Optimal regulating market bidding strategies in hydropower systems

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Abstract - The transmission system operator (TSO) is responsible for keeping the balance between production and consumption in the power system. In order to do so, the TSO continuously trades power on the so-called regulating market. Hydropower is a flexible power source well suited for trading on the regulating market because it allows fast changes in generation output.

This paper describes an optimization model for creating optimal bidding strategies for the regulating market from a hydropower producer perspective. The model is based on nonlinear stochastic programming, where the regulating market prices are considered stochastic variables. The model also includes generation of regulating power price scenarios and scenario tree construction.

Keywords - Power market, hydropower, stochastic optimization, stochastic process, bidding strategy

1 Introduction

RE-REGULATION of electricity markets increases the need for planning tools considering trading of power and financial contracts under uncertainty. Often, these tools are based on stochastic optimization where multi-dimensional stochastic processes are used to represent uncertain parameters, such as market prices, reservoir inflows, electrical load etc. Examples of such models can be found in [1], [2], [3], [4] and [5].

When submitting bids to the market, a bidding strategy is applied. Quite a lot of research has been done considering bidding strategies on electricity markets. A literature review covering strategic bidding strategies can be found in [6], and also [1] gives a comprehensive literature review apart from the model presented in the paper. Most research regarding bidding strategies on electricity markets has focused on the ahead market, the regulating market has not been subjected to the same amount of research. A few papers have been published on the subject, e.g. [7], [8] and [9]. These papers though use constant or linear bidding strategies, whereas this paper presents a nonlinear supply function. Applying a nonlinear function allows for increased flexibility for the supply function, resulting in better approximation of the real bids. An alternative is to use a number of discrete bids, but this require integer variables in the optimization, which, makes the problem harder to solve [10].

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1.1 *Market structure*

The model described in this paper prerequisite a market structure where a day-ahead spot market and a regulating market are present.

1.1.1 *Spot market*

On the electricity spot market actors can place hourly bids, consisting of a price per MWh and a quantity in MWh, before the delivery period. Often, the delivery period is 24 hours, and the bids are submitted the day before the day of delivery.

When the market has closed, supply and demand curves are constructed for each hour in the coming 24-hour period to define a market price and a traded quantity for each hour. Thus, the prices and quantities for all hours are set at the same time.

1.1.2 *Regulating market*

The transmission system operator, TSO, is responsible for keeping the physical balance between production and consumption in the power system. In systems where the TSO does not own any production units, the TSO must continuously purchase or sell power to handle imbalances between production and consumption. This trading is performed on the so-called regulating market. The regulating market is often organized so that actors can submit bids for a specific hour minutes before the start of the hour. When regulation power is needed, the TSO activates the most favorable bid.

The Nordic countries Sweden, Norway and Finland have a common regulating market [11], to which bids for upward or downward regulation can be submitted by producers and consumers. The upward regulation price a specific hour is set by the bid with the highest price accepted that hour. For downward regulation, the price is set by the accepted bid with the lowest price during the hour. All accepted upward regulation bids are paid the same price, and all accepted downward regulation bids are paid the same price. Most of the hours, no bids of one (or occasionally both) of the regulation types were called during the hour. The corresponding price is then undefined. Fig. 1 shows examples of spot and regulating prices from the Nordic market.

Since actors on the regulating market do not have to submit their bids until minutes before the start of the hour, a planning is performed according to new information (e.g. updated weather forecasts, prices from previous hours etc.) before the bids are submitted. This implies that there will be a re-planning process generating bids to the market just ahead of the start of each hour.

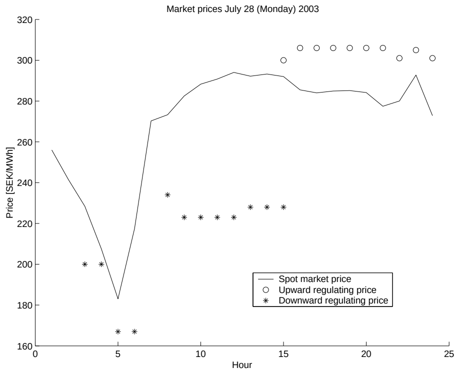

Figure 1: Examples of spot and regulating market prices

1.2 Hydropower

Hydropower is a power source highly suitable for balancing fast variations in production or consumption in the power system because of its possibilities of fast regulation.

