SIMULATION AND OPTIMIZATION OF MARKETS FOR ELECTRICITY AND EL-CERTIFICATES

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Abstract – The paper describes a quantitative stochastic optimization model for the simultaneous balance in the markets for el-certificates (green certificates) and electric power. In the model solution, the storage of el-certificates is managed optimally and certificate prices are given by the shadow price on the certificate storage. Case studies show how simulated prices are affected by the capacity for renewable electricity generation and by the value of certificates at the end of the study period.

Keywords: El-certificates, green certificates, electricity markets, price simulations

1 INTRODUCTION

The purpose of the EU's RES-E directive [1] is to increase renewable electricity generation. This can improve security and diversification of energy supply and promote environmental protection and social and economic cohesion. A major goal is to reduce CO2 emissions from electricity generation. Previously, the Commission has been in favour of using compliance markets for "green certificates" as a tool for promotion of renewable electricity. Under a green certificate scheme the environmental attributes of electricity generated from renewable sources can be traded as a separate commodity independently from the underlying electricity.

Several EU countries have introduced such systems already, for instance UK, Sweden, Italy and Belgium. The Netherlands had a scheme for some years, but recently scrapped this system for a feed-in system. Compliance markets for green certificates are also found outside Europe, e.g. in Australia and Texas.

The Swedish green certificate scheme (the term el-certificates is used in Sweden) was introduced in 2003, and a common Swedish-Norwegian market for el-certificates is planned from 2006. The market for el-certificates arises because the consumers are required to buy a number of certificates that are proportional to their consumption of electrical energy. The authorities decide which technologies can obtain green certificates and the ratio of certificates to electricity that is required for each year. Examples of technologies that can be certified in the Swedish scheme include wind power, existing small-scale hydropower, all new hydropower and bio-fired power plants. The certificates have a defined size of 1 MWh renewable electricity production. A producer of 1 MWh wind power will be paid for 1 MWh in the electricity spot market plus 1 MWh in the certificate market. So far, prices of el-certificates in the Swedish market have been at more or less the same level as electricity prices (200-250 SEK /MWh). This must however be considered to be incidental. UK green certificate prices (ROC prices) have been higher than the Swedish certificate prices. Not much investment in new renewable capacity has been seen in Sweden so far limiting the supply of certificates to output from existing plants. The el-certificate scheme only has been defined to last until 2010. This has given a short payback time for long term investments in renewable electricity production. It is proposed to prolong the scheme and thereby give better investment incentives.

It is the producers of renewable electricity that obtains certificates and it is the consumers of electricity that must buy certificates via their electricity supplier. The market for el-certificates is therefore an integrated part of the electricity market. The increased support for renewable electricity should lead to increased investments in renewable electricity generation and thereby increased supply of electricity. This will reduce producer prices for electricity. There will also be a negative shift in the demand curve for electricity since consumers must buy certificates for a part of the consumed amount corresponding to a tax on electricity consumption. In sum it is not obvious that the total electricity price to the consumer will increase even though they have to buy certificates. The demand for certificates is given by electricity demand. Electricity prices and the prices for el-certificates are equally important for the renewable electricity production. Thus, the price of el-certificates will be heavily affected by the electricity market. But the short run variations in the electricity prices will probably not affect the market for certificates significantly if banking of certificates is permitted.

In order to find an optimal strategy e.g. for trades in the el-certificate market, the players need quantitative tools that can forecast prices. Accurate price forecasts are also required for the system to function optimally. Since the markets for electricity and el-certificates are integrated, such tools should ideally include both markets simultaneously. For governmental agencies it is important to evaluate how different market designs and

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ambition levels for el-certificates affect e.g. the socio-economic cost of the system and the market stability. Among others they have to decide:

- What percentage of the consumption must be "green" electricity? How should certificate requirements be phased in?
- To what degree is it allowed to store certificates from one period to the next?
- Is it allowed to borrow certificates from the subsequent period, and if so, how many?
- What is the penalty for not non-compliance to certificate requirements for a period?
- Which technologies should obtain certificates?

