Application of Benders Decomposition to an Equilibrium Problem

Jordi Cabero, Álvaro Baíllo, Santiago Cerisola and Mariano Ventosa

Abstract— **In this paper we propose an algorithm for addressing the medium term hydrothermal coordination problem faced by a generation company when operating in an electricity market. This problem is formulated and solved as a stochastic Linear Mixed Complementarity Problem. In order to deal with realistic problems, we propose the use of Bender's decomposition technique. A numerical example in the context of the Spanish electricity market illustrates the potential of the algorithm we propose.**

Index Terms— **Market equilibrium, complementarity problem, stochastic programming, Benders decomposition.**

I. Introduction

POWER generation companies willing to sell the electricity produced by their units in a wholesale market are required either to submit offers with trading horizons ranging from a few hours to several months, or to sign long-term bilateral contracts with wholesale purchasers (e.g. load serving entities, LSEs).

In this context, generation companies face a much higher degree of risk exposure due to the uncertain behaviour of their competitors and to the threat of new entry. The urgent need to answer these and other related questions has triggered an unprecedented research effort to develop conceptual models devoted to the analysis of competition in wholesale electricity markets. Differences between models can be identified in a variety of attributes, such as the hypotheses formulated by the modeller/analyst about the agents' behaviour, the specific purpose of the ongoing analysis, the characteristics of the underlying power system, the detail with which the elements of the power system are represented, the organization of the corresponding wholesale electricity market or the technique used to obtain numerical results (see [1] and [2] for extensive reviews).

Equilibrium models have been extensively applied to the case of the power industry during the last few years in order to predict its evolution under the new regulatory framework. In particular, they have been used to explore possible market outcomes that may result under an industrial structure with a limited number of relevant agents (oligopoly). The wide range of applications that have been proposed confirms the interest of this approach.

In an equilibrium model, a problem is formulated in which each agent is assumed to choose his best strategy based on certain conjectures about the behaviour of the relevant rest of the world. In practice, the possibility of obtaining numerical results for large study cases reinforces the validity of a modelling approach.

J. Cabero, Á. Baíllo, S. Cerisola and M. Ventosa are with the Instituto de Investigación Tecnológica, Escuela Técnica Superior de Ingeniería, Universidad Pontificia Comillas de Madrid, Spain (e-mail: jordi.cabero@iit.upco).

Static equilibrium models have been profusely used under the assumption that agents decide their strategies simultaneously and are not able to react to their rivals' decisions. Proposals have ranged from the basic Cournot model (in which agents' strategies are represented in terms of quantities and the equilibrium price is given by the inverse demand function) to more refined versions that include increasingly complex representations of the players' strategies such as supply function equilibrium (SFE) models.

Cournot's oligopoly model has proven to be useful for a diversity of purposes. In particular, it has frequently been used to support market power studies. [3] Some of these market power analyses incorporate simple Cournot models merely to illustrate researchers' assertions. [4] Other authors [5] rely on the Cournot model to obtain a numerical evaluation of the potential for market power in a certain electricity industry.

Currently, equilibrium models based on Cournot's conjecture are usually formulated as complementarity problems (CP, [6, 7]) or, alternatively, as a system of variational inequalities, VI, [8, 9]. In this manner, modellers benefit from the existence of powerful commercial solvers capable of solving large-scale CPs [10] and from the fact that modelling algebraic languages such as GAMS have been specially adapted to this special kind of problems [11]. Both approaches, CP and VI, have been successfully applied to perform market power analysis in which the transmission network plays a central role [4, 12].

From a different perspective, researchers willing to assess the hydrothermal coordination problem of a generation company operating in a wholesale electricity market have also frequently adopted the Cournot framework under a deterministic perspective, [13-15].