In the Nordic countries there are substantial amounts of hydropower. Thereby, there exist relatively good possibilities for compensating imbalances in the Nordic power system by using hydropower.

2 Objective and motivation

2.1 Objective

The aim of this paper is to develop a tool that produces optimal bidding strategies for trading on the regulating market from a hydropower producer perspective. Results from the model are bidding strategies for upward and downward regulation.

2.2 Motivation

When the production uncertainties increases in the system, e.g. because of increasing amounts of wind power, the regulating market becomes more important for the operation of the power system and thereby more interesting for the market actors. This specially apply to hydropower owners.

Many models have been developed to support trading on the power market, but most of them focuses on the spot market and do not consider the regulating market. Hence, there is a potential need for tools for trading on the regulating market.

3 Model

The main model described in this paper is based on nonlinear multistage stochastic optimization [12], where the prices on the regulating market are considered stochastic variables. The benefit of using a stochastic model compared with a deterministic model is that the variance, or risk, is taken into account.

In this paper, a price-taking producer is assumed, implying that actions taken by the actor will not affect the market prices. Further, the model assumes different prices for upward and downward regulation.

In order to avoid repetition, the distinction between upward and downward regulation are not addressed in all sections below. Where the upward and downward regulation are treated differently, this is indicated with the super scripts d (downward) and u (upward) respectively. A list of notations with all definitions of sets, parameters and variables can be found in appendix.

Letting ζ denote the stochastic vector of upward and downward regulating prices, $\zeta = (C^d\ C^u)$, the stochastic programming model described in this paper can on general form be written as

$$\begin{array}{rl} \max & d^T x + E_\zeta[T(\zeta)^T x] \\ \text{s.t.} & A^T x = b \\ & g(x,\zeta) = 0 \\ & x \geq 0, \end{array} \tag{1}$$

where x is a vector containing all the variables. The stochastic properties of the problem are encapsulated in the vector $T(\zeta)$ and the nonlinear functions $g(x,\zeta)$. The problem defined by (1), consists of the parts described in Table 1, where the objective function and the different types of constraints are presented. In the following sections, the different parts and properties included in the model are described.

		Linear or Nonlinear	**Deterministic or Stochastic**
Obj.	Value of stored water	Linear	Deterministic
	Income from selling on regulating market	Linear	Stochastic
	Cost for buying on regulating market	Linear	Stochastic
Constr.	Hydrological coupling	Linear	Deterministic
	Power production	Linear	Deterministic
	Load balance	Linear	Deterministic
	Bidding strategies	Nonlinear	Stochastic
	Downward restriction	Linear	Deterministic
	Initial values	Linear	Deterministic

Table 1: Objective function parts and types of constraints in the optimization problem

3.1 Regulating power market

3.1.1 Generation of price scenarios

An important property of the regulating market is the market prices, which, can be regarded as stochastic variables. In the developed optimization model, the prices are represented by a scenario tree, created by generation of price scenarios. Thus, price scenarios must be generated, which, should resemble the real behavior of the prices. If no regulation of a certain type was needed during the hour, the corresponding price is undefined. This is in the developed model represented by setting the price to zero.

The model used to generate scenarios is based on the methods presented in [13], where an auto regressive integrated moving average (ARIMA) process is used to model the regulating market prices. The ARIMA process, $\{D_k\}$, is defined by the following equation [14]:

$$(1-B)^d\Phi(B)(D_k - m_D) = \Theta(B)Z_k, \qquad (2)$$

where $\Phi(B)$ and $\Theta(B)$ are polynomials of grade p and q respectively, and Z_k is a white noise sequence, WN$(0, \sigma_Z)$. B denotes the back shift operator.

