ECONOMIC DISPATCH ALGORITHMS FOR THERMAL UNIT SYSTEM INVOLVING COMBINED CYCLE UNITS

Feng Gao, *Student Member*, *IEEE* and Gerald B. Sheble, *Fellow*, *IEEE*
Iowa State University
Ames, IA, USA

Abstract — The economic dispatch (ED) function allocates total demand among the available generating units to minimize the total generation cost. Cost curves of conventional thermal units can be modeled as convex functions. Equal incremental cost is criterial to solve the traditional ED problem. Economic dispatch involving combined cycle units is a non-convex optimization that cannot be handled by traditional equal incremental cost criterion. This paper discusses several techniques to solve this problem. The techniques considered include Complete Enumeration (CE), Merit Order Loading (MOL), Genetic Algorithm (GA) and a proposed Hybrid Technique (HT). This hybrid technique is not only feasible but it is also relatively fast to find ED solution for thermal unit system including Combined Cycle Units. This work includes the use of mutation prediction to enhance the efficiency and accuracy of Genetic Algorithm (GA). This paper includes the results for a test case based on a sample utility data set.

Keywords: Economic Dispatch (ED), Combined Cycle Units (CC Units), Non-Convex Optimization, Complete Enumeration (CE), Merit Order Loading (MOL), Genetic Algorithm (GA), Hybrid Technique (HT)

1 INTRODUCTION

The economic dispatch (ED) function allocates total demand among the available generating units to minimize the total generation cost. This activity is often executed on a minute-by-minute basis at each control center or Independent System Operator (ISO) [1]. Cost curves of conventional thermal units can be modeled as convex functions. Equal incremental cost is criterial to solve the traditional ED problem [2]. Combined cycle units utilize both gas turbines and steam turbines to produce electrical energy. The waste heat from the combustion turbines is directed into a boiler just as steam from the boiler is used to power steam turbines. Combined cycle units are of relatively high efficiency, have fast ramp rates and exhibit other beneficial features compared to conventional thermal units [3]. This has enabled the combined cycle units to become the technology of choice for many new power facilities. Economic dispatch involving combined cycle units is characterized as non-convex optimization that cannot be simply handled by equal incremental cost criterion. B. Lu and M. Shahidehpour [3] solved a more general problem considering several temporal periods and the unit-commitment problem. However, there is a drawback of assuming that each configuration has a convex function. Sheble [4] proposed a real-time ED algorithm - merit order loading (MOL) based on the theory of linear programming and the method was applicable for conventional thermal units. Ongsakul [5] made a modification for MOL and sorted CC units based on the unit incremental cost at the highest outputs, but an example with only CC units was provided. Sheble et al. [6] proposed to use genetic algorithm (GA) and refined genetic algorithm (RGA) methods to solve ED problems with non-convex cost curve considering valve point effects. Although genetic algorithms can solve non-convex optimization problem, it can not be mathematically guaranteed to find optimal solution.

This paper examines the cost functions of combined cycle units and presents solutions from several techniques. The techniques applied include Complete Enumeration (CE), Merit Order Loading (MOL), Genetic Algorithm (GA) and a proposed Hybrid Technique (HT). This hybrid technique is not only feasible but it is also relatively fast to find ED solution as shown in test case. A sample test case from a Midwestern utility is solved with each technique.

2 COST CURVE ANALYSIS OF COMBINED CYCLE UNITS

Typically, a combined cycle plant consists of several combustion turbines (CTs) and an HRSG/steam turbine (ST) set. Based on different combinations of CTs and STs, a combined cycle unit can operate in multiple configurations. Each combination of CTs and STs can be regarded as a state. Each state has its own unique cost curve characteristic.

Assuming a combined cycle unit consists of two combustion turbines and one HRSG/steam turbine [7]. Distinct configurations are shown in Table 1:

State	Composition	Minimum Power	Maximum Power
1	1 CT	P_{1min}	P_{1max}
2	2 CT	P_{2min}	P_{2max}
3	1 CT + 1 ST	P_{3min}	P_{3max}
4	2 CT + 1 ST	P_{4min}	P_{4max}

Table 1: The states of a combined cycle unit

The incremental cost curves are not monotonically increasing with generation and are depicted in Figure 1.