However, a number of drawbacks seem to question the applicability of Cournot model. Firstly, it relies on the demand function to determine equilibrium prices, given that generators' strategies are expressed in terms of quantities and not in the form of offer curves. This seems to deviate from the reality of many electricity markets that are based on the offer and bid curves submitted by participants. Secondly, demand in short-term electricity markets is characterized by its inelasticity, which leads to extremely high prices in the Cournot framework. Additionally, Cournot model assumes that each participant knows the decisions taken by his rivals and the shape of the demand function. Some authors also highlight that Cournot is a static model in which players assume that their rivals will not react to their decisions, thus disregarding the fact that electricity markets are based on the repetitive interaction of participants through a variety of market mechanisms.

In an effort to overcome the limitations of Cournot model and at the same time profit from its good properties, some authors have applied an extension of this model in which each agent expresses its strategy in the form of a quantity but conjectures the strategy followed by each of its rivals in terms of a supply function [16]. Under this representation, the sensitivity of the price with respect to the quantity decided by a certain agent is lower than in the standard Cournot model, given that it depends not only on the slope of the inverse residual demand function, but also on the slope of the supply function conjectured for the rest of agents. As a consequence, lower prices typically result. Moreover, the conjectured slopes of such a model can be adjusted to yield prices similar to those observed in reality [17]. We adopt this particular approach in this paper too.

Commercial software specifically devoted to the solution of CPs presents limitations with respect to the size of the problems that can be solved. Due to this some authors have suggested resorting to the formulation of the equivalent quadratic optimization problem with the purpose of using more powerful optimization software [18]. However, this impedes modelling certain relevant aspects of the operation of the generation units.

Due to these size limitations, researchers have typically restricted their attention to the analysis of deterministic situations (see [19] for an exception). In this paper, in contrast, we address the problem of computing the equilibrium of an electricity marketplace under uncertainty and assuming a non-anticipative decision process. In order to cope with the problem sizes that might arise in this context we develop a Benders decomposition algorithm. The master problem comprises the market equilibrium optimality conditions and approximates the operation costs and constraints of each generation company by means of the typical outer linearization provided by Benders' optimality and feasibility cuts, which significantly reduces the size of the linear mixed complementarity problem (LMCP) that has to be solved. In order to obtain these cuts, a linear subproblem is formulated for each generation company which, given the strategy of production suggested by the master equilibrium problem for that company, yields the minimum-cost operation plan for the company's generation units in the presence of uncertainty, including features such as the management of hydro reserves between planning periods.

The purpose of this paper is to show that a decomposition approach is a satisfactory method of tackling an equilibrium situation when it is formulated as a linear mixed complementarity problem. Hence, we focus on the formulation of the corresponding master problem and subproblems, as well as on the design of the algorithm. This paves the way for addressing more complicated equilibrium models for which commercial solvers may turn out to be ineffective.

The paper is organized as follows. Section II presents the equilibrium problem of interest, explains how to decompose it and describes the particular application of Benders algorithm to this situation. Section III illustrates the performance of the proposed algorithm for several numerical examples. Finally, section IV discusses the conclusions that can de drawn from the results obtained.

II. Benders Decomposition of an Electricity Market Equilibrium Model

In this section we describe the equilibrium problem that we intend to solve and present the decomposition algorithm developed for that purpose. Although in the numerical example we will consider uncertainty with respect to certain production factors (e.g. fuel costs, resource availability, etc.) in this section we omit all reference to uncertainty. The reason is that the explicit consideration of uncertainty in the formulation of the problem adds little to the main contribution of this paper (the decomposition of the market equilibrium problem into a pure equilibrium problem and a number of operation subproblems). Indeed, when we introduce uncertainty in our decomposition framework, the algorithm we propose remains unchanged and the only modification is that each of these problems has to be formulated as the corresponding deterministic equivalent problem. Nevertheless, we include some observations that are relevant when uncertainty is considered.

A. *The Equilibrium Model*

We consider a wholesale market in which the supply side is represented in detail whereas the demand side is modelled in an aggregate manner by means of the demand function, $q = D(p)$, which expresses the total quantity q that is purchased at each price p. We assume the demand function to be a linear function.

