A LINEAR TIME MODEL OF THE VOLTAGE SOURCE CONVERTER FOR STATCOM APPLICATIONS

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Abstract – This paper provides a new model for a two levels Voltage Source Converter (VSC) considering distorted voltages at the Point of Common Coupling. By means of a linear time model it is possible to consider the harmonic distortion in the input voltage and solving the equations in a piecewise way. Concatenating their solutions allows to fully determine the whole fundamental current cycle and DC link voltage. A particular study about the STATCOM operation of the VSC is made in this work, showing the influence of voltage harmonic distortion on the injected reactive currents and average DC capacitor voltage. Simulations show that significant deviations with respect to a sinusoidal voltage supply case exist.

Keywords: VSC, Linear Time Model, STATCOM model.

1 INTRODUCTION

The three-phase Voltage Source Converter (VSC) is becoming an important part of the electric power system since it is the main block of new Flexible AC Transmission Systems (FACTS). Thus, it is important to develop models to assess the behavior of this new component of the power system. Several models have been proposed for this purpose that can be summarized as follows:

- Continuous time models [1]. These models do not take into account the switching nature of the VSC, represented as an ideal DC to AC converter.
- Time averaging models [2]. In spite of taking into account the switching process, they are not able to represent the harmonic interaction.
- Linear time varying models [3]. These models represent the VSC as a piecewise linear circuit with a determined switching operation. In this description it is possible to consider the harmonic interactions. An approximate solution based on the assumption of a system voltage containing only a fundamental positive sequence component is presented in [3]. However, this assumption is not suitable for distribution systems, where the high presence of non-linear loads is turning the voltage more and more distorted with harmonic components.

This paper presents a linear time varying model of the VSC for STATCOM applications considering the effect of source voltage harmonics. This effect cannot be neglected because the DC capacitor of the VSC is quite sensible to them. The reason is that the harmonic impedance of this capacitor decreases when the harmonic order increases. This paper is organized as follows. First, the formulation of the problem is presented. Then, the steady-state computation is calculated from the previous equations. Some simulations comparing the solutions for the sinusoidal and distorted voltage supply conditions are presented. Finally, the main conclusions are pointed out.

2 PROBLEM FORMULATION

The VSC considered in this paper is the basic converter shown in Figure 1, a three-phase two-level inverter with autocommutated power switches. The inverter is connected to the distribution system through an inductor. The power losses of the converter plus the inherent losses of the inductor are represented by the resistor connected in series with the coupling reactor.

The VSC is a periodically-switched power device whose time evolution can be represented by a sequence of linear circuits. The transition between these circuits depends on the switching pattern. This basic model is sufficient for representing most of VSC applications such as shunt or series connected devices or even hybrid connected devices [4].
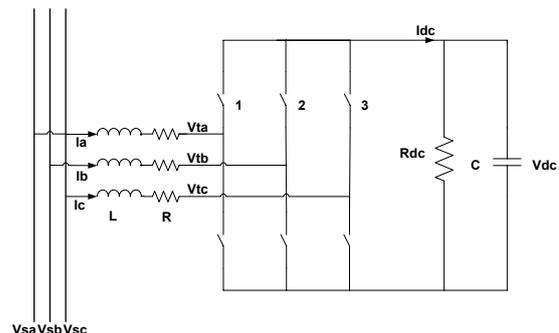

Figure 1: The basic VSC circuit.

The position of the valves, which have been considered ideal switches, determines the eight possible states of the inverter, each being associated to a voltage at the inverter terminals. The voltage represented in the $\alpha\beta$-frame and the possible states with the corresponding position of the valves are represented in Figure 2 and

Table 1 respectively. As it can be advised states 0 and 7 are the same, so seven sets of differential equations are necessary in order to build the complete model of the inverter.