The ARIMA model is not alone suitable for modelling of regulating price scenarios because of the occasions when the price is undefined and thereby set to zero. Therefore, in this paper the prices are modelled according to

$$C_k = (D_k + m_D)E_k, \qquad (3)$$

where

C_k	is the regulation power price
D_k	is a variable modelled with an ARIMA process
E_k	is a binary stochastic variable, i.e. $E_k \in \{0, 1\}$.

D_k describes the "normal" behavior of the prices when they are not zero, and E_k decides when the prices are zero or not. The parameters of the ARIMA model are estimated using the methods described in [13]. Data used when estimating the parameters consists of hourly mean values of the prices, where the undefined prices are excluded, for some time period. This time period should not be too short in order to make the parameter estimation robust [14].

Instead of modelling E_k, the time lengths that E_k is zero, T_0, and one, T_1, respectively are considered. T_0 and T_1 are given by

$$\begin{array}{ll} E_k = 0, & \forall\, k \in \{t, \ldots, t+T_0\} \\ E_k = 1, & \forall\, k \in \{t, \ldots, t+T_1\}. \end{array} \qquad (4)$$

From historical data, the distributions, $F_{T_0}(k)$ and $F_{T_1}(k)$ of these time lengths can be estimated.

When simulating a regulating power price scenario, D_k and E_k are generated for all stages in the planning period. The set $\{D_k, k \in K\}$ is generated directly using the ARIMA model and $\{E_k, k \in K, \}$ is generated by randomizing the stochastic variables T_0 and T_1 until all stages are covered.

The assumption of different prices for upward and downward regulation imply that two scenario generation models have to be derived. These two models are though based on the same mathematics.

3.1.2 *Price scenario tree*

As earlier mentioned are the regulating market prices regarded stochastic variables. This is modelled by using a set of scenarios $\{\xi_s\}_{s\in S}$, which are generated according to (3). Since different price for upward and downward regulation is assumed, each scenario consists of realizations of both prices. Thus,

$$\xi = \{(c_k^d \;\; c_k^u), \forall\, k \in K\}. \qquad (5)$$

The set of scenarios, $\{\xi_s\}_{s\in S}$ defines a set of nodes, N, representing a scenario tree. The nodes in the tree represents decisions and the branches represents possible outcomes of the two dimensional stochastic process. Initially, the node probabilities, $p(n)$ are equal for all nodes at a specific stage k.

To assure that the stochastic properties are represented correctly, many scenarios must be generated. However, when the number of scenarios increases, the number of variables and constraints in the optimization problem also increases, making the problem difficult to solve in reasonable time. This phenomenon is often referred to as the "curse of dimensionality" of stochastic programming problems. One strategy to handle this is to generate a large number of scenarios, and then reduce the number of scenarios in the tree used when solving the optimization problem. The method used for reducing the number of scenarios is presented in [15] and [16]. The method uses the Kantorovich metric to evaluate what scenarios to eliminate or to bundle to assure that the stochastic properties of the tree are not changed more than necessary, or that given a tolerance, assuring that as many scenarios as possible are reduced without violating the tolerance criteria. The output of the scenario reduction model is a reduced tree with adjusted node probabilities.

3.1.3 *Re-planning process*

The re-planning performed by the actors on the regulating market implies a multistage programming problem. For each hour in the planning period, decisions are made what bids to submit. When using a scenario tree, the decisions will be made in the different nodes in the tree. Thereby, the variables in the optimization problem will be node dependent.

3.2 *Hydropower*

3.2.1 *Hydrological constraints*

The developed model handles a multi-reservoir hydro system. This implies that the hydro station structure must be managed since the water flows from one station to the ones downstream. Thus, the stations are not operated separately, but are connected in a certain structure.

The contents in a reservoir at the end of a specific hour is dependent on the contents at the end of the previous hour. In the developed model, this dependency will occur between successive nodes in the scenario tree. Thus, each node will have reservoir contents, $x_i(n)$, which is dependent on the contents in the parent node. Denoting the parent node of n with $\tilde{n}$, the hydrological constraints can be written as

$$\begin{aligned} x_i(n) = x_i(\tilde{n}) - \sum_{j\in J_i} u_{ij}(n) - s_i(n) + \\ \sum_{k\in\Omega_i}(\sum_{l\epsilon J_k} u_{kl}(n) + s_k(n)) + w_i, \quad \forall\, n \in N. \end{aligned} \qquad (6)$$

The inflows to the reservoirs, w_i, are assumed deterministic and constant over the time period.