Several of the technologies that obtain certificates (wind power and hydropower, for example) have substantial natural variation. The consumption of electricity, and thus the demand for certificates, also varies considerably with the temperature. The future price of certificates will therefore be uncertain, depending on the storage and borrowing possibilities, among other factors.

In the present paper, we describe a stochastic model of a hydro-thermal power system that includes el-certificates. The model establishes an operating strategy for the whole system and carries out simulations that include el-certificates market. We believe that this model can be applied to a common Norwegian and Swedish certificate market without much modification.

The paper is organized as follows. An overall overview of our modelling approach is provided in Section 2, while the mathematical model is described in Section 3. Section 4 describes simulation results for different cases and Section 5 concludes.

2 MODEL OVERVIEW

2.1 El-certificates

The new el-certificate model that is described in the present paper is based on the Swedish certificate system, but we expect that the model can be adapted to other similar system as well, including a future common Scandinavian certificate system. The main assumptions and characteristics of the implementation are:

- There is only one market for certificates.
- One certificate is issued for each MWh renewable electricity generation, including some hydropower.
- The balance between "production" and "consumption" of certificates are settled once a year.
- The consumption of certificates is proportional to the electricity consumption.
- There is a user-specified penalty for lack of certificates that is proportional to the certificate deficit.
- Certificates may sold to the regulator at a minimum price at any time.
- Banking and or borrowing of certificates may be al-lowed within user specified limits.
- The certificate debt is eliminated when the deficit is penalized. It is also possible to accept the penalty and save the certificates to future years. We can therefore interpret the penalized amount as a purchase of certificates.

In the Swedish certificate system the penalty for lack of certificates at time of settlement is actually a function of the observed certificate prices for the obligation period. This property of the market is not implemented into our model yet, as also commented in Section 3.5.

2.2 Computer implementation

The new computer model for the integrated power and el-certificate markets is based on an extension of an existing program for stochastic scheduling in hydro-thermal power systems. This program uses stochastic dual dynamic programming to find optimal operating strategies, and is similar to the models in [2] and [3], except for aggregation of power stations described below.

The objective is to minimize the expected operating cost for the whole power system considered over a certain study period. The operating cost includes such terms as cost of thermal generation, income from sales and the value of remaining water at the time horizon. The expectation is taken over a set of scenarios for inflow, temperature and wind generation.

To be able to handle a system with many reservoirs, like the Scandinavian, some simplifications are needed. The system is divided into geographically separated subsystems (areas), mainly based on borders between countries, variation in precipitation and known bottlenecks in the transmission system (se example in Section 4).

In each subsystem, an aggregate power system model is used, as shown in Figure 1. Hydro generation is mostly described by lumping power stations into one power station with one reservoir with stochastic inflow, but units can also be described in greater detail.

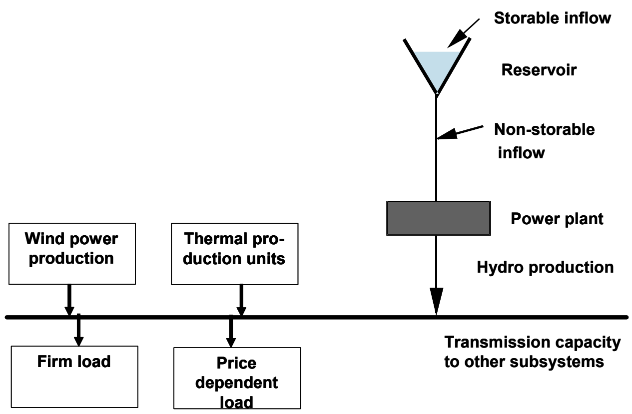

Figure 1: Subsystem model

Thermal generation units are described by their capacity and marginal generation costs. In practice similar thermal units may be aggregated to blocks of units. The wind power generation and the demand for electricity are stochastic. The load modelling is flexible and both fixed loads and different types of price dependent loads can be included. Since load reductions are modelled as buyback, the cost minimization formulated in Section 3 is equivalent to the maximization of socio-economic surplus, and from the first welfare theorem of economics we know that the solution of this kind of optimization problem is identical to the competitive market equilibrium without externalities [4].