The VSC gating signals control the three upper switches while the lower valves are gated in a complementary fashion. This switching strategy avoids the short-circuit of the DC capacitor.
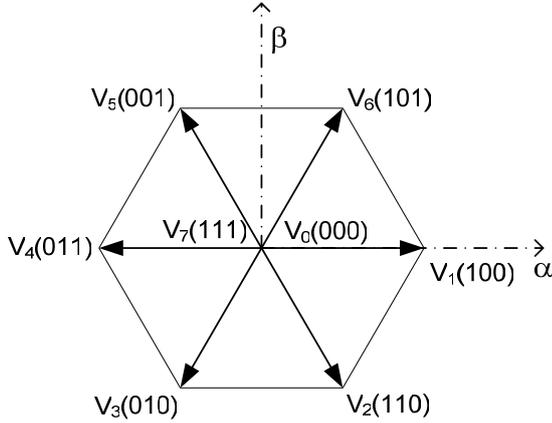

Figure 2: Feasible voltage space vectors.

Vector	S_1	S_2	S_3	V_{ab}	V_{bc}	V_{ca}	$V_{\alpha\beta}$
V_0	0	0	0	0	0	0	$\bar{v}_0 = 0V$
V_1	1	0	0	V_{dc}	0	$-V_{dc}$	$\bar{v}_1 = \frac{2}{\sqrt{3}}V_{dc}\angle 0°$
V_2	1	1	0	0	V_{dc}	$-V_{dc}$	$\bar{v}_2 = \frac{2}{\sqrt{3}}V_{dc}\angle -60°$
V_3	0	1	0	$-V_{dc}$	V_{dc}	0	$\bar{v}_3 = \frac{2}{\sqrt{3}}V_{dc}\angle -120°$
V_4	0	1	1	$-V_{dc}$	0	V_{dc}	$\bar{v}_4 = \frac{2}{\sqrt{3}}V_{dc}\angle -180°$
V_5	0	0	1	0	$-V_{dc}$	V_{dc}	$\bar{v}_5 = \frac{2}{\sqrt{3}}V_{dc}\angle -240°$
V_6	1	0	1	V_{dc}	$-V_{dc}$	0	$\bar{v}_6 = \frac{2}{\sqrt{3}}V_{dc}\angle -300°$
V_7	1	1	1	0	0	0	$\bar{v}_7 = 0V$

Table 1: Switches states and their corresponding space vectors.

A linear time varying model is presented in [3], being the basic theoretical background given in [5], [6]. Continuing these works, an extended model that considers harmonic distortion in the source voltage is developed. It has been considered that the system is piecewise linear with transition between each state determined by the control commands, as explained in the next section.

Standard linear techniques can be employed to solve the corresponding equations associated to each state, since each interval is represented by a linear circuit. In order to apply the solving method, the set of differential equations can be expressed in either the *abc* or the *αβ* reference frame, being the proposed model formulated in the former one for convenience.

The set of differential equations representing the VSC model is as

$$\frac{d}{dt}\begin{bmatrix} i_a \\ i_b \\ i_c \\ v_{dc} \end{bmatrix} = A_{S_1 S_2 S_3}\begin{bmatrix} i_a \\ i_b \\ i_c \\ v_{dc} \end{bmatrix} + B\begin{bmatrix} v_{sa} \\ v_{sb} \\ v_{sc} \end{bmatrix} \qquad (1)$$

With

$$A_{S_1 S_2 S_3} = \begin{bmatrix} P & \frac{1}{3L}Q_{S_1 S_2 S_3} \\ S_{S_1 S_2 S_3} & -\frac{1}{R_{dc}C} \end{bmatrix} \quad B = \begin{bmatrix} -\frac{1}{3L}Q \\ 0 \end{bmatrix} \qquad (2)$$

$$P = \begin{bmatrix} -\frac{R}{L} & 0 & 0 \\ 0 & -\frac{R}{L} & 0 \\ 0 & 0 & -\frac{R}{L} \end{bmatrix} \quad Q_{S_1 S_2 S_3} = Q\begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix}$$

$$\qquad (3)$$

$$Q = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \quad S_{S_1 S_2 S_3} = \frac{1}{C}\begin{bmatrix} S_1 & S_2 & S_3 \end{bmatrix}$$