3.2.2 Generation curves

The discharge-generation curve of a hydropower plant is generally nonlinear and non-concave, which may cause problems in optimization problems. The curves for the different power plants are modelled with piecewise linear curves as in Fig. 2, constructed by using the best local efficiency points as break points. The generation in power plant i, P_i, for node n can thereby be calculated as

$$P_i(n) = \sum_{j \in J_i} \mu_{ij} u_{ij}(n), \quad \forall\, n \in N. \tag{7}$$

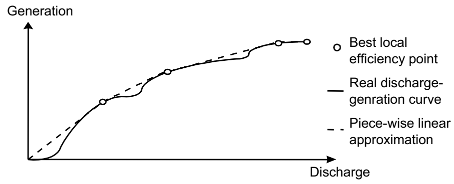

Figure 2: Hydropower generation curve

3.3 Bidding strategy model

In the developed model the bidding strategies, or supply functions, are modelled using continuous functions, where the quantity of the bid is a function of the price. The supply function decided in a specific node in the scenario tree is subjected to prices in the child nodes, and will thereby decide the quantities traded on the market in these nodes. Thus,

$$q(n) = f_{\tilde{n}}(c), \quad \forall\, n \in \{n_1, \ldots, n_{\#N}\}, \tag{8}$$

where $f_{\tilde{n}}(c)$ denotes the supply function of the parent node, $\tilde{n}$.

The bidding function can be chosen somewhat arbitrarily. Though, to assure the solution of the optimization problem to be global, the bidding function should be, if not linear, at least a convex function [10].

3.3.1 Upward regulation

In each node, a decision is made represented by the supply function. Then the bidding strategy is subjected to a number of outcomes of the stochastic prices in form of scenarios branching out of the current node. Some of these realizations may become extreme with very high prices. The bidding function is applied to all of the children of the node, and thereby also to the extreme prices. For upward regulating this can cause some problems. Assuming all children of a specific node has normal prices, the bidding function may be increasing. If the same function is applied to an extreme price, it might be physically impossible to generate the power that the function returns. This can be avoided by designing the supply function with the possibility of a threshold level. This means that the traded quantity will after a certain level not increase even though the price becomes high. The typical shape of such a function is presented in Fig. 3. This is also the typical shape of a supply curve for a power producer.

In the developed model, the function

$$f(c) = \beta \frac{c}{1 + \alpha c} \tag{9}$$

is used for modelling the supply function for upward regulation. In the optimization problem, the parameters α and β are variables and the prices c are parameters. The quantities in the child nodes are decided by the supply function of the parent node. Thus, the constraints in the optimization problem can be written as

$$q^u(n) = \beta^u(\tilde{n}) \frac{c_n^u}{1 + \alpha^u(\tilde{n}) c_n^u}, \quad \forall\, n \in N. \tag{10}$$

Rewriting the expression yields

$$q^u(n)(1 + \alpha^u(\tilde{n}) c_n^u) = \beta^u(\tilde{n}) c_n^u, \quad \forall\, n \in N, \tag{11}$$

which is a quadric expression. Thereby, the function defined in (9) is convex.

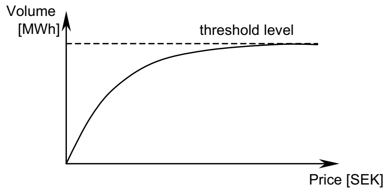

Figure 3: Upward regulating supply function

3.3.2 Downward regulation

The downward regulation function should asymptotically go towards zero as the price goes towards infinity. This is the typical shape of a demand curve for an actor on the power market. In the developed model, the function

$$f(c) = \beta \frac{1}{1 + \alpha c} \tag{12}$$

is used for representing the downward regulation bidding strategies. This is, similarly to (9), also a convex function.