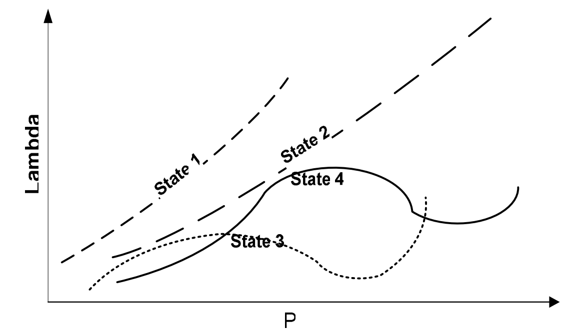

Figure 1: Incremental Cost Curves of Combined Cycle Units

3 ECONOMIC DISPATCH PROBLEM FORMULATION

The classical ED problem formulation is as follows:

Minimize: $$F = \sum_{i=1}^{n} f_i(P_i) \quad (1)$$

Subject to:

$$\sum_{i=1}^{n} P_i = P_D \Rightarrow g(P_i) = \sum_{i=1}^{n} P_i - P_D = 0$$
$$P_i \geq \underline{P_i} \quad (2)$$
$$P_i \leq \overline{P_i}$$

Where:

P_i: generation of unit i

f_i: cost of unit i

F: total cost of n units

P_D: total demand

$\underline{P}_i$: generation lower limit of unit i

$\overline{P}_i$: generation upper limit of unit i

n: the number of units

When all individual cost curves $f_i(P_i)$, $i=1,\ldots,n$, are convex, the objective function of the ED problem is also convex, because the summation of convex functions is also a convex function. Since all constraints are linear, the Karush-Kuhn-Tucker (KKT) condition guarantees that the problem has only one global minimum.

We form the Lagrange function as follows:

$$L = F(P_i) - \lambda\left(\sum_{i=1}^{n} P_i - P_D\right) \quad (3)$$

This fact allows us to find the solution by applying first order necessary conditions, which are in this case:

$$\frac{\partial L}{\partial P_i} = 0 \Rightarrow \frac{\partial f_i(P_i)}{\partial P_i} = \lambda, \quad i = 1,\ldots,n$$
$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow \sum_{i=1}^{n} P_i - P_D = 0 \quad (4)$$

Inequality constraints may be handled by checking whether the resulting solution is against them, and for any violation, setting up another equality constraint which binds the given decision variable to the limit which was violated.

Because F is convex, and g is linear, we can be certain that application of the above equations will result in the unique minimum cost solution.

If all of the inequality constraints are not binding, then $\frac{\partial f_i(P_i)}{\partial P_i} = \lambda$ induces the "equal incremental cost criterion" which is a simple and powerful rule to solve traditional ED problem.

4 COMPARISONS OF SOLUTION METHODS

The condition required to apply the equal incremental cost criterion is that all curves are monotonous increasing. With combined cycle units, monotonous decreasing intervals are typically ignored and only the monotonous increasing intervals participate in economic dispatch to apply the equal incremental cost criterion. When cost curve sections are ignored, the solution may not be optimal. Thus, methods that incude these segments should be examined. Such methods include: complete enumeration, merit order loading, genetic algorithm and a hybrid technique with convex optimization and complete enumeration are applied to find the global optimal solution.

4.1 Complete enumeration

Complete enumeration means searching every possible solution within feasible region to find the optimal one.

The searching scheme of complete enumeration is shown in Figure 2. The basic idea is to discretize generation level of each unit, sequentially change generation output of one unit while keeping all of others constant until all of possible trials are examined. The algorithm will seek the minimal cost within all feasible solutions.