Variables S_1, S_2 and S_3 are the gating signals: "1" if upper switch is conducting and "0" if not. The three gating signals determine the relationships between *AC* and *DC* side quantities [7]. Thus, seven possible A_{S1S2S3} matrices exist, A_{111} and A_{000} are identical, depending on the states shown in Table 1. An exact solution to (1) exists over any time interval during which no switching occur, but its evaluation is not straight forward. However, in case of sinusoidal voltage it is possible to modify the equations in order to obtain an exact but easy solution of them [3]. In case of considering the harmonic distortion of the supply voltage, it is not possible to find out simple solutions and the traditional method to solve first order differential equations must be applied. Each switching interval, using the state variables formulation of (1), can be represented as

$$\frac{d}{dt}X = AX + Bu \qquad (4)$$

where $X=[i_a\ i_b\ i_c\ v_{dc}]^T$ and $u=[u_{sa}\ u_{sb}\ u_{sc}]^T$. The solution of this equation is composed by the homogeneous and particular solutions associated to the natural and forced responses respectively:

$$X(t) = X_h(t) + X_p(t) \qquad (5)$$

These solutions take the following form for each circuit associated to a switching interval:

$$X_h(t) = e^{A_{s_1 s_2 s_3} t} X(0)$$

$$X_p(t) = \int_0^t e^{A_{s_1 s_2 s_3}(t-\tau)} Bu(t-\tau)d\tau \qquad (6)$$

where *X(0)* are the initial conditions at the beginning of the analyzed interval.

Using this formulation it is possible to consider a distorted source voltage by modifying the term *u* adequately. In the proposed analysis it has been considered a source voltage distorted with a fifth harmonic:

$$v_{sa}(t) = \sqrt{2}U_1 \cos(wt+\alpha_1) + \sqrt{2}U_5 \cos(5wt+\alpha_5)$$

$$v_{sb}(t) = \sqrt{2}U_1 \cos(wt+\alpha_1 - \frac{2\pi}{3}) + \sqrt{2}U_5 \cos(5(wt - \frac{2\pi}{3})+\alpha_5)$$

$$v_{sc}(t) = \sqrt{2}U_1 \cos(wt+\alpha_1 + \frac{2\pi}{3}) + \sqrt{2}U_5 \cos(5(wt + \frac{2\pi}{3})+\alpha_5)$$

$$(7)$$

In fact, this is one of the major harmonics of the distribution power systems, because the increasing presence of non-linear loads. However, the proposed formulation can be used in case of considering more than one harmonic component.

Since the solution of the VSC equations depends on the switching technique, a specific switching pattern must be specified before the analysis can be carried out. For high-power applications, because of the power losses associated to the valve commutations, only two strategies with reduced commutation frequency are economically feasible:

- Strategy-I: switching at 3 or 9 times line frequency for independent control of DC voltage and reactive power compensation levels.
- Strategy-II: line frequency switching, yielding a DC voltage related to reactive power compensation level.

Analysis of the Strategy-I is carried out since Strategy-II operation may be considered as a subclass of Strategy-I operation. For Strategy-I operation switching at three times line frequency, two possible switching strategies can be selected. To simplify the analysis, the switching functions used are those shown in Figure 3. Since these switching functions determine the voltage at the VSC terminals, it is possible to control both the amplitude and the phase. These variables are controlled throughout the duty cycle and the phase angle respectively, being the switching instants τ_1 and τ_2 a function of them:

$$D = 1 - \frac{\tau_2 - \tau_1}{\pi/3}$$

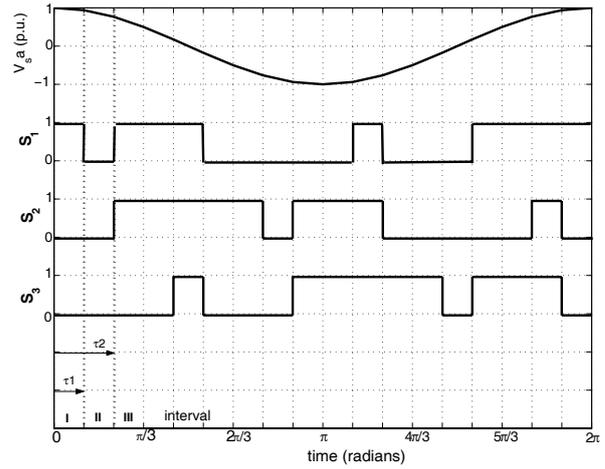$$\phi = \frac{\tau_2 + \tau_1}{2} - \frac{\pi}{6} \qquad (8)$$

Figure 3: Converter switching function.