The subsystem structure of the model is similar to the structure of the much used EMPS model [5], but since stochastic dual dynamic programming is used to establish the operating strategy, the transmission can be handled directly. However, it is necessary to establish the marginal value of water stored at the time horizon using a separate tool, such as the EMPS model or some other long-term model.

3 MATHEMATICAL MODEL

3.1 Symbols

We consider a study period of several years that is divided into T time intervals, typically of length one week. We define the following variables, mostly related to time step $t \in \{1,...,T\}$:

A	Number of subsystems
$\alpha_{i,j}$	Loss fraction in transmission from subsystem *i* to subsystem *j*.
B_t	Amount of certificates purchased
B	Set of generation units excluding hydro power
$c_{B,t}$	Penalty cost for lacking certificates at settlement
$c_{S,t}$	Price for sale of certificates
$D_{i,t}, D'_{i,t}$	Energy demand in area *i*, fixed and scenario-dependent
$D_{G,i,t}$, $D'_{G,i,t}$	Amounts of el-certificates associated with energy demand
G_t	Amount of el-certificates in stock at end of interval *t*
$g_{i,j,t}$	Quantity generated or consumed according to the *j*-th market option in area *i*.
$\gamma_{i,t}$	Fraction of "green hydro" in area *i*.
$\mu_{i,j,t}$	Fraction of "green power" for market option *j* in area *i*.
$P_{i,t}$	Hydro generation in area *i*
$q_{i,t}$	Water through plant *i*
$Q^{\max}$	Maximum discharge through plant *i*
$s_{i,t}$	Spill in area *i*
S	Set of load types
S_t	Amount of certificates sold
T	Number of time steps within the planning horizon
t	Time interval index (weeks)
τ	Week number for the annual settlement of certificates
$u_{i,t}$	Water release from reservoir *i*
$V_{i,t}$	Stored energy in area *i* at end of time step
$v_{i,t}$	Storable inflow in area *i*
$w_{i,t}$	Inflow that cannot be stored, area *i*.
$y_{i,j,t}$	Energy transmitted from are *i* to area *j*

Limits for variables are indicated by underlining/overlining the variable name, or by a superscript such as "max".

3.2 Reservoirs and hydropower generation

For each area *i*, *i*=1,...,*A*, and each time step *t*, *t*=1,...,T, the (lumped) reservoir balance is:

$$V_{i,t+1} + u_{i,t} + s_{i,t} = V_{i,t} + v_{i,t} \ , \tag{1}$$

subject to

$$\underline{V}_{i,t} \le V_{i,t} \le \overline{V_{i,t}} \tag{2}$$

$$s_{i,t} \ge 0; u_{i,t} \ge 0 \tag{3}$$

The amount of water run through the turbine is $q_{i,t} = u_{i,t} + w_{i,t}$ if $u_{i,t} + w_{i,t} \le Q^{\max}$ and $Q^{\max}$ otherwise. We model the hydropower generation in area *i* as:

$$P_{i,t} = f_i(q_{i,t}) \tag{4}$$

where $f_i(\cdot)$ is a concave, piecewise linear function, reflecting the reduced marginal efficiency at high plant output.

3.3 Inflow and demand

The power demand is represented as the sum of two parts, $\{D_{i,t}\}$ and $\{D'_{i,t}\}$, where $\{D'_{i,t}\}$ is a stochastic demand component that we associate with the different scenarios for inflow, wind power generation etc. Wind generation is part of $\{D'_{i,t}\}$, as a negative contribution. Time series for $\{v_{i,t}\}$, $\{w_{i,t}\}$ and $\{D'_{i,t}\}$ for all i are established in the aggregation process for each area. From these series pieces of length *T* are taken to make up inflow scenarios. The model simulations start at a common initial state, simulating the inflow scenarios in parallel.

The non-storable inflow $\{w_{i,t}\}$ may have physical origin, but mainly it arises in the aggregation process when data for each area is built, reflecting minimum releases etc. For simplicity of notation the scenario index has been dropped in all equations. We also mention that a first-order autoregressive model is established from inflow data for inflow prediction one week ahead during the strategy computation.

