TRANSFER CAPABILITY COMPUTATION WITH SECURITY CONSTRAINTS

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Abstract – In this paper, a computational approach for transfer capability, which can consider steady security constraints (contingencies), is presented. The security constrained transfer capability problem is solved by a nonlinear interior point method in a united optimization framework. In the security constrained transfer capability calculations, FACTS is included. Numerical examples on the IEEE 30 bus system is given to illustrated the approach.

Keywords: Transfer capability, Security constraints, FACTS, Nonlinear interior point method

1 INTRODUCTION

It has been well recognized that in the operation of electric power markets, determination of the transfer capabilities of the transmission system is a very important analysis function. Transfer capability of electric power systems is limited by a number of different mechanisms, including thermal, voltage and stability constraints, which is characterized by so-called Available Transfer Capability [1, 2]. The comprehensive definition of the transfer capability is referred to [2].

The Transfer Capability (TC) computation methods in literature can be classified into, (a) DC power flow calculation method (or linear method) (b) repeated power flow calculation method (c) continuation power flow method (d) OPF and security-constrained OPF methods. The DC power flow calculation method has been implemented in a commercial software product called MUST [3]. The advantage of such method is its simplicity in terms of formulation & computation. In the method, voltage and voltage stability limits are not considered. It has been recognized that neglecting the reactive power & voltage influence in TC may generate errors that in certain conditions could drive the computation to be wrong or at least give inaccurate results. The repeated power flow calculation method has been proposed [4]. The repeated power flow calculation method is heuristic in nature. The computational effort of the method is significantly higher compared with other methods. The continuation method for TC has been reported and implemented in commercial software products [5]. The advantage of the continuation method is that various operating limits such as thermal, voltage & voltage stability limits can be fully taken into account. The disadvantage of this method is that adjustment of generation, transformer tap positions, FACTS controls, etc in TC calculations would be very difficult if not impossible. Similar to the DC power flow calculation method and repeated power flow calculation method, continuation power flow method could not be used in an integrated contingency-constrained analysis framework. In such a situation, sequential heuristic TC calculations are used instead. Solution of TC by a successive linear programming based OPF has been proposed [6]. Non-linear interior point OPF algorithms for TC calculations have also been proposed [7, 8, 9]. However, most OPF based TC methods are based on single state calculations. It is recognized that the single state optimization based approaches have difficulty to deal with control actions such as preventive and/or corrective controls.

The deficiencies of the current TC computational methods are:

(1) lack of couplings between base case and contingencies;
(2) lack of adequate consideration of reactive power/voltage effects and/or voltage stability effects;
(3) lack of modeling of FACTS controllers in transfer capability determination.

In order to handle the deficiencies of the current TC computation methods, in this paper the TC computation problem will be formulated as a general contingency-constrained optimization problem and will be solved by the nonlinear interior point optimization algorithms. The TC computational method proposed has the following features:

(1) Considering various operating limits and contingency constraints;
(2) Incorporating corrective or/and preventive control actions in the united framework;
(3) Modeling of FACTS controllers;
(4) Solving simultaneously the base case and contingencies in a united optimization framework.

2 A UNIFIED TRANSFER CAPABILITY COMPUTATION METHOD WITH SECURITY CONSTAINTS

A unified transfer capability computation problem with security constraints may be formulated as

Objective function:

$$\max f(\mathbf{y}) = \lambda$$

or

$$\min\ -f(\mathbf{y}) \tag{1}$$

Subject to the following constraints:

Base case constraints:

$$\mathbf{g}_0(\mathbf{y}_0)=\mathbf{0} \tag{2}$$

$$\mathbf{h}_0^{\min} \le \mathbf{h}_0(\mathbf{y}_0) \le \mathbf{h}_0^{\max} \tag{3}$$