From the diagram it can be observed that Strategy-II operation occurs if the switching times τ_1 and τ_2 are set to the same value.

This procedure can be expanded to other switching techniques such as PWM, although in this case it is also necessary to determine the commutation times.

3 STEADY-STATE CALCULATION

Steady state occurs when all DC and AC quantities return to their initial values after one period, i.e.

$$X(2\pi) = X(0) \qquad (9)$$

This allows finding a complete solution for an entire period by concatenating the different solutions of each of the resulting linear circuits represented by (6).

The initial value of the state variables, *X(0)*, is a function of the VSC duty cycle and phase angle, thus related to the switching times τ_1 and τ_2 according to (8). Considering the voltage distortion in the system, significant differences will appear with respect to the sinusoidal case with only a fundamental component. Figure 4 shows how the phase *a* current and the capacitor DC voltage as a function of the fifth harmonic content. In this case, the fifth harmonic voltage phase has been maintained constant, $\alpha_5 = -\pi/2$ *rad*, while its amplitude increases up to 0.1 per unit. The operating point of the converter is defined by *D=0.65* and *Φ=0.5°*. This figure shows the sensitivity of these magnitudes with respect to the harmonic voltage content in the point of common coupling (PCC). Therefore, these important differences can affect the STATCOM operation leading to a wrong response, being not possible to neglect the effect of voltage harmonics.

The effect shown in the time domain can be also appreciated in case of analysing other variables as the initial values that corresponds to the steady-state solution, *X(0)*. In this sense, it is interesting to analyze different operating points obtained by varying the duty

cycle D with a constant phase angle Φ in the v_{dc}-i_q plane. Although the proposed model is formulated in the *abc* reference frame, it is possible to obtain the current in the *dq* reference frame by applying the adequate matrix transformation. The reason for this transformation is that the current i_q is directly related to the reactive power injected by the STATCOM. Previous works [1,3] use it to explain the different operating points of this device. As it has been previously shown, the steady-state behavior strongly depends on these two variables. These results are presented in Figure 5. Each curve corresponds to a different voltage supply conditions with a fixed phase angle $\Phi=1.5°$. The points that have been marked up in the curves represent different duty cycles, being the minimum voltage of each curve associated to the maximum duty cycle D=1 p.u. The voltage supply condition of the simulated cases are shown in Table 2.
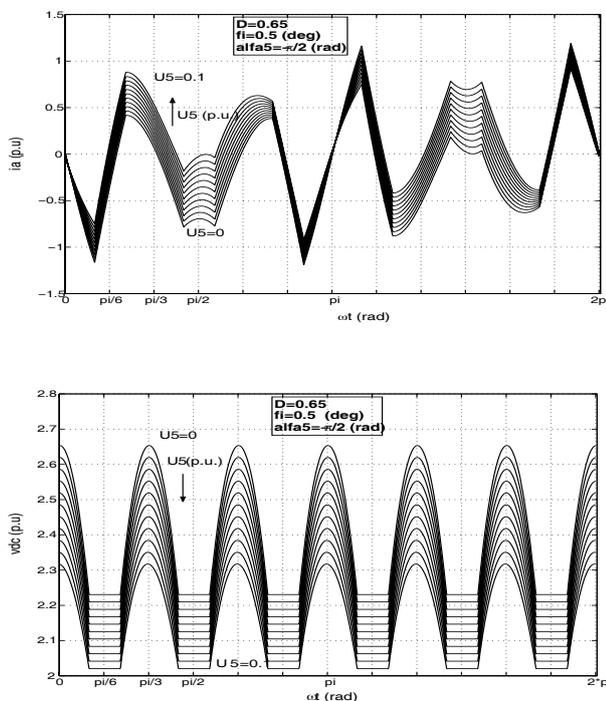