The supply side is constituted by E (risk-neutral) suppliers with different production technologies and cost structures that compete to sell the product. We assume that each supplier e expresses its strategy in terms of a quantity, q_e. We also assume rational behaviour in the sense that each supplier has the objective of maximizing its (expected) profit $z_e(q_e)$:

$$\max_{q_e} z_e(q_e) = pq_e - c_e(q_e) \tag{1}$$

where $c_e(q_e)$ is the cost function of supplier e. This cost depends on the production facilities that the supplier uses to provide the quantity q_e. We assume that agent e makes internal decisions, y_e, in order to supply the amount q_e at the lowest possible cost:

$$(SP_e) \quad \begin{array}{ll} \min_{y_e} c_e(q_e) = d_e y_e & \\ T_e q_e + W_e y_e = h_e & y_e \in Y_e \end{array} \tag{2}$$

where d_e is the vector of unitary costs associated with the internal decisions made by agent e, assuming that the costs of these decisions can be expressed as linear functions. The equations in (2) represent, in an aggregated manner, the subproblem constraints, such as minimum and maximum power output, dynamics of hydro units reservoirs, etc. We use conformable dimensioned matrices T_e, W_e and h_e to maintain the traditional notation of two-stage models, due to the decomposition approach that will be later proposed. Notice that some of these constraints link internal decisions y_e, with the total amount sold, q_e.

Under these assumptions the maximization problem (1) may be equivalently formulated as follows:

$$\max_{q_e} z_e(q_e) = pq_e - d_e y_e$$
$$T_e q_e + W_e y_e = h_e \qquad (3)$$
$$y_e \in Y_e$$

The concept of equilibrium arises from the simultaneous consideration of the maximization problem of every supplier and the inverse residual demand function, $p = D^{-1}(q) = D^{-1}\left(\sum_e q_e\right)$, which establishes a link between the decisions made by all the suppliers (Fig. 1).

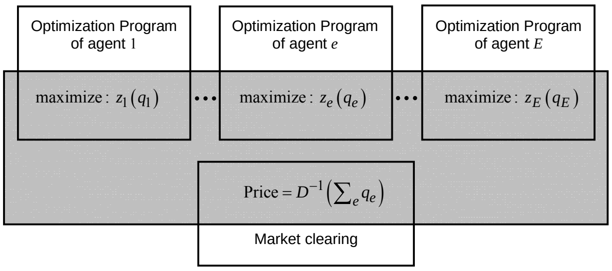

Fig. 1. Market Equilibrium

In order to compute a numerical solution for this problem we must formulate the first-order optimality conditions for each agent. These optimality conditions, together with the inverse residual demand function constitute a linear mixed complementarity problem that, in principle, can be solved using a commercial solver. [10]

However, in practice, commercial solvers such as PATH present limitations when addressing the resolution of large-scale problems such as the deterministic equivalent problem that arises from the formulation of a stochastic model. A natural approach then consists of adapting decomposition strategies to the previously described situation. In particular, this paper proposes the use of the Benders decomposition technique in order to replace the cost function of problem (1) by a collection of cuts that outer approximates it.

B. *Benders Decomposition of the Equilibrium Model*

According to the theory of linear programming duality, problem (2) can be equivalently formulated as follows:

$$c_e(q_e) = \max_{\pi_e} \pi_e (h_e - T_e q_e)$$
$$\pi_e W_e = d_e \qquad (4)$$

where π_e is the corresponding vector of dual variables. Exploiting the fact that the optimal solution of problem (4) lies in a vertex of the feasible region, a simple enumeration of the collection of possibilities will give the optimal solution.

$$c_e(q_e) = \max_j \pi_e^j (h_e - T_e q_e) \quad j:1,...,J \qquad (5)$$

Problem (3) may now be reformulated as follows:

$$\max_{q_e, y_e} z_e(q_e) = pq_e - \theta_e$$
$$\theta_e \ge \pi_e^1 (h_e - T_e q_e)$$
$$\cdots \qquad (6)$$
$$\theta_e \ge \pi_e^J (h_e - T_e q_e)$$

where each of the constraints that set a lower bound on the value of θ_e is usually referred to as Benders optimality cut.