The supply function defined by (12) will be subjected to a number of prices, of which some might be zero. If at least one price is zero, the only feasible solution of the optimization problem will be to set $\beta_n^d = 0$. To avoid this problem, the bidding function will only be applied in the nodes where the downward regulation price is greater than zero. When the price is zero, the traded quantity in that node will be set to zero. Thus,

$$q^d(n) = \begin{cases} \beta^d(\tilde{n}) \dfrac{1}{1 + \alpha^d(\tilde{n}) c_n^d} & \text{if } c_n^d > 0 \\ 0 & \text{if } c_n^d = 0 \end{cases}. \tag{13}$$

3.4 Downward regulation restrictions

A power producer acting on the downward regulation market must be able to decrease the production if the submitted bids are accepted by the TSO. This sets limits on the quantities of electricity that a producer can purchase on the downward regulation market. Assuming a producer only active on the spot market having bids accepted on that market for a specific hour, can at the most decrease the production with the same amount of power that is sold on the spot market that hour. Thus, constraints on the form

$$q^d(n) \le q^{spot}(k_n), \quad \forall\, n \in \{n_1, \dots, n_{\#N}\}, \tag{14}$$

where $q^{spot}(k_n)$ denotes the amount of power sold on the spot market the hour k_n corresponding to the node n, must be implemented in the optimization.

3.5 Load balance

To assure the balance between production and traded quantities, constraints on the following form must be added to the problem:

$$\begin{aligned} q^u(n) - q^d(n) + q^{spot}_{k_n} = \\ \sum_{i \in I} P_i(n), \quad \forall\, n \in \{n_1, \dots, n_{\#N}\}. \end{aligned} \tag{15}$$

3.6 Initial values

The situation being present at the start of the optimization is modelled using initial values for some of the variables reservoir contents, discharge and spillage. Thus,

$$\begin{array}{rcll} x_i(0) & = & x0_i, & \forall\, i \in I \\ u_{ij}(0) & = & u0_{ij}, & \forall\, i \in I,\ j \in J_i \\ s_i(0) & = & s0_i, & \forall\, i \in I. \end{array} \tag{16}$$

3.7 Variable limits

Some of the variables in the optimization problem might be subjected to limitations. The variables limits in the developed model are the following:

$$\begin{array}{lllll} 0 & \le & x_i(n) & \le\ \overline{x}_i & \forall\, i \in I,\ n \in N \\ 0 & \le & u_{ij}(n) & \le\ \overline{u}_{ij} & \forall\, i \in I,\ j \in J_i,\ n \in N \\ 0 & \le & s_i(n) & & \forall\, i \in I,\ n \in N \\ 0 & \le & s_i(n) & & \forall\, i \in I,\ n \in N. \\ 0 & \le & q^u(n) & & \forall\, n \in N. \\ 0 & \le & q^d(n) & & \forall\, n \in N. \end{array} \tag{17}$$

The supply function parameters α and β are unconstrained, but there exists implicit constraints since the traded quantities are non-negative.

3.8 Objective function

The objective of the optimization is to maximize the expected income including the value of stored water. The expected income from selling on the upward regulating market can be formulated as

$$z^u = \sum_{n \in N} p(n) q^u(n) c^u_n. \tag{18}$$

The corresponding expected cost for purchasing on the downward regulating market can be calculated as

$$z^d = \sum_{n \in N} p(n) q^d(n) c^d_n. \tag{19}$$

The value of the water stored in the reservoirs after the planning period must also be considered. In order to do so, the future production equivalent of the different stations must be estimated. In this paper it is assumed that the future power generation will be performed at best efficiency, i.e. in the first segment of the piecewise linear generation curve, see Fig. 2. The hydrological structure of the system must also be considered since the water flows from one station to the ones downstream. Thus, the expected value of the water left in the reservoirs at the end of the planning period can be calculated as

$$z^{stored} = c_w \sum_{n \in L} p(n) \sum_{i \in I} \sum_{j \in \Gamma_i} \mu_{j1} x_i(n). \tag{20}$$

The total objective function thereby becomes

$$z = z^u - z^d + z^{stored}. \tag{21}$$

4 Case study

A case study was performed using the developed model described in this paper. The scenario generation model was implemented in Matlab and the optimization problem was solved using GAMS and the NLP solver CONOPT. The GAMS software package SCENRED was used for the scenario reduction.

