major contribution of this paper is a further development of the concept of I^H and I^{MP} by extending the method with the use of the Monte Carlo simulation technique. Hence, enabling to solve more complicated models compared to the analytical approach. One of the advantages of the method is that it is possible to implement in already existing network reliability simulations. This is achieved by keeping record of a relatively small number of additional data from the simulations. The major advantages are ease of implementation and that the method becomes computationally cost effective. The proposed simulation based index is evaluated against generally known indices and discrepancies are discussed.

Notation and index:

h System reliability.
p_i Component reliability.
D At least one minimal cut set containing component i is failed.
J System failed.
i Component number.
s System.
λ Failure rate [f/yr.].
C_s Expected total yearly interruption cost [SEK/yr].
K_i System cost caused by component i [SEK].
T Total simulation time.
n_i Number of system failures caused by component i.
N Total number of system failures.

2 ANALYTICAL BASED INDICES

This section is dedicated to a short description and analysis of three generally known and analytically based indices and two analytically based indices that utilize reliability worth as a measure of reliability.

2.1 Birnbaum's reliability importance, I^B

Birnbaum's measure of component importance is a partial derivative of system reliability with respect to individual component failure rate [3]. It can be argued that this method is a form of sensitivity analysis. The index gives an indication of how system reliability will

change with changes in component reliability. Birnbaum's reliability importance is defined as:

$$I_i^B(t) = \frac{\partial h(\mathbf{p})}{\partial p_i} \quad (1)$$

where h is the system reliability depending on all component reliabilities $\mathbf{p}$ (and system structure) and p_i component i's reliability. A drawback with the method is that the studied component's reliability does not affect the importance index (for the studied component). Noteworthy is the relationship with Birnbaum's structural importance. The structural importance can be calculated from I^B by setting all component reliabilities to ½ (p=0.5) [3].

One issue regarding this index is that it cannot be used in order to predict several changes at the same time, i.e. reliability changes in several components at a time [4]. However, the index can be used to determine effects of changes, which is not possible for all indices.

2.2 Fussell-Vesely's measure of importance, I^{FV}

Given system failure, Fussell-Vesely's measure of component importance [3] is the probability that at least one failed minimal cut set contains the studied component, as defined by:

$$I_i^{VF} = \frac{P(D)}{P(J)} \quad (2)$$

where $P(D)$ is the probability that at least one minimal cut set containing component i is failed and $P(J)$ is the probability that the system is failed. An interpretation of this index is the answer to the question: If the system fails, what is the probability that the studied component will be involved in the failure? A drawback with Fussell-Vesely's index is that it does not take into account the component's contribution to system success [5].

2.3 Criticality Importance, I^{CR}

The criticality importance, I^{CR}, is based on I^B, but places the focus on the component's criticality for the system. Given a failed system at time t, this is the probability that component i is critical for the system and is failed at that time [3]. That is the probability that the system is failed because of component i's failed status at time t. The criticality importance is suitable for preventive maintenance decisions since it puts focus on probable events rather than final events (which cause system failure). The index is defined as follows:

$$I_i^{CR} = \frac{I_i^B(1-p_i)}{P(J)} \quad (3)$$

where I^B is defined in (1), p_i component i's reliability and $P(J)$ is the probability that the system is failed.

2.4 The interruption cost based importance index, I^H

The concept of I^H is to study the interruption cost with respect to component reliability [1]. The method is based on the concept of I^B, which is extended for assessment of multiobjective networks (e.g. networks that serve several load points). I^H uses total interruption costs instead of probabilities as a measure of system reliability (the interruption costs do however depend on probabilities). Note that the analysis is performed on component failure rate instead of component reliability. The interruption cost based index is defined as follows:

$$I_i^H = \frac{\partial C_s}{\partial \lambda_i} \qquad [SEK/f] \quad (4)$$

where C_s [SEK/yr] is total yearly customer interruption cost and λ_i [f/yr] component i's failure rate. The index identifies components that are critical for the system with respect to their individual impact on total interruption cost with changes in component failure rate [1]. One interpretation of I^H is that it corresponds to the total expected interruption cost (for all load points) that would occur if component i failed. Hence, if there were one maintenance action available, which would result in the same absolute change in failure rate for any component in the network.. I^H would then be the natural index to use for a prioritization of what component the action should be performed on.