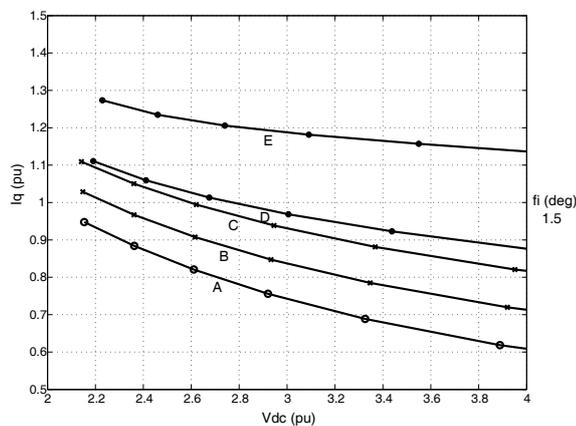**Figure 4:** Ia and Vdc evolution versus U5 for 5= - $\pi/2$ rad, D = 0.65 and ϕ = 0.5°.

Figure 5: Steady state v_{dc}-i_q curves comparison for sinusoidal and distorted voltage supply conditions.

It can be seen that the voltage distortion can strongly affect the behaviour of the STATCOM, because the operating curves in that case are quite different from the case of sinusoidal and balanced conditions. Moreover, not only the amplitude of the voltage harmonics is a relevant parameter in the study, but also their phase angle. This fact can be noticed if the resulting operating curves for cases B and D, with the same amplitude of the fifth harmonic but with different phase angle, are compared. As a consequence, if the control algorithm of the STATCOM does not take into account this issue, in case of harmonic voltage distortion in the PCC it is possible to inject a reactive power different than intended.

Case	A	B	C	D	E
U_5 (p.u)	0	0.05	0.1	0.05	0.1
α_5 (rad)	0	0	0	$\pi/6$	$\pi/6$

Table 2: Voltage supply conditions for the v_{dc}-i_q curves represented in Figure 5.

4   FREQUENCY DOMAIN ANALYSIS

The previous section points out the influence of the harmonic distortion on the initial values of the state variable, X, that have been used to solve the problem. However, these initial values do not reflect the overall behaviour of the STACOM. For this purpose, once the solution of the steady-state has been obtained, the phase currents and the DC voltage from one cycle can be build up from the partial solutions of each interval. Then, it is possible to obtain the behaviour of the STACOM in the frequency domain by applying the Fourier analysis to the obtained waveforms in the time domain. Both the phase currents and the capacitor voltage are not sinusoidal nor DC quantities. Nevertheless, it is interesting to compute their fundamental harmonic and DC components to point out the difference between the sinusoidal and distorted conditions of the voltage supply. These values have been computed by applying a FFT algorithm to the aforementioned variables computed in the time domain.

In order to better asses the influence of amplitude and phase angle of the supply voltage harmonics, simulations have been performed for a fixed operating point, $D=1$ and $\Phi=0$. Figures 6 and 7 show the variations of the fundamental harmonic of the current i_q and the DC value of capacitor voltage respectively. Note that both variables can be either greater or lower than their corresponding value for the sinusoidal supply conditions depending on the phase angle of the harmonic. It can be also noticed that the sensitivity of these variables, v_{dc}