The system analyzed consists of three hydro stations. The structure of the hydro system is presented in Fig. 4 and system data are presented in Table 2. The production equivalents of the piecewise linear generation curves are presented in Table 3.

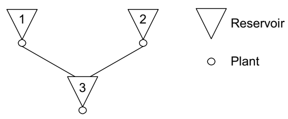

Figure 4: Hydropower system in case study

The aim of the case study was to generate bidding strategies for the first hour in a 24 hour period. In order to generate regulating market price scenarios, spot market prices must be available. Prices from the Nordic Power Exchange, Nord Pool, for January 8, 2003 (Wednesday), were used in the case study.

Station number	Max. res. contents $\overline{x}_i$ [he]	Local inflow w_i [he/h]
1	2 668	24
2	57 950	68.6
3	241 800	6.8

Table 2: Hydro data for system in case study

Station number	Segment 1		Segment 2	
	μ_{i1} [MWh/he]	$\overline{u}_{i1}$ [he]	μ_{i2} [MWh/he]	$\overline{u}_{i2}$ [he]
1	0.340	24	0.312	3
2	0.450	100	0.415	60
3	0.330	100	0.301	60

Table 3: Power generation data

To estimate the ARIMA and binary processes used when generating price scenarios, regulating market prices are needed. For this, Swedish regulating market prices for

2003 were used. The estimated ARIMA(2,1,2) polynomials became the following:

$$\begin{aligned}\Phi^u(B) &= 1 - 0.4786B + 0.2433B^2 \\ \Theta^u(B) &= 1 - 0.7104B - 0.0009B^2 \\ \Phi^d(B) &= 1 + 0.0524B + 0.3072B^2 \\ \Theta^d(B) &= 1 - 0.0730B + 0.0126B^2.\end{aligned} \tag{22}$$

The white noise standard deviations for the two ARIMA models were

$$\begin{aligned}\sigma_Z^u &= 185.9 \\ \sigma_Z^d &= 140.5.\end{aligned} \tag{23}$$

The distribution functions of the binary processes are displayed in Fig. 5.

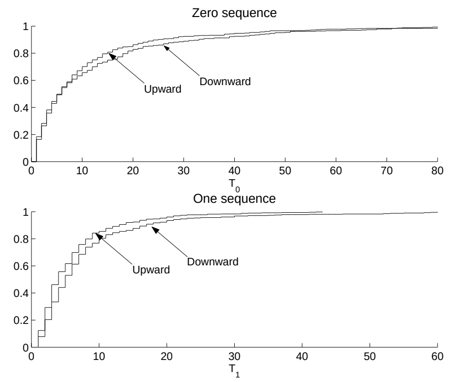

Figure 5: Distribution functions of zero and one sequences

The original scenario tree included 2449 nodes, which were reduced to 1201 nodes, representing about 90% of the information in the original tree. The value of stored water was set to 330 SEK/MWh. It is assumed that the actor has sold 83.7 MWh each hour on the ahead spot market, corresponding to full production in the first segments of the three hydropower stations.

The results of the optimization, consisting of the supply functions for the first hour of the planning period, are presented in Fig. 6. In this particular case, the upward regulation supply function is close to linear, while the corresponding function for downward regulation shows a nonlinear decreasing behavior.

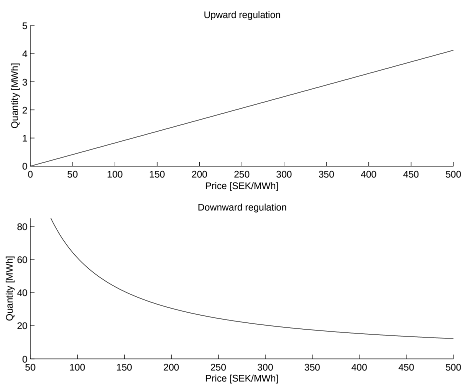

Figure 6: Bidding strategies in case study

5 Discussion

This paper describes a model based on nonlinear stochastic optimization and stochastic processes for generating bidding strategies for the regulating market from a hydropower producer perspective. The model regards the regulating power market prices as stochastic variables, which are represented by a scenario tree in the optimization model.