Once we have reformulated the profit maximization problem for each agent as in (6), we may derive its optimality conditions. Bunching all the optimality conditions together with the demand or price equation leads to an LCP that can be solved with specific software.

Obviously, the computation of the complete collection of vertices of the feasibility region is impracticable, so an iterative algorithm is performed that sequentially obtains them. This iterative algorithm alternates between an equilibrium problem, which plays the role of the master problem in Benders decomposition, and a set of E minimization subproblems (one for each agent), as illustrated in Fig. 2.

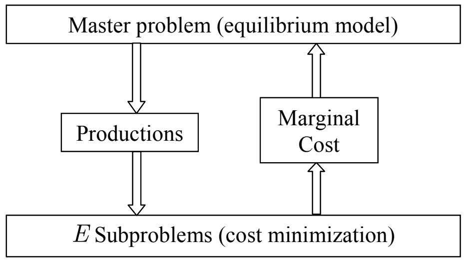

Fig. 2. Algorithm Description

C. *Expressions for the Optimality and Feasibility Cuts*

Let us assume that in the j-th iteration of the algorithm the master equilibrium problem has suggested a production level q_e^j for agent e. Agent e then solves the cost minimization subproblem (4) and obtains as optimal dual solution π_e^j. This dual solution is then used to build an optimality cut that improves the outer approximation of agent e's cost function in the master problem. This optimality cut is given by:

$$\theta_e \ge \pi_e^j (h_e - T_e q_e) \qquad (7)$$

After some algebra we obtain an alternative formulation for this constraint that is more adequate for further developments of the method:

$$\begin{aligned} \theta_e &\ge \pi_e^j (h_e - T_e q_e) \\ &= \pi_e^j \left(h_e - T_e q_e - T_e q_e^j + T_e q_e^j\right) \\ &= \pi_e^j \left(h_e - T_e q_e^j\right) - \pi_e^j \left(T_e q_e - T_e q_e^j\right) \\ &= \theta_e^j - \pi_e^j \left(T_e q_e - T_e q_e^j\right) \end{aligned} \qquad (8)$$

where θ_e^j is the value of the objective function obtained for problem (4) given the production suggested by the master problem, q_e^j.

Assume now that a certain production decision suggested by the master problem, q_e^i, renders the subproblem (5) infeasible. In this case, the dual variables obtained after applying phase I of the Simplex algorithm to the subproblem, π_e^i, are useful to generate a feasibility cut. This is a constraint that eliminates the last proposed solution (the infeasible one) while maintaining the collection of optimal solutions [20]. Such a feasibility cut takes the following form:

$$0 \ge \theta_e^i - \pi_e^i \left(T_e q_e - T_e q_e^i\right) \qquad (9)$$

where θ_e^i denotes the sum of infeasibilities of the subproblem once the Phase I of the simplex algorithm has finished.

The optimality and feasibility cuts derived in this section are used to approximate the recourse function (production cost function and constraints) of each agent in the master equilibrium problem. In this manner the computational requirements of the equilibrium problem are alleviated and the market equilibrium representation can be enriched through, for example, the introduction of uncertainty.

D. Optimality Conditions for the Master Problem

After a number of iterations of the algorithm a partial approximation of each agent's recourse function is available in the form of I_e feasibility cuts and J_e optimality cuts:

$$\max_{q_e} z\left(q_e\right) = pq_e - \theta_e$$

$$0 \geq \theta_e^i + \pi_e^i T_e\left(q_e^i - q_e\right) \qquad i:1,...,I_e \tag{10}$$

$$\theta_e \geq \theta_e^j + \pi_e^j T_e\left(q_e^j - q_e\right) \quad j:1,...,J_e$$