Contingency constraints:

$$\mathbf{g}_i(\mathbf{y}_i)=\mathbf{0} \tag{4}$$

$$\mathbf{h}_i^{\min} \le \mathbf{h}_i(\mathbf{y}_0) \le \mathbf{h}_i^{\max} \tag{5}$$

i = 1, 2, …, Nc

where subscripts 0 and i indicate base case and contingencies, respectively. Nc is the total number of contingencies. $\mathbf{g}_0(\mathbf{y}_0)$ and $\mathbf{h}_0(\mathbf{y}_0)$ are base case equalities and inequalities, respectively. While $\mathbf{g}_i(\mathbf{y}_i)$ and $\mathbf{h}_i(\mathbf{y}_i)$ are equalities and inequalities respectively for contingency i. $\mathbf{y}=[\mathbf{x},\mathbf{u},\lambda]^T$ is the system variable vector. $\mathbf{u}$ is the control variable vector with preventive control actions. λ is a scalar parameter, which represents the loading factor. Modeling of FACTS controllers in power system network analysis can be found in [10 –12].

Without loss of generality, preventive control actions may be formulated as

Preventive control actions:

$$\mathbf{u}_0=\mathbf{u}_i \qquad (i=1,2,\ldots Nc) \tag{6}$$

where $\mathbf{u}_0$, $\mathbf{u}_i$ are base case and contingency control vectors respectively.

The problem in (1) – (6) is a unified security constrained transfer capability computation problem. In this problem, bus load may be represented by

$$P_d=\lambda P_d^0 \tag{7}$$

$$Q_d=\lambda Q_d^0 \tag{8}$$

where P_d^0 and Q_d^0 are base case bus active and reactive load powers, and it is assumed that a constant power factor is maintained.

3 SOLUTION OF THE UNIFIED TRANSFER CAPABILITY PROBLEM

3.1 Solution of the unified security constrained transfer capability problem by nonlinear interior point method

Mathematically, the unified transfer capability computation problem is an optimization problem, which may be solved by nonlinear interior point methods.

The nonlinear OPF problem given in (1) – (5) can be solved by the Nonlinear Interior Point Methods [12 - 14], which include three important achievements in optimization. Those achievements are Fiacco & McCormick's barrier method for optimization with inequalities, Lagrange's method for optimization with equalities and Newton's method for solving nonlinear equations [13-15].

By applying Fiacco & McCormick's barrier method, the unified OPF problem (1) – (6) can be transformed into the following equivalent OPF problem,

Objective:

$$\min\begin{Bmatrix} -f(\mathbf{y})-\mu\sum_{j=1}^{Nh}\ln(sl_{0j})-\mu\sum_{j=1}^{Nh}\ln(su_{0j}) \\ -\mu\sum_{i=1}^{Nc}\sum_{j=1}^{Nh}\ln(sl_{ij})-\mu\sum_{i=1}^{Nc}\sum_{j=1}^{Nh}\ln(su_{ij}) \end{Bmatrix} \tag{9}$$

Subject to the following equality constraints:

$$\mathbf{g}_0(\mathbf{y}_0)=\mathbf{0} \tag{10}$$

$$\mathbf{h}_0(\mathbf{y}_0)-\mathbf{sl}_0-\mathbf{h}_0^{\min}=\mathbf{0} \tag{11}$$

$$\mathbf{h}_0(\mathbf{y}_0)+\mathbf{su}_0-\mathbf{h}_0^{\max}=\mathbf{0} \tag{12}$$

$$\mathbf{g}_i(\mathbf{y}_i)=\mathbf{0} \tag{13}$$

$$\mathbf{h}_i(\mathbf{y}_i)-\mathbf{sl}_i-\mathbf{h}_i^{\min}=\mathbf{0} \tag{14}$$

$$\mathbf{h}_i(\mathbf{y}_i)+\mathbf{su}_i-\mathbf{h}_i^{\max}=\mathbf{0} \tag{15}$$

$$\mathbf{u}_0-\mathbf{u}_i=\mathbf{0} \tag{16}$$

i = 1, 2, …, Nc

where $\mu>0$, $\mathbf{sl}_0>\mathbf{0}$, $\mathbf{su}_0>\mathbf{0}$, $\mathbf{sl}_i>\mathbf{0}$ and $\mathbf{su}_i>\mathbf{0}$. Nh is the number of double sided inequalities.