and i_q, with respect to the voltage harmonic amplitude is quite dependent on the phase angle of that harmonic. In the proposed simulations the results reveal that the bigger differences occur for phase angles around $\pm\pi/2$ while smaller ones are found for phase angles close to zero. Moreover, in cases such as the analyzed one where the imposed control variables make the STATCOM reactive power have a reduced value, it is possible that the harmonic voltages reverse the reactive power flow. This fact can be appreciated in Figure 6. In case of sinusoidal voltage supply, the current demanded by the STATCOM has a negative value, meaning that there is a reactive power absorption, but for some phase angles of the fifth harmonic introduced in the supply, the current is positive, meaning a reactive power injection.
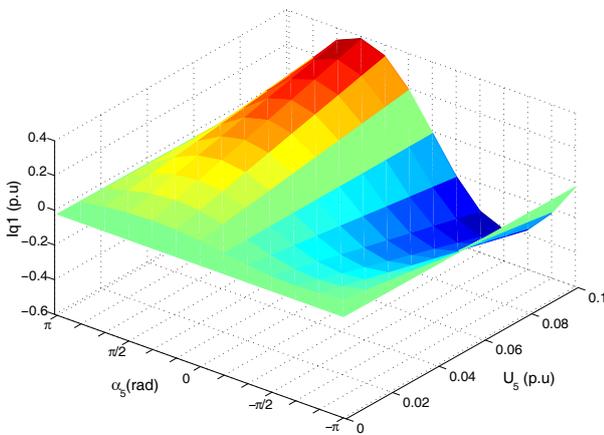

Figure 6: Fundamental component of the current i_q, I_{q1}, as a function of the fifth harmonic for D=1 and Φ =0.
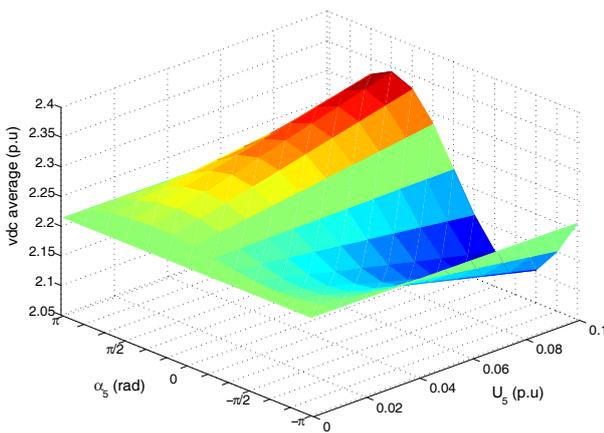

Figure 7: DC component of the capacitor voltage v_{dc}, V_{DC}, as a function of the fifth harmonic for D=1 and Φ =0.

Once it has been shown how the amplitude and the phase angle of the supply voltage harmonics affect the capacitor voltage and the reactive power injection, it is interesting to analyze the behaviour of the phase variables for a given distortion. It has been studied a case with a $U_5=0.05$ p.u. and $\alpha_5=\pi/2$ rad for different operating points of the STATCOM. The following results

have been obtained for the phase fundamental current, reactive power injection and total harmonic distortion shown in figures 8, 9 and 10 respectively. The following comments can be pointed out:

- The harmonic distortion does not affect the fundamental component of the phase current in a homogenous way. Figure 8 shows that in case of control phase angles lagging the fundamental supply voltage, reactive power injection operation, the fundamental component under distorted conditions is lower than in case of sinusoidal supply ones. However, the behaviour is contrary in case of leading control phase angles.

- In case of analyzing the results for the fundamental reactive power injected by the STATCOM shown in Figure 9, it can be seen that the surface defining the different operating points for the distorted supply conditions is always below that of the sinusoidal case. That means that in case of reactive power absorption, i.e. positive values of the control angle Φ, the device consumes more reactive power in the distorted case than in the sinusoidal one. But in case of analyzing the operating points related to reactive power injection, the behavior is the opposite, that is, the STATCOM injects less reactive power in the distorted case. If the control algorithm does not take into account the harmonic distortion, then it is possible a missoperation of the compensating device.