It is interesting now to formulate the optimality conditions of such a maximization problem. The Lagrangian function has the following form:

$$\begin{aligned} L_e\left(q_e,\theta_e,\lambda_e,\mu_e\right) &= pq_e - \theta_e \\ &+ \sum_{i\in I_e} \lambda_e^i\left[\theta_e^i + \pi_e^i T_e\left(q_e^i - q_e\right)\right] \\ &+ \sum_{j\in J_e} \mu_e^j\left[\theta_e^i + \pi_e^j T_e\left(q_e^j - q_e\right) - \theta_e\right] \end{aligned} \tag{11}$$

where λ_e^i is the vector of Lagrange multipliers corresponding to feasibility cuts and μ_e^j is the vector of Lagrange multipliers corresponding to optimality cuts.

Deriving the Lagrangian function with respect to decision variables we obtain the following optimality conditions:

$$\frac{\partial L_e}{\partial q_e} = \frac{\partial p}{\partial q_e} q_e + p - \sum_{i\in I_e} \lambda_e^i \pi_e^i T_e - \sum_{j\in J_e} \mu_e^j \pi_e^j T_e = 0 \tag{12}$$

$$\frac{\partial L_e}{\partial \theta_e} = -1 - \sum_{j\in J_e} \mu_e^j = 0 \tag{13}$$

These optimality conditions are completed with the optimality and feasibility cuts as well as with the complementarity slackness requirements:

$$\begin{aligned} \theta_e^i + \pi_e^i T_e\left(q_e^i - q_e\right) \leq 0 \quad i:1,...,I_e \\ \theta_e^j + \pi_e^j T_e\left(q_e^j - q_e\right) - \theta_e \leq 0 \quad j:1,...,J_e \end{aligned} \tag{14}$$

$$\begin{aligned} \left[\theta_e^i + \pi_e^i T_e\left(q_e^i - q_e\right)\right] \perp \lambda_e^i \quad i:1,...,I_e \\ \left[\theta_e^j + \pi_e^j T_e\left(q_e^j - q_e\right) - \theta_e\right] \perp \mu_e^j \quad j:1,...,J_e \end{aligned} \tag{15}$$

with $\lambda_e^i,\ \mu_e^j \geq 0$.

The term $\frac{\partial p}{\partial q_e}$ of equation (12) expresses the sensitivity of the price to changes in agent e's production. As mentioned before, this term can be adjusted so that the model provides prices that are similar to those observed in reality. Such an approach has received different names, including conjectural variations [16], implicit residual demand [17] and strategic parameter [21].

The collection of optimality conditions for the set of agents, together with the inverse demand function yields a complementarity problem that plays the role of master problem in our decomposition algorithm (Fig. 3).

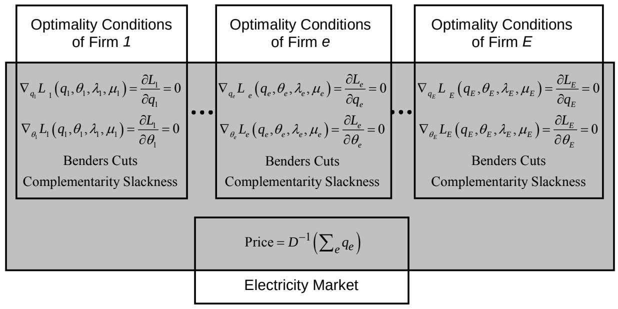

Fig. 3. Market equilibrium as a mixed complementarity problem

As can be seen, in each iteration of Benders algorithm the master problem incorporates a new optimality or feasibility cut as well as the corresponding Lagrange multiplier and complementarity slackness condition. If the algorithm requires several hundred iterations to converge, this adds little to the computational requirements of the master equilibrium problem, given that current solvers can cope with problems with several thousand equations.

E. Summary of the algorithm

We now present a summary of the decomposition algorithm that we propose in this paper.