3.4 Power balance, thermal units and renewable sources

The energy balance for area *i* in time step *t* becomes:

$$P_{i,t}+\sum_{j\in B}g_{i,j,t}-\sum_{j\in S}g_{i,j,t}$$
$$+\sum_{j\notin i}(1-\alpha_{j,i})y_{j,i,t}-\sum_{j\notin i}y_{i,j,t}=D_{i,t}+D'_{i,t} \tag{5}$$

We require $0\le g_{i,j,t}\le g^{\max}_{i,j,t}$ and $0\le y_{i,j,t}\le y^{\max}_{i,j,t}$. The transmission losses are taken as linear, modelled by the α coefficients. This can be improved by splitting the y variables and using a piecewise linear representation.

3.5 El-certificates model

It is assumed that there is a single market for el-certificates. Let the stock of certificates at the end of time interval t be G_t. The balance equation for certificates becomes:

$$G_t-G_{t-1}=\sum_{i=1}^{A}\left(\gamma_{i,t}P_{i,t}+\sum_{j\in B}\mu_{i,j,t}g_{i,j,t}\right.$$
$$\left.-\sum_{j\in S}\mu_{i,j,t}g_{i,j,t}-D_{G,i,t}-D'_{G,i,t}\right)-S_t+B_t \tag{6}$$

Constraints are:

$$\underline{G_t}\le G_t\le\overline{G_t} \tag{7}$$
$$\underline{S_t}\le S_t\le\overline{S_t} \tag{8}$$
$$\underline{B_t}\le B_t\le\overline{B_t} \tag{9}$$

The first term after the equality sign reflects the creation of certificates from “new” hydro generation within the area. The relative amount of “new” hydro is given by the γ coefficients. The second after the equality sign shows the amount of certificates issued to thermal generation, while the third term represents the use of certificates by consumers that buy directly from the market. It is here stipulated that a fraction $\mu_{i,j,t}$ of the consumption requires el-certificates.

In practice, the limits $\underline{G_t}$ and $\overline{G_t}$ for the stock of el-certificates are set rather wide, except in settlement weeks, where there will be a more strict minimum requirement, such as $\underline{G_t}=0$. If this level is not met, a penalty is introduced, modelled as buying a quantity B_t at a high price, $c_{B,t}$. Usually the maximum quantity for buying, $\overline{B}_t$, is set to zero except in settlement weeks.

A major simplifying assumption in our model is that $c_{B,t}$ is a fixed quantity (one value for each year) known in advance, as mentioned in Section 2. In practice, this marginal penalty will often be set from historical values last year, say, so that the decisions during the study period may actually influence $c_{B,t}$. This case would be much more difficult to treat mathematically. It should also be noted that it is allowed that B_t can be larger than the minimum necessary value during settlement. In any case, the certificate inventory is increased by the amount B_t, according to (6). Sales S_t means that certificates are bought back by the issuing authority, the corresponding price $c_{S,t}$ may be regarded as a minimum certificate price.

Finally, at the time horizon T, it is necessary to specify a value function for remaining certificates: $\psi(G_T)$. For simplicity of presentation we here assume that the time horizon coincides with the last settlement time, so that $T=\tau+(n-1)\cdot 52$, where n is the number of years in the study period. We take $\psi(\cdot)$ as zero below $\underline{G_T}$ and as a piecewise linear function that has to be specified above $\underline{G_T}$. Ideally, $\psi(\cdot)$ should be established by some analysis with a longer time horizon

3.6 Objective function

As mentioned in Section 2, the operating strategy is designed to minimize expected cost, where the expectation is taken over scenarios for inflow, temperature and wind power. For the el-certificates, there will for each scenario and time interval be additional terms

$$-c_{S,t}S_t+c_{B,t}B_t-\delta_{t,T}\psi(G_T), \tag{10}$$

where $\delta_{t,T}=1$ if $t=T$ and zero otherwise.