The index is focused on failure rate. Reliability importance measures are generally focused on component availability (i.e. failure and repair rate combined). To apply the concept of I^H to repair times might prove to be more straightforward than failure rates and would complement I^H. This is due to one interesting aspect: in general it is easier to estimate how repair time changes with different actions than how maintenance actions affect the failure rate, and hence predicted system effects of these repair rate related actions might be more precise.

2.5 Maintenance potential, I^{MP}

Analogous with Birnbaum's importance index I^H is not affected by the studied component's failure rate but "only" by component repair time and the position of the component and all other components in the system. Hence, the concept of maintenance potential [1] is introduced. Maintenance potential corresponds to the total expected yearly cost that is incurred by the specific component's failures. Another interpretation of this measure is the expected system cost reduction that would occur in the case of a perfect component, i.e. no failures for the studied component (hence maintenance potential). Another way to express this measure is the expected total interruption cost that the studied component will cause (alone or together with other components) during one year.

Maintenance potential is defined as:

$$I_i^{MP} = I_i^H \lambda_i \qquad \text{[SEK/yr]} \qquad (5)$$

where I^H [SEK/f] is defined in (4) and λ [f/yr] component i's failure rate.

2.6 Concluding remark

The reliability worth approach distinguishes I^H and I^{MP} from the more classical indices in more ways than just the multiobjective approach. One additional major difference is that the initiation of an interruption can be penalized and that the length of an interruption does not necessarily have to have linear consequences with respect to time.

3 A TEST SYSTEM

In order to further evaluate the presented indices a small test system is analyzed. Figure 1 displays the topology of the system. The model assumes faultless automatic breakers, which isolates failures without affecting the rest of the network. Even though the model is simple, its properties will prove difficult for the most commonly used reliability importance indices. The test system has independent components with exponentially distributed failure and repair times. Tables 1 and 2 present data used for the system. Minimal cut sets are defined for the systems as follows: {1,2} for load point 1 and {3} and {1,2} for load point 2.
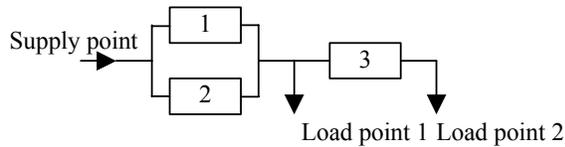
Figure 1: Test system.

Component number	Failure Rate [failures/yr]	Mean Down Time [h/failure]	Component type
1	0.02	480	Transformer
2	0.03	480	Transformer
3	0.01	6	Cable

Table 1: Component reliability input data for the test system.

Load point	kW	SEK /kW	SEK /kWh	SEK /inter.	SEK/h	Type of customer
LP1	10000	15	60	150000	600000	Light industry
LP2	10000	10	35	100000	350000	Agricultural

Table 2: Load point input data.

Table 3 presents the evaluated indices for the test system. Birnbaum's, Fussell-Vesely's and the criticality indices are all calculated for each load point, since they only are applicable to two state systems.

	Component number		
Index	1	2	3
I^B LP1	0.001644	0.001096	x
I^B LP2	0.001644	0.001096	0.999998
I^{FV} LP1	1	1	x
I^{FV} LP2	0.208243	0.208243	0.791758
I^{CR} LP1	1	1	x
I^{CR} LP2	0.208241	0.208241	0.791756
I^H [SEK/f]	750408	500272	2199996
I^{MP} [SEK/yr]	15008	15008	22000

Table 3: Reliability importance indices, 'x' denotes that the index is not applicable for the studied component and load point.