- As a periodically switched circuit, the STATCOM is a non-linear device that generates current harmonics. These current harmonics are affected by the supply voltage harmonics. This effect is analyzed in Figure 10 where the total harmonic distortion of the phase current is represented. For the studied case, the results show that the distortion is lower in case of the distorted supply conditions than in the sinusoidal ones.
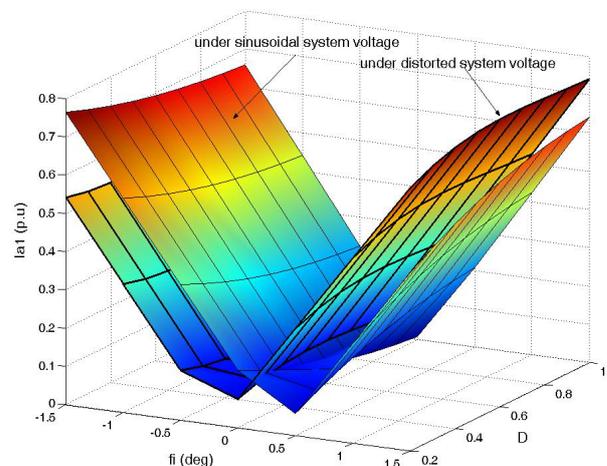

Figure 8: Fundamental component of the phase current for sinusoidal and distorted supply conditions.

Note that the comments that have been pointed out can be applied only to the analyzed example. Each particular case may lead to different conclusions depending on the harmonic spectrum of the supply voltage and the STATCOM parameters.
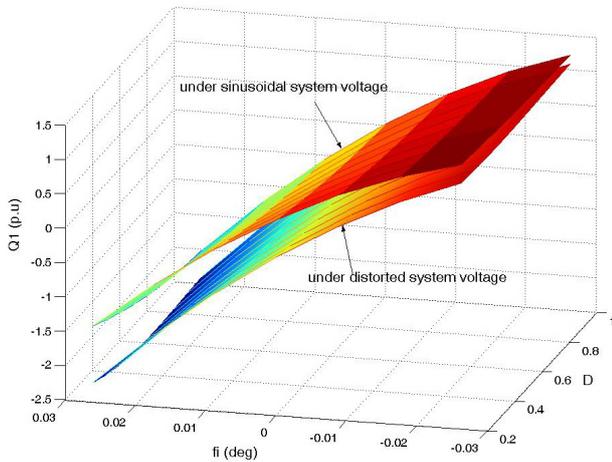

Figure 9: Fundamental injected reactive power for sinusoidal and distorted supply conditions.
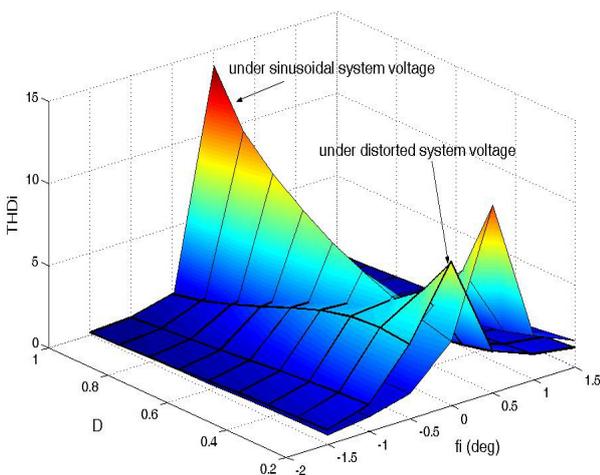

Figure 10: Total harmonic distortion of the phase current for sinusoidal and distorted supply conditions.

5   CONCLUSIONS

This paper has proposed a linear time model of a VSC for STATCOM applications. The VSC has been represented by a number of linear circuits associated to different states of the inverter switches. The state of the switches is determined by the control algorithm of the VSC, that computes the instants when the switches have to be operated. The differential equations corresponding to each switching interval have been integrated in the phase domain, so that it is possible to include the effect of the voltage harmonics.

The increasing number of non-linear loads connected to distribution systems is turning the voltage more and more distorted. The results show that the effect of harmonic voltage distortion in the supply voltages cannot be neglected since the STATCOM operation is strongly influenced.

APPENDIX

The data used for the calculation of all curves is summarized in Table 3. Individual switch resistances are included in the ac side resistance while switch forward voltage drops and switching losses are neglected.

Quantity	Value (p.u.)
L	29.7
R	21.0
C	2.5
W	8.0
R_{dc}	2.0
U_{sa}	24.7

Table 3: System data.

ACKNOWLEDGMENTS

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