3.7 Solution method

The resulting system model becomes a stochastic linear model. As mentioned earlier, it is optimized by stochastic dual dynamic programming (SDDP); see [3] and [4]. In this method, the problem is decomposed into small linear programs, one for each time step and each inflow case. The operating strategy is given by the estimated expected future cost functions at the end of each time interval. For each t, these are represented by hyperplanes in the reservoir states $\{V_{i,t}\}$ (and possibly also inflow states, not described here). The sets of hyperplanes, or cuts, are built iteratively, the coefficients computed from the dual variables of the water balances, as described in [2].

In our case, G_t, the stock of el-certificates, becomes a new state variable, so that each cut receives an additional term $\beta_t G_t$. The coefficient β_t is derived from dual variables of the certificate balance (6), much in the same way as the coefficients for reservoirs. In most ways, the certificates balance is similar to a reservoir balance, but one should note that it introduces an extra coupling between the subsystems, and that the coefficients are different from unity.

4 CASE STUDY

The new integrated certificate and electricity market model is tested on the Nord Pool (Norway, Sweden, Finland and Denmark) electricity market for different implementations of the Swedish el-certificate market.

4.1 Reference case description

The reference case describes the Nord Pool electricity market with a Swedish el-certificate market as implemented for the years 2003 and 2004. The subsystems are shown in Figure 2.

The certified generation for the Swedish system in our reference case model is given in Table 1. The actual installed capacities in the Swedish system may deviate from our values but this is not essential for our testing of the model. The different generation types are distributed into the modelled subsystems according to their physical location. In the model this capacity is split into different units according to their marginal generation costs. Certified hydro generation corresponds to 2.8 % of the Swedish hydro power generation.

Generation type	Annual generation(GWh)
Bio based generation (installed capacity)	4970
Hydro power generation (average generation)	1870
Wind power (average generation)	650
Sum certified generation	7490

Table 1: Summarized input data for certified generation in the Swedish system.

All Swedish consumers, except power intensive industry, are obliged to buy el-certificates for 7.4 % and 8.1 % of their electricity consumption for the year 2003 and 2004, respectively.

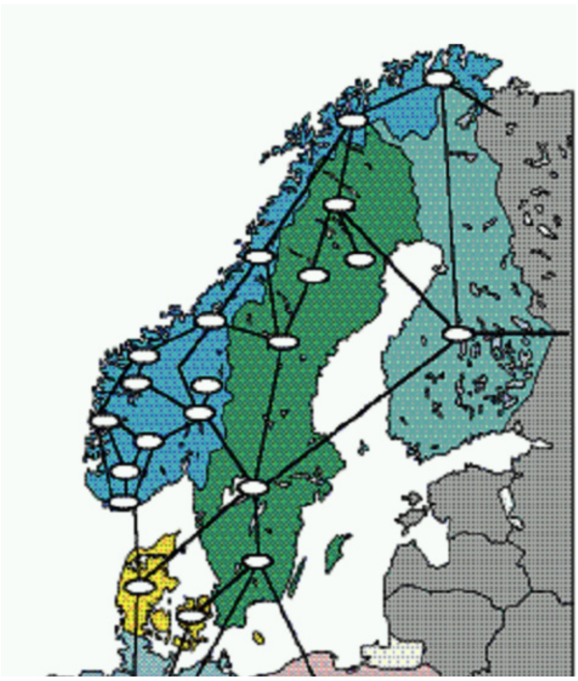

Figure 2: Model of the Nord Pool electricity market where each node represent a subsystem

The penalty for lack of certificates by end of year 2003 is 242 NOK/MWh and 333 NOK/MWh for 2004. The penalties are corrected for taxes. The minimum price for certificates is 60 NOK/MWh for 2003 and 50 NOK/MWh for 2004. Because banking is allowed in the Swedish system, certificates have a value by the end of 2004. In the implementation the end value of certificates is described by a piece-wise linear description with marginal values of certificates in the rage from 250 to 150 NOK/MWh corresponding to zero and 10 000 GWh of certificates by the end of 2004. Borrowing of certificates is not allowed. In our case simulations certified consumption is modelled price independent. Marginal costs for certified biomass generation are so low that they in practice produce at all times. Settlement dates for certificates are defined to be the last week in each year and we only simulate for the years 2003 and 2004.