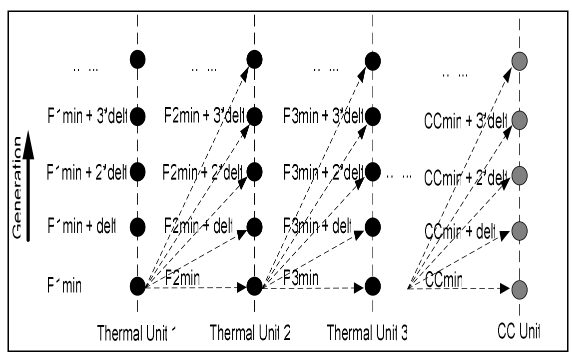

Figure 2: Searching Scheme of Complete Enumeration

Procedure:

1. Assume totally ($N+M$) units (N thermal units and M CC units) participate in economic dispatch. Set demand = D. *Delta* is the step size of the changing of generation;
2. Set i=1, j=0;
3. If $i > N+M$, go to step 7;
4. For unit i, set $P_i = P_{i,\,min} + j{*}Delta$, if $P_i > P_{i,\,max}$,

go to step 6;

5. if $\sum_{i=1}^{N+M} P_i = D$, store P_i and go to step 6; otherwise $j = j + 1$, go to step 4;
6. $i = i + 1$, go to step 3;
7. Within all of the solutions, select the lowest total cost and corresponding generation of each unit as the optimal solution.

4.2 Merit Order Loading

Merit order loading [4] is a fast and fit algorithm method for conventional thermal units with convex cost curves. The process is to sort the unit-segments into ascending sequence by breakpoints for piece-wise linear incremental cost curves.

The monotonously decreasing segments of incremental cost curves of combined cycle units can be handled. Reference [5] modified the merit order loading method and sorted the unit-segments based on the unit incremental cost at the highest/lowest outputs. This approach was not supported mathematically. There are conditions when this approximation is not correct. That reference did show one set of conditions when, it did provide an optimal solution.

An illustration of the order of four combined cycle units has been shown in Figure3. The unit dispatch sequence by MOL method is 1, 2, 3 and 4.

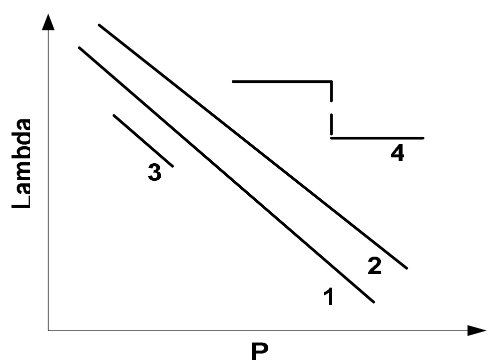

Figure 3: Dispatching Sequence of CC units by MOL

Procedure:

1. Assume totally N thermal units and M CC units participate in economic dispatch. Set demand = D;
2. Sort the unit-segments into ascending sequence by upper breakpoint for piece-wise linear ICs of thermal units;
3. Sort monotonous increasing sections of piece-wise linear ICs of CC unit into ascending sequence by upper breakpoint;
4. Sort monotonous decreasing sections of piece-wise linear ICs of CC unit into ascending sequence by maximal generation point (minimal incremental cost);
5. Dispatch the unit-segment by adding each incremental unit-segment generation into the total generation;
6. Increment the unit-segment index until all demand D is met.

4.3 Genetic Algorithm

The Genetic Algorithm is a classical form of genetic search which basically consists of initialization, selection, crossover, mutation and evolution.

This paper applied RGA proposed by Sheble et al [6] into ED with CC units. Reference [6] discussed some techniques in order to enhance the efficiency and accuracy of SGA, such as mutation prediction, elitism, interval approximation and penalty factors. Mutation prediction is applied in this paper and is discussed in detail.

Mutation is an important operator and does consume a large percentage of the computation time. In GA, the random number generator needs to be called each time for each bit to determine whether to carry out mutation operation or not. By reducing the number of times that the random number generator is called, computing time can be minimized. This is the motivation of mutation prediction.

In order to understand the mechanism of mutation prediction, two strategies need be compared. Assume there are totally m bits in one-generation chromosomes.

First strategy: for each bit, toss a coin. If head, mutate; otherwise do not mutate. Assume the probability of head is Q, the probability of tail should be 1-Q, *i.e.* Q represents the probability of mutation of each bit.