Table 3 shows that all indices identify component 3 as the most important component (for I^B, I^{FV} and I^{CR} from the load point 2 perspective). It is interesting to note that I^B "values" component number 3 by a factor 10^3 more than the other components (for load point 2), while I^{FV}, I^{CR} and I^H place all of the components in approximately the same relative range and I^{MP} almost equals all three components. One important thing to note for both Birnbaum's index and I^H is their emphasis on component 1 compared to component 2, due to the failure rate which is 50% higher for component 2. Both these indices points out that the system depends more on component 1 than on 2. That is, the system depends more on the more reliable component in a parallel setup. However, from a maintenance perspective this result might be somewhat misleading, since improvements in absolute numbers are more likely to be found for the weaker component. On the other hand I^{FV}, I^{CR} and I^{MP} all values both the components in parallel equal, which of course also is sound, since both the components have to fail in order to fail the system.

4 IDENTIFIED COMMON PROBLEMS FOR RELIABILITY IMPORTANCE INDICES

Most importance indices, like those presented in *2.1-2.3*, are created for two state system models. This is a system perspective that is not always suitable for electrical networks, since they usually have several supply and load points (which can function more or less independently). In this sense transmission and distribution networks can be stated to be multi-objective, with several goals to fulfill, e.g. to deliver energy to all load points. One approach to the problem of calculating component importance for networks with multiple supply and load points is to study small parts of the network at a time, e.g. by studying a customer load point at a time. By dividing the network into these smaller groups it becomes possible to calculate traditional component importance indices. One problem with this approach is how to determine the importance relationship between components in different branches as well as for shared components. One solution to the problem is to use I^H and/or I^{MP}, which are indices that utilize customer

interruption costs as a measure of reliability. The concept of customer interruption costs as a measure of reliability, also recognized as reliability worth, can be found in [2].

The presented analytically calculated indices have a number of common properties regarding model detail depth. To a certain extent the analytical approach can adapt time varying failure and repair rates. Nevertheless, the analytical approach becomes inherently hard to calculate for variations due to events in the network. Events that for example include complex reliability dependencies. To model large and complex structures with an analytical approach is a complicated procedure, but definitely possible as for example shown with the reliability tools RADPOW [6] and AREP [7]. To produce the derivative of models built with these tools with respect to component reliability, for example in order to calculate I^B and I^H, is even harder. One approach to a numerical estimate of the derivative is to perform a sensitivity analysis on the models with respect to the component reliability, for example performed in [1], which is a computational costly approach.

To conclude this section, we state that the above described indices have their limited capability to capture the reality of electrical networks and if these indices are to be produced with the help of analytically based reliability tools, for electrical networks, a computational costly sensitivity analysis has to be performed. Most of the difficulties mentioned in this section can be addressed with simulation techniques, maybe with the exception of computation time.

5 SIMULATION BASED INDICES

In this section we will discuss one recently developed index and propose one new index. Numerical results for both these indices could easily be gained as results from reliability simulations. For some already existing simulation software these indices should be easily implemented.

5.1 Failure criticality importance index, I^{FC}

The failure criticality index (I^{FC}) was developed in order to obtain a reliability index from already existing reliability simulation routines. The basic concept is to divide the number of system failures caused by component i in (0,t) with the number of system failures in (0,t) [8], as defined in:

$$I_i^{FC} = \frac{n_i}{N} \qquad (6)$$

where n_i is the number of system failures caused by component i and N is the total number of system failures. "Caused" should here be interpreted as; the final event that fails the system. For example if we study the test system and if component 1 fails followed by component 2, that also fails, n_2 and N would be incremented with 1, and n_1 would not be incremented.

One of the major advantages with this method is that it calculates a component reliability importance index from simulations at a small additional cost in computation time. The method does not require any extra simulation cycles, only logging of failures for system and components (i.e. n and N).

The authors of I^{FC} also propose another related measure where the denominator is replaced with the number of the studied component's total failures in (0,t). This alternative gives an indication of the percentage of component failures that are critical for the system.

One important characteristic of this index is that it measures the importance in number of failures. This is a different approach than the classical availability based indices. For I^{FC} one failure is as important as another, even if the failures bring the system down for different durations. This is addressed with more indices, indices that take time into consideration [8].

The index has been applied to a distributed control system designed for a power generation plant [8].