The model simulations are done for 70 different scenarios. Each scenario represents a given future of wind power generation, hydro inflow and temperatures. These scenarios are generated using observations for the historical period 1931 to 2000.

4.2 Reference case simulations

Figure 3 shows simulated el-certificate prices for 2003 and 2004. The simulated certificate prices for 2003 are equal to 242 NOK/MWh, which is the penalty price, for all simulates scenarios. The “consumption” of certificates in the system is larger than “production”. Thus, some of the obliged consumers have to take the penalty with 100% certainty. The penalty price is lower for 2003 than for 2004 and it is therefore optimal to take the penalty by the end of 2003 rather than in 2004. The price for 2003 should therefore be equal to the penalty for 2003 from week 1. For year 2004, week number 53-104, the prices are different for the different scenarios. If the expected price in 2004 is less than the price in week 52, then it would be optimal to buy fewer certificates in week 52, and vice versa. Therefore, we expect that the simulated average price for 2004 is approximately equal to the 2003 price and this is also the case. The maximum price is limited by the penalty for the second year and the minimum price is given by the end storage marginal value.

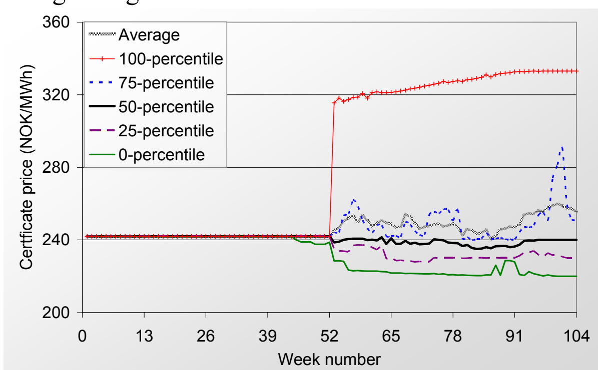

Figure 3: Simulated certificate prices for the reference case.

Figure 4 shows the simulated variations in certified generation, which are mainly caused by variations in wind power generation and hydro generation. Seasonal variations are mainly caused by in variations in biomass generation.

Figure 5 shows simulated percentiles for the development of the certificate storage. The storage shows the accumulated balance between “production” and “consumption” of certificates. The sharp storage increase in week 52, i.e. the end of 2003, corresponds to the penalty taken and can be seen as a buyback of certificates from the regulator. The players in the certificate market can take the penalty and store available certificates for later use. This is also exactly what happened in Sweden.

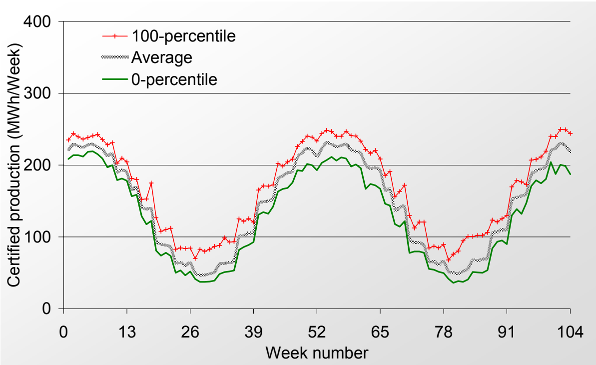

Figure 4: Simulated "production" of certificates for the reference case.

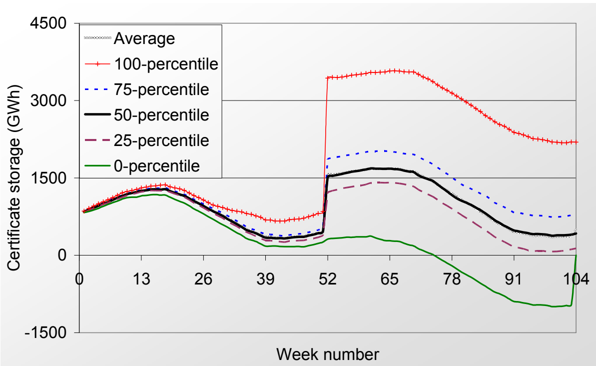

Figure 5: Simulated development of certificate storage

4.3 Other simulations

The model is also tested on different other cases that are variations of the reference case, cf. Table 2: .