Define a random variable X which is the number of heads (mutations) in m trials. We know X satisfies Binomial Distribution.

$$\Pr ob(X = k) = C_m^k Q^k (1-Q)^{m-k} \qquad (5)$$
$$k = 0,1,\ldots m$$

The expected value of X is

$$E(X) = mQ \qquad (6)$$

Which means the average number of mutations in m bits is mQ.

Second strategy: for the same case of tossing a coin above, define another random variable Y which is the number of trials when head (mutation) first appears. We know Y satisfies Geometric Distribution.

$$\Pr ob(Y = h) = (1-Q)^{h-1} Q \qquad (7)$$
$$h = 1,2,\ldots, m$$

The expected value of Y is

$$E(Y) = 1/Q \qquad (8)$$

Which means the average number of trials when head first appears is $1/Q$. Therefore within m bits, the average number of heads (mutations) is

$$\frac{m}{E(Y)} = \frac{m}{1/Q} = mQ \qquad (9)$$

It is easy to show that equations (6) and (9) give the same result. However, the second strategy calls the random number generator only one time, whereas, the first one calls random number generator one time per bit.

Therefore, mutation prediction should be carried out as follows: with 1/Q as the expectation parameter, randomly generate a series of numbers satisfying Geometric Distribution. These random numbers indicate positions of bits which need to be mutated.

4.4 Hybrid Technique

The hybrid technique originates from the idea that conventional convex thermal units and CC units can be divided into two different groups, for conventional units group, convex optimization methods such as Lambda Iteration are applied; for CC units, Complete Enumeration is applied. This proposed technique combines Lambda Iteration with Complete Enumeration is shown in Table 2. Lambda Iteration is able to find the optimal cost for thermal units part as well as Complete Enumeration for CC units part. The global optimum can be reached whatever the order of the CC units considered within the algorithm is. Essentially HT is to build a composite generation cost function for all of thermal units, then apply CE to solve a lower dimension non-convex optimization.

Conventional Units		CC Units		Total
Lambda Iteration		Complete Enumeration		Solution
Generation Sum	+	Generation Sum	=	Generation
Cost Sum	+	Cost Sum	=	Cost
Demand served	+	Demand served	=	Demand

Table 2: Illustration of Hybrid Technique

Procedure:

1. Assume totally N thermal units and M CC units participate in economic dispatch. Set demand = D, i = 0 and P_0 is the summation of generation lower limit of all M CC units, ΔP is step size of the changing of generation.
2. Calculate the cost of all of CC units with demand $P = P_0 + i * \Delta P$. If $P >$ the summation of generation upper limit of all M CC units, go to step 6.
3. Using Lambda Iteration to dispatch generation ($D - P$), and calculate the cost of each thermal unit.
4. Calculate the total cost of CC units and thermal units.
5. $i = i + 1$, go to step 2.
6. Find the minimal total cost among all of the solutions, it is the optimal solution.

5 APPLICATION

The case study is a test system with twelve thermal units and one combined cycle unit.

The CC units consist of two combustion turbines and one HRSG steam turbine. There are totally four operation states. Piece-wise linear cost curves are provided and breakpoints are shown in Table 3.

	State 1		State 2		State 3		State 4	
	MW	BTU/Hr	MW	BTU/Hr	MW	BTU/Hr	MW	BTU/Hr
1	0	600	0	1200	0	600	0	1200
2	60	950	120	1900	95	950	190	1900
3	90	1150	180	2300	145	1150	290	2300
4	110	1280	220	2560	167.5	1280	335	2560
5	130	1437	260	2874	189	1437	378	2874
6	150	1601	300	3202	210	1601	420	3202
7	170	1775	340	3550	245	1775	490	3550
8	180	1872	360	3744	265	1872	530	3744
9	200	2056	400	4112	295	2056	590	4112

Table 3: I/O data of a combined cycle unit

Incremental cost curves of the CC unit are shown in Figure 4. State 1&2 essentially are thermal unit states. Correspondingly, incremental cost should be monotonously increasing. The “One CT” and “Two CTs” curves are almost monotonously increasing, except a few points which should be bad data. The “One CT and One HRGS” and “Two CTs and One HRGS” curves corresponding to state 3&4 are not monotonously increasing any more, because state 3&4 are combined cycle unit states.