5.1.1 Applicability for electrical networks

I^{FC} has a potential use in electrical networks, the major reason for this is that the index is developed (and suitable) for large systems with a degree of complexity that makes analytical solutions hard to obtain [8]. Another reason is that simulation methods are already developed for many electrical networks and that this index most likely is reasonably easy to implement into the simulation methods. However this index shares the problem with the classical reliability indices, i.e. it is not developed for multiobjective systems (several load points).

5.1.2 I^{FC} applied to the test system

The index has been applied to the test system and the results of the simulation are presented in Table 4. The small differences in I^{FC} between component 1 and 2 are an effect of the simulation, they share the same expected value (which is an effect of that they have equal average repair times). Note that this equal importance between component 1 and 2 is not a shared property with Fussell-Vesely and the criticality indices. If we for example would reduce the expected repair time drastically for component 1, the primary result of this reduction would be a drop of importance of component 2 (failures of component 2 would not cause as many system failures). This should be compared with I^{FV} and I^{CR} which both would change the importance for both the components to the same lower value. This example emphasizes the special characteristic of I^{FC}, that is the focus on the causing component, in this example the more reliable component is regarded as more important.

	Component number		
Index	1	2	3
I^{FC} LP1	0.498	0.502	0.0032
I^{FC} LP2	0.0032	0.0033	0.9935

Table 4: Reliability importance index I^{FC}. Note that the sum for each column equals 1.

5.2 Proposed new index, I^M

This index is based on the concepts of I^H and I^{MP}, combined with the failure criticality index. It is an index that is derived from simulations that calculate customer interruption costs. The idea is to achieve this at a low additional calculation cost, by keeping track of a relatively low number of events (component failures and related system costs).

The index, I^M, is calculated by designating the total interruption cost caused by an interruption to the finally causing component, i.e. if the component is the final cause of failed delivery to load point(s), the studied component is held responsible for the whole interruption cost. The accumulated cost over time for the component is then divided with the total simulation time in order to get an expected interruption cost per time unit (year). The index is defined as follows:

$$I_i^M = \frac{K_i}{T} \qquad [\text{SEK/yr}] \qquad (7)$$

where K_i is the total accumulated interruption cost over the total simulation time T for component i.

The interruption cost perspective of the index allows us to identify the components that are likely to cause the most costs in terms of interruption. Hence, the index gives us an indication of what components should be prioritized for maintenance actions (or in some cases re-design of the structure that results in the high value of I^M). Moreover, I^M gives information on components that do not cause much interruption cost for the network. It might be beneficial to reduce preventive maintenance for these components. It is however important to note that a relatively low value of I^M might be due to low component failure rate and that the network (total inter-ruption cost) might be sensitive for small changes in these failure rates. Hence precaution should be taken regarding what components that get reduced attention.

It might seem somewhat unreasonable to formulate an index as previously defined i.e. by holding the component that trip the (sub)system responsible for the whole event, as similarly defined in *5.1*. Nevertheless, since simulation-runs generally include many events, this should not be an issue. However, the major reason for just "blaming" one component is that the measure becomes non-ambiguous. Consequently minimal cut sets are not needed in order to calculate I^M. In a com-

plex network with advanced mechanisms it might not be possible to deduce minimal cut sets. Hence, for a more complex system the proposed index, I^M, might be a suitable measure.

5.2.1 Alternative approach

One alternative approach to the suggested method is to assign the interruption cost to all components in the failed minimal cut set, not just to the component that caused the failure. The drawback with such an approach is that it is necessary to calculate all minimal cut sets and to keep track of which is failed, which is not necessary for the previously defined method.

5.2.2 I^M applied to the test system

We simulate the test system with the same properties as used for the analytical calculations. This includes assuming independent components with exponentially distributed failure and repair times. The applied simulation technique is event driven. Results from the simulation can be seen in Table 5.

Note that the sum for component 1 and 2, 15003 SEK/yr, is very close to the individual components maintenance potential (I^{MP}), 15008 SEK/yr. This is an effect of the definitions of the indices. The maintenance potential correspond to the total amount possible to save on the studied component which in this case correspond to the total interruption cost of the cut set components. This corresponds with the results for component 3 where the value for I^M almost exactly corresponds to the value of I^{MP}.