The lagrangian function for equalities optimization of problem (9) – (16) is

$$\begin{aligned} L = & -f(\mathbf{y})-\mu\sum_{j=1}^{Nh}\ln(sl_{0j})-\mu\sum_{j=1}^{Nh}\ln(su_{0j}) \\ & -\mu\sum_{i=1}^{Nc}\sum_{j=1}^{Nh}\ln(sl_{ij})-\mu\sum_{i=1}^{Nc}\sum_{j=1}^{Nh}\ln(su_{ij}) \\ & -\boldsymbol{\lambda}\mathbf{g}_0^T\mathbf{g}_0(\mathbf{y}_0)-\boldsymbol{\pi}\mathbf{l}_0^T(\mathbf{h}_0(\mathbf{y}_0)-\mathbf{sl}_0-\mathbf{h}_0^{\min}) \\ & -\boldsymbol{\pi}\mathbf{u}_0^T(\mathbf{h}_0(\mathbf{y}_0)+\mathbf{su}_0-\mathbf{h}_0^{\max}) \\ & -\sum_i^{Nc}\boldsymbol{\lambda}\mathbf{g}_i^T\mathbf{g}_i(\mathbf{y}_i)-\sum_i^{Nc}\boldsymbol{\pi}\mathbf{l}_i^T(\mathbf{h}_i(\mathbf{y}_i)-\mathbf{sl}_i-\mathbf{h}_i^{\min}) \\ & -\sum_i^{Nc}\boldsymbol{\pi}\mathbf{u}_i^T(\mathbf{h}_i(\mathbf{y}_i)+\mathbf{su}_i-\mathbf{h}_i^{\max}) \\ & -\sum_i^{Nc}\boldsymbol{\lambda}\mathbf{u}_i^T(\mathbf{u}_0-\mathbf{u}_i) \end{aligned} \tag{17}$$

where $\mu>0$, $\mathbf{sl}_0>\mathbf{0}$, $\mathbf{su}_0>\mathbf{0}$, $\mathbf{sl}_i>\mathbf{0}$ and $\mathbf{su}_i>\mathbf{0}$. $\boldsymbol{\lambda}\mathbf{g}_0$, $\boldsymbol{\pi}\mathbf{l}_0$ and $\boldsymbol{\pi}\mathbf{u}_0$ are dual variable vectors to equalities (10), (11) and (12), respectively. $\boldsymbol{\lambda}\mathbf{g}_i$, $\boldsymbol{\pi}\mathbf{l}_i$ and $\boldsymbol{\pi}\mathbf{u}_i$ are dual variable vectors to equalities (13), (14) and (15), respectively. $\boldsymbol{\lambda}\mathbf{u}_i$ is dual variable vector to equalities (16), which represent the constraints of the preventive controls. In (17), transformer tap ratios are treated as continuous variables.

The Karush-Kuhn-Tucker (KKT) first order conditions for the Lagrangian function of (17) are:

$$\nabla_{\mathbf{y}_0} L_\mu = -\nabla_{\mathbf{y}_0} f(\lambda) - \nabla_{\mathbf{y}_0}\mathbf{g}_0(\mathbf{y}_0)^T \boldsymbol{\lambda}\mathbf{g}_0 - \nabla_{\mathbf{y}_0}\mathbf{h}_0(\mathbf{y}_0)^T \boldsymbol{\pi}\mathbf{l}_0 - \nabla_{\mathbf{y}_0}\mathbf{h}_0(\mathbf{y}_0)^T \boldsymbol{\pi}\mathbf{u}_0 - \sum_i^{Nc} \nabla_{\mathbf{y}_0}\mathbf{u}_0^T \boldsymbol{\lambda}\mathbf{u}_i \quad (18)$$