Figure 5 shows cost curves of all four states of the combined cycle units. All of the four curves are monotonously increasing. However, the “One CT” and “Two CTs” curves corresponding to state 1&2 are convex, instead, the “One CT and One HRGS” and “Two CTs and One HRGS” curves corresponding state 3&4 are neither convex nor concave.

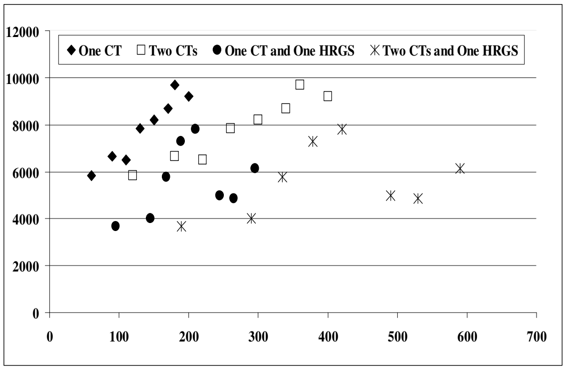

Figure 4: Incremental Cost Curve of a CC unit

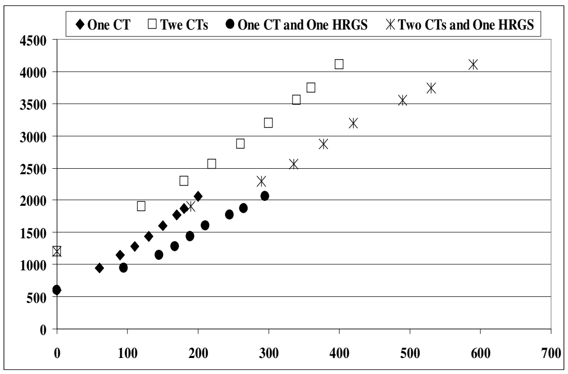

Figure 5: Cost Curve of a CC unit

All four methods are applied to a demand equal to 2500 MW for this first comparison. The CC unit is run at state 4 ("Two CTs and One HRGS"). Total calculated cost and the CPU time are shown in Figure 6 and 7 respectively. (Computer configuration: CPU P4 1.3GHZ, 384M Memory)

Total costs of GA and HT are a little higher than that of CE and MOL. GA is the most time-consuming method. The proposed HT is not only feasible but also relatively fast to find ED solution. Similar results are obtained for other demand levels.