	Component number		
Index	1	2	3
I^M [SEK/yr]	7487	7516	22008

Table 5: The proposed reliability importance index I^M.

The sum of all components' indices adds up to total expected yearly interruption cost, for the whole system.

6 THE SIMULATION INDEX, I^M, APPLIED TO A LARGE NETWORK

The proposed simulation based index, I^M, has been applied to a distribution network in the Stockholm area, here referred to as the Birka system, see Figure 2 [6].

6.1 The Birka system

The system includes a 220/110kV station (Bredäng) and one 110/33kV, 33/11kV station (Liljeholmen). These two stations are connected with two parallel 110kV cables. From the Liljeholmen station there are two outgoing 33 kV feeders, Högalid (HD) and Stock-holm railway (SJ), there are also 32 outgoing 11kV feeders (LH11), here represented by one average set of components (28-35). The model includes 58 compo-nents divided into four types: circuit breakers, cables, transformers and bus bars. In the network, every com-ponent has a specific failure rate and repair rate. In total, this network serves approximately 38 000 customers

where the load point SJ consists of one customer, that is the railway, the load point LH11 represent one average load point of 32 actual outgoing feeders, which in total serve 14 300 customers and the load point HD that feed 23 400 customers [9]. The model has exponentially distributed failure and repair times and independent components, which are shared properties with the model in [1].
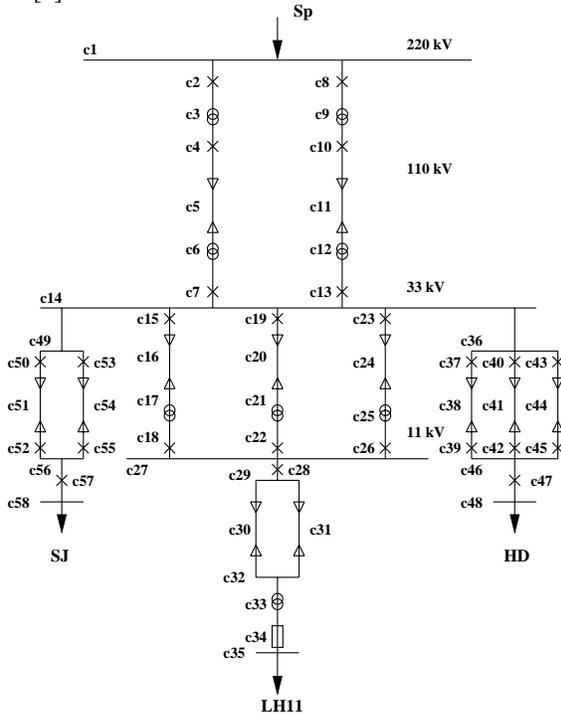
Figure 2: The Birka system [6].

6.2 Simulation results

Figure 3 presents the importance of the components involved in terms of average caused interruption cost per year. Note that some of the components have an importance of 0 SEK/yr, this is due to the fact that events that require three independent components to be failed at the same time has an extremely low probability. For this system these events generally do not happen, even with simulation times in the magnitude of billions of years. In Figure 3 it can be seen that the most important components (from a I^M perspective) are 14 and 1, followed by 8 and 2. It is interesting to note that there are a number of components that cause a large part of the interruptions. The six most critical components (10% of the population) cause 48% of the interruption cost. And the 12 most critical components (20% of the population) cause 76% of the interruption cost. This is close to the 20/80 rule, i.e. 20% of the population cause 80% of the trouble.
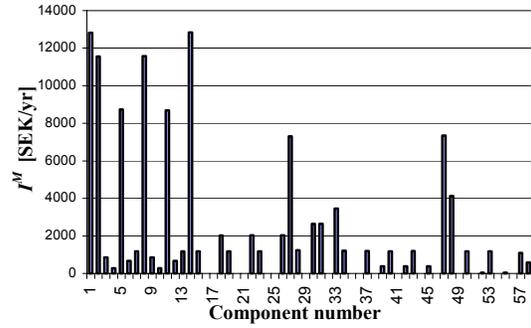
Figure 3: Component importance, I^M, where component 1 and 14 are identified as most important.