$$\nabla_{\boldsymbol{\lambda}\mathbf{g}_0} L_\mu = -\mathbf{g}_0(\mathbf{y}_0) = \mathbf{0} \quad (19)$$

$$\nabla_{\boldsymbol{\pi}\mathbf{l}_0} L_\mu = -(\mathbf{h}_0(\mathbf{y}_0) - \mathbf{sl}_0 - \mathbf{h}_0^{\min}) \quad (20)$$

$$\nabla_{\boldsymbol{\pi}\mathbf{u}_0} L_\mu = -(\mathbf{h}_0(\mathbf{y}_0) + \mathbf{su}_0 - \mathbf{h}_0^{\max}) \quad (21)$$

$$\nabla_{\mathbf{sl}_0} L_\mu = \mu\mathbf{e} - \mathbf{SL}_0 \mathbf{\Pi L}_0 \quad (22)$$

$$\nabla_{\mathbf{su}_0} L_\mu = \mu\mathbf{e} + \mathbf{SU}_0 \mathbf{\Pi U}_0 \quad (23)$$

$$\nabla_{\mathbf{y}_i} L_\mu = -\nabla_{\mathbf{y}_i} f(\lambda) - \nabla_{\mathbf{y}_i}\mathbf{g}_i(\mathbf{y}_i)^T \boldsymbol{\lambda}\mathbf{g}_i - \nabla_{\mathbf{y}_i}\mathbf{h}_i(\mathbf{y}_i)^T \boldsymbol{\pi}\mathbf{l}_i - \nabla_{\mathbf{y}_i}\mathbf{h}_i(\mathbf{y}_i)^T \boldsymbol{\pi}\mathbf{u}_i + \sum_i^{Nc} \nabla \mathbf{u}_i^T \boldsymbol{\lambda}\mathbf{u}_i \quad (24)$$

$$\nabla_{\boldsymbol{\lambda}\mathbf{g}_i} L_\mu = -\mathbf{g}_i(\mathbf{y}_i) = \mathbf{0} \quad (25)$$

$$\nabla_{\boldsymbol{\pi}\mathbf{l}_i} L_\mu = -(\mathbf{h}_i(\mathbf{y}_i) - \mathbf{sl}_i - \mathbf{h}_i^{\min}) \quad (26)$$

$$\nabla_{\boldsymbol{\pi}\mathbf{u}_i} L_\mu = -(\mathbf{h}_i(\mathbf{y}_i) + \mathbf{su}_i - \mathbf{h}_i^{\max}) \quad (27)$$

$$\nabla_{\mathbf{sl}_i} L_\mu = \mu\mathbf{e} - \mathbf{SL}_i \mathbf{\Pi L}_i \quad (28)$$

$$\nabla_{\mathbf{su}_i} L_\mu = \mu\mathbf{e} + \mathbf{SU}_i \mathbf{\Pi U}_i \quad (29)$$

$$\nabla_{\boldsymbol{\lambda}\mathbf{u}_i} L_\mu = -(\mathbf{u}_0 - \mathbf{u}_i) \quad (30)$$

$$\nabla_\lambda L_\mu = -\frac{\partial f(\lambda)}{\partial \lambda} - \nabla_\lambda \mathbf{g}_0(\mathbf{y}_0)^T \mathbf{e} - \nabla_\lambda \mathbf{g}_i(\mathbf{y}_i)^T \mathbf{e} \quad (31)$$

i = 1, 2, …, Nc

where $\mathbf{SL}_0 = diag(sl_{0j})$, $\mathbf{SU}_0 = diag(su_{0j})$, $\mathbf{\Pi L}_0 = diag(\pi l_{0j})$, $\mathbf{\Pi U}_0 = diag(\pi u_{0j})$, $\mathbf{SL}_i = diag(sl_{ij})$, $\mathbf{SU}_i = diag(su_{ij})$, $\mathbf{\Pi L}_i = diag(\pi l_{ij})$, $\mathbf{\Pi U}_i = diag(\pi u_{ij})$.