Case	Deviation from Reference case
B	(Reference case)
C1	The value of certificates is zero at the end of the second year.
C2	The value of certificates is 2500 NOK/MWh at the end of the second year.
D1	Biomass based electricity generation is increased by 2 TWh/year
D2	Biomass based electricity generation is decreased by 1 TWh/year
E	No certificate system

Table 2: Case description

Figure 6 shows the simulated average electricity spot prices for one of the modelled subsystems in Sweden.

Certified hydro storage final value is not corrected for the end certificate values. Therefore, different end values of certificates give different hydro generation during the study period and also slightly different electricity market balance. When certificate prices are high more certified hydro will be produced and this will lower the electricity prices. This is the reason why the case C1 price is higher price than for case C2 except for a short period prior to spring inflow season. This caused by higher curtailment probability.

Case D1 has on average a lower electricity price than the reference and D2 has a higher price. This is as expected because these cases also represent new (D1) and reduced electricity generation (D2).

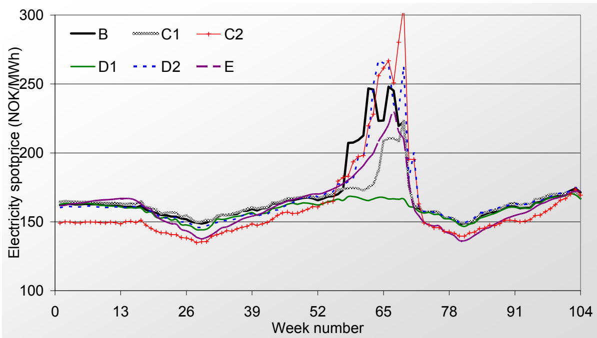

Figure 6: Simulated average electricity prices for a subsystem in Sweden.

Figure 7 shows simulated average storage levels of certificates for the different cases. Case C1 has zero certificate storage in week number 104; this is as expected because the end value of certificates is set to zero. The storage level of case C2 goes outside the plot in week 52 because the end value of certificates for this case is set very high. The players take as much penalty as possible in the first year in order to have as much storage as possible by the end of the second year. For Case D1 there is no penalty taken because there is a surplus of certified generation as seen in Figure 7. Case D2 has less certified generation than the reference case, this is compensated by additional penalty taken in week 52.

Figure 8 shows simulated average certificate prices for the different cases. Case C2 is not included in the figure because the simulated prices for this case are outside the range of the Y-axis. The prices for this case are of course equal to the end value of 2500 NOK/MWh for all weeks in the study period. The price for case D1, increased certified generation, is equal to 220 NOK/MWh for the whole study period for all simulated scenarios. This price is given by the marginal value of certificates at the end of the study period.

The prices for case C1 is a bit lower than the reference and case D2. This is not as expected because from a theory it could be shown that the prices should be the same for these three cases. An important decision for these three cases (B, C1 and D2) is the amount of penalty taken by the end of the first year. We have observed that this decision is very sensitive to the statistical properties of the stochastic model used in the strategy part of the model. The probability of extremes for accumulated values over many time periods in the stochastic model is too low. The results also include sampling uncertainty because the optimal strategy is simulated for only 70 scenarios.

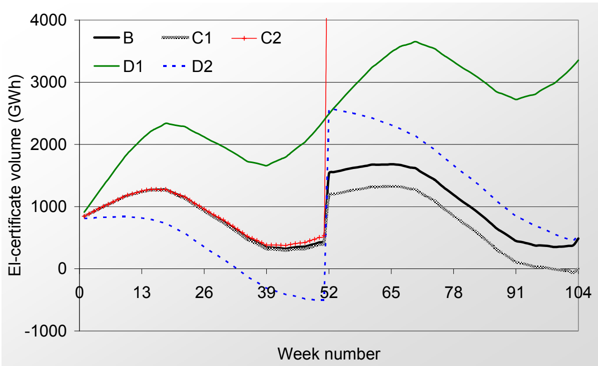

Figure 7: Simulated average certificate storage.