6.3 I^M evaluated against I^{MP} and I^{FC}

The result from the simulation can be compared with the analytically calculated importance index I^{MP} for the system and with the previously discussed simulation based index, I^{FC}. In Figure 4 the analytical based index I^{MP} is calculated. If we compare I^{MP} and I^M we see a quite high correlation, i.e. close to the same values for many components. Differences, as for example for component 5 and 11, are explained by differences in reliability models and by the differences in the definitions of the two indices (this is especially true for components in parallel structures). However, both the methods, applied to respective model, identify 14, 1, 8 and 2 as the most important components.
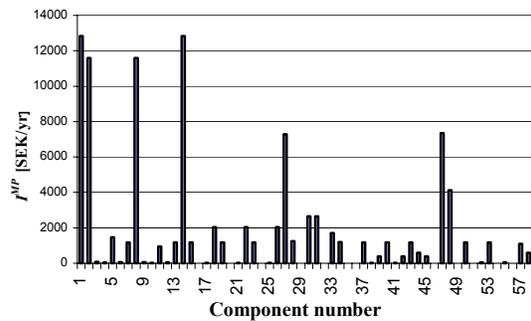
Figure 4: I^{MP} for the whole Birka system, where component 1 and 14 are identified as most important.

Results of calculations of I^{FC} for load point LH11 are presented in Figure 5. It is interesting to note the emphasis on component 34, which is explained by the fact that I^{FC} is calculated from the single load point perspective (LH11) and that the index is based on the number of failures (and not the duration).
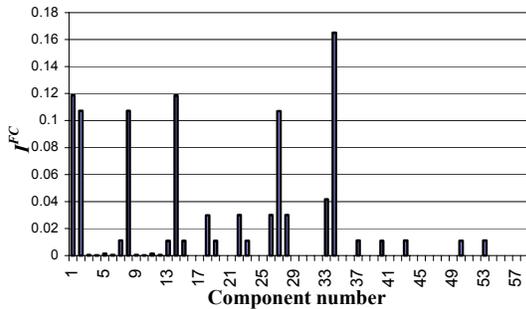
Figure 5: I^{FC} for load point LH11, where component 34 is the most important component.

I^{FC} calculated for HD position component 48 as the most important component while for load point SJ component 14 is the most important. The data presented for I^{FC} put emphasis on the previous discussion regarding two state systems and their applicability to networks with several load points.

6.4 Conclusion

This section has shown an applied example of the proposed index, I^M. In comparison with other indices I^M give congruous and interesting results that could be useful for reliability related activities in the network.

7   DISCUSSION

Computation time is a common objection to simulations, i.e. simulations are costly in terms of computation time. There are techniques for reduction of calculation times without loss of precision, different variance reduction techniques for example. Still, simulations quite often turn out costly in terms of calculation time. However, if we compare simulations techniques with analytical based calculations for the calculation of reliability importance indices, the question regarding computation time is not clear, since analytical-based reliability methods in general are difficult to adjust for the calculation of such indices. One conclusion might be that simulations are suitable for component importance calculations in complex systems. Another issue regarding simulations is their repeatability and consistency. Rare events with high impact can have a huge effect on simulations. However, being aware of these issues regarding simulation we must acknowledge their value. Simulation based calculations enable us to develop models with higher resolution for larger systems in a more straightforward manner compared to the analytical approach.

8   CONCLUSION

All of the indices presented have their own area of application, where they are especially suited for their purpose. The purpose of the proposed index, I^M, is to establish a connection between component reliability and its effect on system level. Hence it gives decision makers support with the tasks of resource allocation for example. The method encourages reliability importance index calculations during computation of reliability simulations. The interruption cost is used as a measure of reliability; this enabling an analysis of component importance for several load points with one index for the whole system.

The limitations of I^M are related to simulations, i.e. repeatability and computation time. However, one of the strongest advantages comes from simulations and that is the level of achievable model detail depth. Regarding the issue of computation time the index is suitable for implementation in already existing simulation routines at a low additional computational cost.

The conclusion of the paper is that the proposed simulation based importance index provides means of improved analysis of complex electrical networks with several load points.

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