The nonlinear equations (18) – (31) in polar coordinates can be solved simultaneously. The simultaneous equations can be linearized and expressed in a compact Newton form

$$\mathbf{A}\ \Delta\mathbf{x} = -\mathbf{b} \quad (32)$$

where $\mathbf{A} = \dfrac{\partial \mathbf{b}}{\partial \mathbf{x}}$. $\mathbf{x} = [\mathbf{x}_0, \mathbf{x}_i, \boldsymbol{\lambda}\mathbf{u}_i, \lambda]^T$. $\mathbf{b} = [\mathbf{b}_0, \mathbf{b}_i, \mathbf{bu}_i, b_\lambda]^T$. $\mathbf{X}_0$ and $\mathbf{X}_i$ are given by

$$\mathbf{x}_0 = [\mathbf{sl}_0, \mathbf{su}_0, \boldsymbol{\pi}\mathbf{l}_0, \boldsymbol{\pi}\mathbf{u}_0, \mathbf{y}_0, \boldsymbol{\lambda}\mathbf{g}_0]^T \quad (33)$$

$$\mathbf{x}_i = [\mathbf{sl}_i, \mathbf{su}_i, \boldsymbol{\pi}\mathbf{l}_i, \boldsymbol{\pi}\mathbf{u}_i, \mathbf{y}_i, \boldsymbol{\lambda}\mathbf{g}_i]^T \quad (34)$$

and $\mathbf{b}_0$ $\mathbf{b}_i$, $\mathbf{bu}_i$ and b_λ are given by

$$\mathbf{b}_0 = [\nabla_{\mathbf{sl}_0} L_\mu, \nabla_{\mathbf{su}_0} L_\mu, \nabla_{\boldsymbol{\pi}\mathbf{l}_0} L_\mu, \nabla_{\boldsymbol{\pi}\mathbf{u}_0} L_\mu, \nabla_{\mathbf{y}_0} L_\mu, \nabla_{\boldsymbol{\lambda}\mathbf{g}_0} L_\mu]^T \quad (35)$$

$$\mathbf{b}_i = [\nabla_{\mathbf{sl}_i} L_\mu, \nabla_{\mathbf{su}_i} L_\mu, \nabla_{\boldsymbol{\pi}\mathbf{l}_i} L_\mu, \nabla_{\boldsymbol{\pi}\mathbf{u}_i} L_\mu, \nabla_{\mathbf{y}_i} L_\mu, \nabla_{\boldsymbol{\lambda}\mathbf{g}_i} L_\mu]^T \quad (36)$$

$$\mathbf{bu}_i = \nabla_{\boldsymbol{\lambda}\mathbf{u}_i} L_\mu \quad (37)$$

$$b_\lambda = \nabla_\lambda L_\mu \quad (38)$$

The security constrained TC problem can be solved iteratively via the Newton equation in (32), and at each iteration the solution can be updated as follows

$$\mathbf{sl}_0[k+1] = \mathbf{sl}_0[k] + \sigma\alpha_p \Delta\mathbf{sl}_0[k] \quad (39)$$

$$\mathbf{su}_0[k+1] = \mathbf{su}_0[k] + \sigma\alpha_p \Delta\mathbf{su}_0[k] \quad (40)$$

$$\mathbf{y}_0[k+1] = \mathbf{y}_0[k] + \sigma\alpha_p \Delta\mathbf{y}_0[k] \quad (41)$$

$$\boldsymbol{\pi}\mathbf{l}_0[k+1] = \boldsymbol{\pi}\mathbf{l}_0[k] + \sigma\alpha_d \Delta\boldsymbol{\pi}\mathbf{l}_0[k] \quad (42)$$

$$\boldsymbol{\pi}\mathbf{u}_0[k+1] = \boldsymbol{\pi}\mathbf{u}_0[k] + \sigma\alpha_d \Delta\boldsymbol{\pi}\mathbf{u}_0[k] \quad (43)$$