5.1 Generation of price scenarios

Generation of the price scenarios is performed using a model based on ARIMA processes and a binary process according to (3). A performance analysis of this model is not with scope of this paper and a more thorough analysis of the regulating market prices is left for future work.

The binary sequences of the upward and downward regulation prices are in the developed model considered uncorrelated. Fig. 1 clearly shows that this is not the case in reality. This is not covered within this paper and is left for future work.

5.2 Bidding strategy function

The supply function used in this paper is rather non-complex, which, limits the fan of possible outcomes of the optimization. It is easy to implement other supply functions, which, can relaxe these limitations. However, the chosen function is suitable from an optimization perspective because of its convexity.

The supply function is assumed to be decided by the generating companies. Though, in reality, the supply function partly is dependent on the bidding protocols of the regulating market. This is not covered in this paper.

Currently, the bids are regarded as a quantity of energy in MWh. In reality, the bids consists of an amount of power in MW. The number of produced MWh will thereby depend on when during the hour the bid was accepted. This is not consider in the presented model.

5.3 Hydropower model

The hydropower discharge-generation curves are approximated by using piecewise linear curves, which, is a suitable approximation in LP-problems. However, the developed model is nonlinear, implying that the optimal solution might be in the interior of the set. This decreases the chance that the optimal discharges will be in the breakpoints of the discharge-generation curves. Modelling these curves using more detailed functions could therefore be advantageous.

Delay times, costs for changing output and start/stop costs are other important aspects of hydropower modelling. The delay time and costs for changing output are rather straightforward to implement in the model, but is not covered in this paper. The start/stop costs are more difficult since the implementation require integer variables. The problem will then become a mixed integer nonlinear programming problem, which, are difficult to solve.

In the presented case study, a small hydropower system is used. Large systems can easily be implemented, but can result in large execution times for solving the optimization problem. These effects have, however, not been analyzed in this paper.

6 Conclusions

When the short-term production or consumption uncertainties increases in the power system, the need for planning tools considering the regulation market increases. An conclusion of this paper is that nonlinear stochastic optimization models, where the regulating market prices are considered stochastic variables and modelled using a scenario tree, are possible to use for generating bidding strategies for the regulating market.

A Nomenclature

A.1 *Indices and Index sets*

K	Stages in planning problem
N	Nodes in scenario tree
J_i	Segments in station i
I	Hydropower stations
Ω_i	Stations directly upstream of i
Γ_i	All stations downstream station i
$\#$	Cardinality operator
$\tilde{n}$	Parent node of node n
k_n	Stage corresponding to node n
L	Leaves in scenario tree

A.2 *Parameters*

$q^{spot}(k)$	Quantity sold on spot market hour k
$\Phi(B)$	AR polynomial of order p
$\Theta(B)$	MA polynomial of order q
D_k	Mean corrected normal price, stage k
Z_k	White noise
σ_Z	White noise standard deviation
E_k	Binary price behavior, stage k
m_D	Mean value of D
T_0, T_1	Length of zero and one sequence, respectively
F_{T_0}, F_{T_1}	Distribution function of T_0 and T_1, respectively
C_k	Stochastic regulation price, stage k
c_k	Realization of C_k
ξ_s	Scenario in scenario tree
$p(n)$	Node probabilities
μ_{ij}	Production equivalent, station i, segment j
w_i	Inflow to reservoir i

A.3 *Variables*

$x_i(n)$	Reservoir content, station i, node n
$u_i(n)$	Discharge, station i, node n
$s_i(n)$	Spillage, station i, node n
$f_n(c)$	Bidding strategy function, node n
$\alpha(n)$	Bidding function parameter, node n
$\beta(n)$	Bidding function parameter, node n
$q(n)$	Quantity traded, node n

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