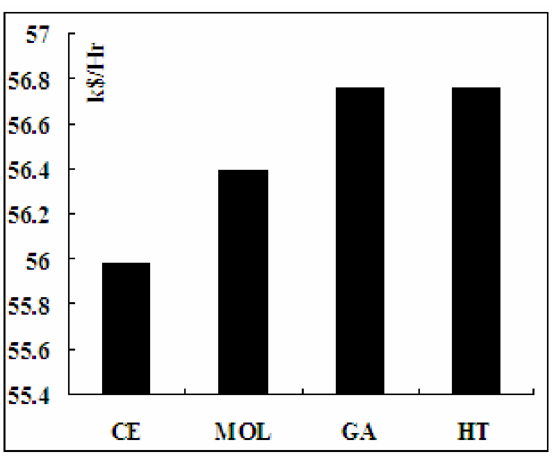

Figure 6: Comparison of Total Cost of Four Methods

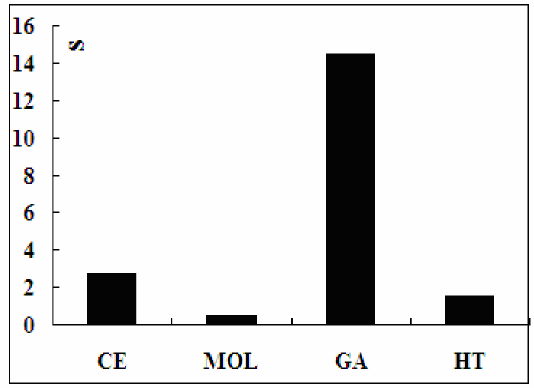

Figure 7: Comparison of CPU Time of Four Methods

To continue the comparison, the demand is increased from 1200MW to 2800MW with a step size 100 MW, all four methods are executed. Using the result of CE as a benchmark, the percentage differences of the calculated total cost between CE and other three techniques are shown in Table 4. The MAPE of HT and MOL is very close to each other. GA has a relative larger error. Remember that GA cannot guarantee to find the global optimal solution.

Demand	MOL	GA	HT
1200	0.000309979	-0.000124466	-6.2233E-05
1300	-4.57114E-05	-9.14227E-05	-4.57114E-05
1400	-0.000267683	-0.000354797	-0.000177398
1500	-7.14742E-06	4.53714E-06	2.26857E-06
1600	-0.000328433	-0.000201094	-0.000100547
1700	0.00101537	0.00195927	0.000979635
1800	0.002245037	0.004580744	0.002290372
1900	0.00272554	0.006448703	0.003224352
2000	0.004366522	0.009192978	0.004596489
2100	0.005451713	0.011499457	0.005749729
2200	0.007044526	0.013188988	0.006594494
2300	0.00751545	0.0150309	0.00751545
2400	0.008276869	0.016553738	0.008276869
2500	0.007346905	0.013927808	0.013900922
2600	0.007826203	0.015257941	0.00762897
2700	0.025135134	0.046870952	0.023435476
2800	0.029736227	0.042204629	0.021102314
MAPE	0.006449674	0.011617201	0.006216661

Table 4: Comparison of Total Cost of Four Methods for Continuous Demand

6 CONCLUSION

This paper first explains the conventional solution and analyzes combined cycle cost curves. Since the cost curves are not convex and the incremental cost curves are not monotonously increasing, the equal incremental criterion can not be applied.

This paper compares several existing methods which can solve the ED problem involving combined cycle units. They are Complete Enumeration, Merit Order Loading and Genetic Algorithm. The differences from originally published methods are presented. A method to mutation prediction of RGA is shown in detail which does reduce computing time tremendously.

Finally this paper proposes a Hybrid Technique combining Lambda Iteration with Complete Enumeration to solve ED with CC Units. Both single demand and continuous demands are tested by the four methods. CPU time is compared. Advantages and disadvantages of each method are compared.

REFERENCES

[1] K. Bhattacharya, M. H.J. Bollen and J. E. Daalder, *Operation of Restructured Power Systems*, Kluwer Academic Publishers, Boston, MA, 2001.

[2] A. J. Wood and B. F. Wollenberg, *Power Generation Operation and Control*, 2nd ed., Wiley, New York, 1996.

[3] B. Lu and M. Shahidehpour, "Short-Term Scheduling of Combined Cycle Units", *IEEE Transactions on Power Systems*, Vol. 19, No. 3, Aug. 2004, pp. 1616-1625.

[4] G. B. Sheble, "Real-Time Economic Dispatch and Reserve Allocation Using Merit Order Loading and Linear Programming Rules", *IEEE Transactions on Power Systems*, Vol. 4, Nov. 1989, pp. 1414–1420.

[5] W. Ongsakul, "Real-Time Economic Dispatch Using Merit Order Loading for Linear Decreasing and Staircase Incremental Cost Functions", *Electric Power Systems Research*, 51 (1999) 167–173.

[6] G. B. Sheble, K. Brittig, **"**Refined Genetic Algorithm-Economic Dispatch Example", IEEE *Transactions on Power Systems*, Vol. 10, Feb. 1995, pp.117 – 124.

[7] M. R. Bjelogrlic**, "**Inclusion of combined cycle plants into optimal resource scheduling", *Power Engineering Society Summer Meeting, 2000*. IEEE, Vol. 1, pp. 16-20 July 2000.