$$\mathbf{sl}_i[k+1] = \mathbf{sl}_i[k] + \sigma\alpha_p \Delta\mathbf{sl}_i[k] \quad (44)$$

$$\mathbf{su}_i[k+1] = \mathbf{su}_i[k] + \sigma\alpha_p \Delta\mathbf{su}_i[k] \quad (45)$$

$$\mathbf{y}_i[k+1] = \mathbf{y}_i[k] + \sigma\alpha_p \Delta\mathbf{y}_i[k] \quad (46)$$

$$\boldsymbol{\pi}\mathbf{l}_i[k+1] = \boldsymbol{\pi}\mathbf{l}_i[k] + \sigma\alpha_d \Delta\boldsymbol{\pi}\mathbf{l}_i[k] \quad (47)$$

$$\boldsymbol{\pi}\mathbf{u}_i[k+1] = \boldsymbol{\pi}\mathbf{u}_i[k] + \sigma\alpha_d \Delta\boldsymbol{\pi}\mathbf{u}_i[k] \quad (48)$$

$$\boldsymbol{\lambda}\mathbf{u}_i[k+1] = \boldsymbol{\lambda}\mathbf{u}_i[k] + \sigma\alpha_d \Delta\boldsymbol{\lambda}\mathbf{u}_i[k] \quad (49)$$

$$\lambda[k+1] = \lambda[k] + \sigma\alpha_p \Delta\lambda[k] \quad (50)$$

i = 1, 2, …, Nc

where k is the iteration count. Parameter $\sigma \in$ [0.995 - 0.99995]. α_p and α_d are the primal and dual step-length parameters, respectively: The step-lengths are determined as follows

$$\alpha p_0 = \min\left[\min\left(\frac{sl_{0j}}{-\Delta sl_{0j}}\right), \min\left(\frac{su_{0j}}{-\Delta su_{0j}}\right), 1.00\right] \quad (51)$$

$$\alpha d_0 = \min\left[\min\left(\frac{\pi l_{0j}}{-\Delta \pi l_{0j}}\right), \min\left(\frac{\pi u_{0j}}{-\Delta \pi u_{0j}}\right), 1.00\right] \quad (52)$$

$$\alpha p_i = \min\left[\min\left(\frac{sl_{ij}}{-\Delta sl_{ij}}\right), \min\left(\frac{su_{ij}}{-\Delta su_{ij}}\right), 1.00\right] \quad (53)$$

$$\alpha d_i = \min\left[\min\left(\frac{\pi l_{ij}}{-\Delta \pi l_{ij}}\right), \min\left(\frac{\pi u_{ij}}{-\Delta \pi u_{ij}}\right), 1.00\right] \quad (54)$$

i = 1, 2, …, Nc

for those $\Delta sl<0$, $\Delta su<0$, $\Delta \pi l<0$ and $\Delta \pi u>0$. α_p and α_d are determined by

$$\alpha_p = \min[\alpha p_0, \alpha p_i] \tag{55}$$

$$\alpha_d = \min[\alpha d_0, \alpha d_i] \tag{56}$$

$$i = 1, 2, \ldots, Nc$$

The Barrier parameter μ can be evaluated by

$$\mu = \frac{\beta \times Cgap}{2 \times Nh \times Nc} \tag{57}$$

where $\beta \in [0.01 - 0.2]$ and $Cgap$ is the complementary gap for the transfer capability calculation problem with security constraints, and it can be determined by

$$Cgap = (\mathbf{sl}_0)^T \boldsymbol{\pi}\mathbf{l}_0 - (\mathbf{su}_0)^T \boldsymbol{\pi}\mathbf{u}_0 + \sum_{i=1}^{Nc}[(\mathbf{sl}_i)^T \boldsymbol{\pi}\mathbf{l}_i - (\mathbf{su}_i)^T \boldsymbol{\pi}\mathbf{u}_i] \tag{58}$$