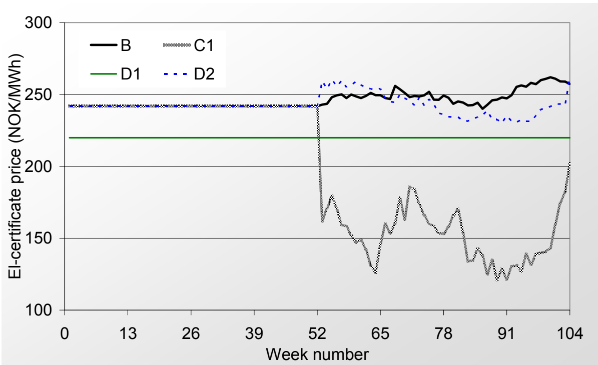

Figure 8: Simulated average certificate prices.

5 CONCLUSION

In this paper we have given a brief overview of the development towards a common obligatory market for el-certificates (green certificates) in Norway and Sweden, and we have shown how we have implemented el-certificates in a quantitative stochastic optimization model for electricity markets. We have also shown some simulation results.

Increased supply of renewable electricity reduced the prices for el-certificates in our simulations, while decreased supply had small effects on prices since this was compensated with increased non-compliance. This result is heavily dependent of the shortage of el-certificates in the reference case. Increased value of stored certificates at the end of the study period increased certificate prices from week 1, while the reduced price of certificates had a small effect on certificate prices towards the end of the study period. But prices were at lower levels during the final year, and we believe this is caused by too little annual variation in the stochastic model compared to the variation in the simulations.

The model is a tool for making price forecasts for el-certificates and electricity. It can also be used to evaluate the socio-economic costs of different designs of the market for el-certificates, and we will include these calculations in future work.

A major advantage with our approach is that the price of el-certificates is given by the shadow price of the certificate balance equation and not by the annual equilibrium in each year. We can therefore take into account that the owners of certificates will store certificates if they believe the price will be higher in future years. As far as we know this model is the only model for the electricity market accounts for the optimal management of the el-certificate storage.

A limitation of the model is that investments are not determined endogenously. Thus, all capacities and the value of el-certificates at the end of the study period are input parameters to the model. The best estimate of the end value of certificates is probably given by the difference between the long run marginal costs for new renewable electricity and the cheapest alternative, e.g. gas-power.

Another limitation is that the user must specify the penalty for non-compliance. In the Swedish system the penalty for a given year is a function of the observed certificate prices. We do, however, plan to include endogenous penalties in future work. Finally, it is hard to find the optimal solution of large stochastic problems, and it is necessary to find a compromise between computational time and accuracy. The properties of numerical results can therefore deviate somewhat from those derived from qualitative economic models.

At present we are doing various sensitivity analyses, such as the value of the penalty of non-compliance, possibilities for borrowing in the certificate market, changed annual settlement date for certificate etc. The model can probably be applied to a common Norwegian and Swedish certificate market without any modifications.

REFERENCES

[1] OJ L 283, 27.10.2001, pp. 33-40, Directive 2001/77/EC of the European Parliament and of the Council of 27 September 2001 on the promotion of electricity produced from renewable energy sources in the internal electricity market.

[2] M. V. F. Pereira, “Optimal stochastic operations scheduling of large hydroelectric systems" Electrical Power & Energy Systems, 11, 3, pp 539—546, 1989.

[3] T. A. Røtting and A. Gjelsvik, “Stochastic dual dynamic programming for seasonal scheduling in the Norwegian power system”, IEEE Transactions on Power Systems, 7, 1, pp 273—279, 1992.

[4] Hal R. Varian, *Microeconomic Analysis*, (3rd ed), New York: W.W. Norton & Company, inc., 1992.

[5] O.J. Botnen, A. Johannesen, A. Haugstad, S. Kroken, O. Frøystein, “Modelling of Hydropower Scheduling in a National/International Context”, in E. Broch and D.K. Lysne (eds.), Hydropower ‘92, Balkema, Rotterdam, June 1992.