Step 0	Set tolerance parameter *tol*. Set $I_e = 0$, $J_e = 0$.
Step 1	Solve Master problem and propose q_e and p .
Step 2	For each agent e Solve Subproblem (SP_e) If infeasible Get dual values $\pi_e^{I_e}$, infeasibility value $\theta_e^{I_e}$ and augment the collection of infeasibility cuts: $I_e = I_e + 1$ If feasible Get dual values $\pi_e^{J_e}$, optimal value $\theta_e^{J_e}$ and augment the collection of optimality cuts: $J_e = J_e + 1$
Step 3	Stopping rule If any subproblem turns out to be infeasible Go to step 1 In other case Check the difference between proposals If dif < tol stop In other case go to step 1

Algorithm 1. Benders decomposition for the equilibrium problem

F. Algorithm Issues

Decomposition algorithms based on outer approximations such as Benders algorithm or Lagrangian relaxation have the disadvantage of presenting an oscillatory behaviour when converging towards the solution. In Benders algorithm, this oscillatory behaviour is more noticeable when in a number of iterations the subproblem turns out to be infeasible and ends up with a reduction in the speed of convergence. One way of mitigating this oscillatory behaviour is to reduce the number of iterations in which the subproblem is infeasible by using sensitivity analysis after the subproblem resolution. Sensitivity analysis provides the limits for right-hand-side (RHS) variation without changes in the optimal basis. In a Benders-type algorithm, in each iteration the master problem solution modifies the subproblem right-hand-side value. In this context, sensitivity analysis can be interpreted as the maximum change in the master problem solution such that the Benders optimality cut corresponding to that solution remains unchanged. This leads to the introduction of a tuning parameter that limits the movement of the master solution. Let the subproblem of agent e be formulated as follows:

$$\begin{aligned} c_e(q_e) &= \min_{y_e} dy_e \\ W_e y_e &= h_e - T_e q_e \end{aligned} \tag{16}$$

and assume that, after problem (SP_e) resolution, the sensitivity analysis of the *RHS* returns l_{RHS} and u_{RHS} as the limits for *RHS* variation. The quantities proposed by the master problem can then be constrained based on these limits in the next resolution of the master problem:

$$l_{RHS} - \varepsilon \le h_e - T_e q_e \le u_{RHS} + \varepsilon \tag{17}$$

We recommend this approach when a feasible solution for the subproblem is already available. However, according to our experience with this type of problems, Benders algorithm may require a large number of iterations before achieving a feasible solution for all the subproblems. For this reason, a good initial starting point may be extremely helpful. One possibility is to formulate a cost-minimization problem with inelastic demand for the whole set of agents in order to determine the quantities that the subproblems evaluate in the first iteration. In this initial cost-minimization problem we can incorporate all the constraints that limit the production process of all the agents, so that the feasibility of the initial solution is guaranteed. After that, if the sensitivity analysis that we propose is used, the algorithm will smoothly evolve from the cost-minimization solution to the equilibrium solution.

III. Numerical Application to the Electricity Market

This section presents the application of the decomposition algorithm described in the previous section to the computation of the equilibrium of a wholesale electricity market with a medium-term time scope (one year).

The case study corresponds to the Spanish wholesale electricity market during year 2004. We consider a one-year time horizon divided into monthly periods. Additionally, we discretize electric market variables (wholesale electricity prices and agents' productions) into load levels (on-peak, shoulder and off-peak); so that each month includes six load levels (three for working days and three for non-working days). We assume that six generation companies take part in this electricity market. These companies on the whole own 78 thermal units (including nuclear and fossil-fuel units) and 16 hydro units. We neglect the transmission network.

We introduce uncertainty in fuel costs (coal, oil and natural gas). To do so we generate 1000 scenarios of each random parameter (Fig. 5 to Fig. 7 in light gray) (see [22] for a general exposition on the generation of scenarios for energy prices). Nevertheless, it is important to point out that the scenario generation process has taken into account the correlation between the considered random parameters.