3.2 Solution Procedure of the Nonlinear Interior Point OPF

The solution procedure of the nonlinear interior point optimization algorithm for the unified security constrained transfer capability problem is summarized as follows:

Step 0: Set iteration count $k = 0$, $\mu = \mu 0$, and initialize the optimization solution
Step 1: If KKT conditions (18) – (30) are satisfied & complementary gap is less than a tolerance, output results. Otherwise go to step 2
Step 2: Form and solve Newton equation in (32)
Step 3: Update Newton solution (39) – (50)
Step 4: Compute complementary gap (58)
Step 5: Determine barrier parameter (57)
Step 6: Set k=k+1, and go to step 1

4 NUMERICAL RESULTS

4.1 Test Systems

Test cases in this paper are carried out on the IEEE 30 bus system. The IEEE 30 bus system has 6 generators, 4 OLTC transformers and 37 transmission lines. For all cases in this paper, the convergence criteria are:

(1) Complementary gap $Cgap \le 5.0e^{-4}$
(2) Barrier parameter $\mu \le 1.0e^{-4}$
(3) Maximum mismatch of the Newton equation $\| \mathbf{b} \|_\infty \le 1.0e^{-4}$ p.u.

4.2 The IEEE 30 Bus System Results

In the simulations, it is assumed that all generators except the generator at bus 1 are using preventive control of active power generation while other control resources are using corrective controls. Bus 1 is taken as slack bus.

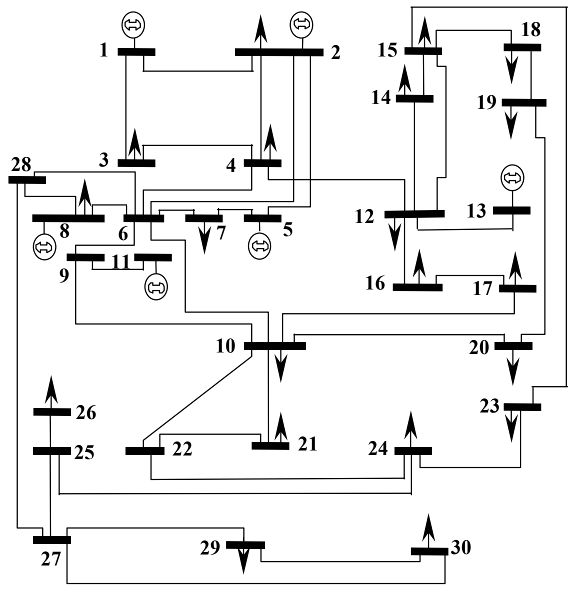

Figure 1: IEEE 30-bus system

In the study, the IEEE 30 bus system was divided into two areas. The two areas are interconnected by intertie lines: 4-12, 6-9, 6-10, and 28-27 while buses 4, 6, 28 belong to the area 1. The power transfer from the area 1 to the area 2 has been investigated. The single state cases are presented as follows,

Case 1: This is a base case for the transfer capability computation.
Case 2: This is similar to Case 1 except that there is an outage of line 5-7.
Case 3: This is similar to Case 1 except that there is an outage of line 24-25.

The transfer capabilities of Case 1 – 3 on the IEEE 30 bus system are shown in Table 1.

Case no	Case 1	Case 2	Case 3
Transmission line outage description	None	Line 5-7	Line 24-25
λ	1.64	1.49	1.63
Number of iterations	14	15	16

Table 1: Transfer Capability Results of Single State Cases

The transfer capability results of cases with security constraints on the IEEE 30-bus system are presented as follows,

Case 4: This is a case for the transfer capability computation including one *N*-1 contingency with line 5-7 outage.
Case 5: This is a case for the transfer capability computation including one *N*-1 contingency with line 24-25 outage.
Case 6: This is a case for the transfer capability computation with two contingencies. The first contingency is the outage of line 5-7 while the second one is the outage of line 24-25.

The transfer capabilities of Case 4 – 6 are shown in

Table 2. From this Table, it can be seen the CPU time for the transfer capability calculations with security constraints is proportional to the total number of base case and contingencies.

Case no	Case 4	Case 5	Case 6
Transmission line outage description	Base case and one *N*-1 contingency with line 5-7 outage	Base case and one *N*-1 contingency with line 24-25 outage	Base case and two *N*-1 contingencies: with line 24-25 outage and line 5-7 outage
λ	1.49	1.63	1.49
Number of iterations	14	14	15
Normalised CPU time	100%	100%	150%

Table 2: Transfer Capability Results of Cases with Security Constraints

Case no	Case 7	Case 8
Transmission line outage description	Base case and one *N*-1 contingency with line 5-7 outage	Base case and two *N*-1 contingencies with line 24-25 outage and line 5-7 outage, respectively
λ	1.75	1.64
Number of iterations	28	32

Table 3: Transfer Capability Results of Cases with FACTS

Case 7	Case 8
Base case: *Shunt converter*: $Vsh = 0.9786$ p.u. $\theta sh = -10.61°$ *Series converters*: $Vse = 0.1829$p.u. $\theta se = 118.05°$	Base case: *Shunt converter*: $Vsh = 0.9596$ p.u. $\theta sh = -7.54°$ *Series converters*: $Vse = 0.1033$p.u. $\theta se = 179.54°$
Contingency with line 5-7 outage: *Shunt converter*: $Vsh = 0.9707$ p.u. $\theta sh = -12.86°$ *Series converters*: $Vse = 0.2503$p.u. $\theta se = -114.30°$	Contingency with line 5-7 outage: *Shunt converter*: $Vsh = 0.9615$ p.u. $\theta sh = -7.51°$ *Series converters*: $Vse = 0.1006$p.u. $\theta se = -179.67°$
	Contingency with line 24-25 outage: *Shunt converter*: $Vsh = 0.9685$ p.u. $\theta sh = -9.85°$ *Series converters*: $Vse = 0.1280$p.u. $\theta se = -145.05°$

Table 4: UPFC Solutions of Cases 7 and 8

Cases 7 and 8 are presented to show the security constrained transfer capability computation with FACTS. Cases 7 and 8 are corresponding to Cases 4 and 6, respectively except that a UPFC is installed between buses 3 and 4. The test results of Cases 7 and 8 are shown in Table 3. The UPFC solutions of Cases 7 and 8 are given by Table 4. In the calculations, the UPFC is using corrective controls. The results indicate that UPFC taking corrective control actions can improve the transfer capability effectively.

A further Case – Case 9 is carried out with 8 contingencies included in the transfer capability computation, the algorithm converges in 15 iterations. The *N*-1 contingencies are outages of lines 2-6, 4-6, 5-7, 6-7, 10-21, 12-15, 12-16 and 24-25, respectively. It is found for this case λ =1.05.

It can also be seen that in the above cases, the more contingencies, the less system transfer capability.

4.3 Discussion of the Results

From these results on the IEEE 30 bus system, it can be seen:

1) numerical results demonstrate the feasibility of the proposed unified optimisation framework for transfer capability computation with security constraints.
2) The computation framework is general, which can simultaneously take voltage, thermal and voltage stability limits as well as any electricity transaction constraints into consideration.
3) The optimisation framework of transfer capability computation with security constraints can be solved by nonlinear interior point methods.
4) In addition, FACTS controllers can be modelled as corrective control devices in the calculations.

5 CONCLUSION

A unified optimization framework for transfer capability computation with security constraints has been proposed in this paper. The unified computation framework is general, which can simultaneously take voltage, thermal and voltage stability limits of base case and contingencies as well as any electricity transaction constraints into consideration. In addition, modelling of FACTS controllers has been considered. The unified optimization framework for transfer capability computation with security constraints has been solved successfully by the nonlinear interior point methods.

Numerical results based on the IEEE 30-bus system have demonstrated the feasibility and effectiveness of the method proposed.

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