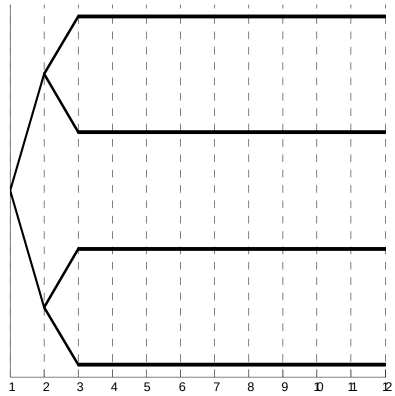

Fig. 4. Scenario-tree skeleton

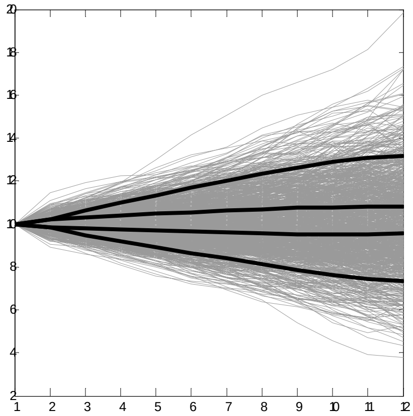
Fig. 5. Coal prices in €/MWh

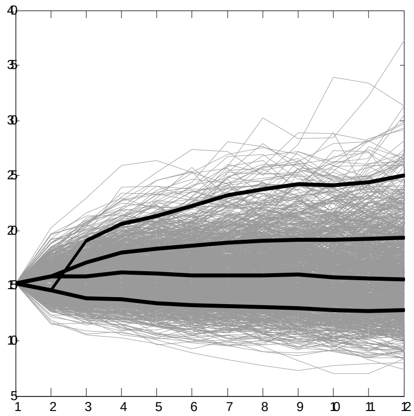
Fig. 6. Oil prices in €/MWh

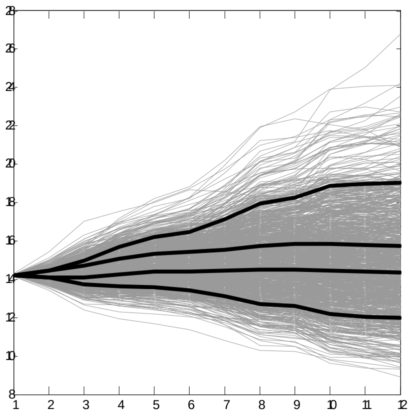
Fig. 7. Gas prices in €/MWh

We have clustered these 1000 multivariate scenarios into a multivariate 4-scenario tree (Fig. 4) using Neural-Gas clustering techniques [23]. We have also represented the resulting multivariate scenario tree for each variable in Fig. 5 to Fig. 7 (in black). The scenarios have probabilities 0.251, 0.318, 0.162 and 0.269, respectively.

As already commented, the equilibrium is divided into two well-differentiated problems:

- A stochastic market equilibrium problem that determines the spot price of the electricity as a result of the production of all the generation companies participating in the market. This problem plays the role of master problem and is formulated and solved as an LCP.
- A stochastic hydrothermal coordination problem for each generation company that derives the minimum-cost operation plan to provide the production profile suggested by the master problem subject to operation constraints (e.g. water reservoir management, take-or-pay contracts and operation limits of the generation units). The subproblem returns the marginal cost of the production plan obtained.

We have implemented the stochastic equilibrium model, the cost minimization subproblems and the algorithm in GAMS language. We have solved a real-size problem using commercial optimizers: PATH 4.3 (equilibrium problem) and CPLEX 7.1 (cost-minimization problem).

TABLE 1
SIZE OF THE EXAMPLES

	1 scenario	4 scenario	4 scenario	
			Master	Subproblem
Variables	8934	32016	3390	29548
Constraints	8934	32016	3390	2200
Non-zeroes	28410	102060	14376	57532

Table 1 presents the size of the considered problems. The two first columns show the size of the 1-scenario and the 4-scenario case studies that we were able to solve in a straightforward manner with PATH. The last two columns show the problem size of the 4-scenario case study, solved as the deterministic equivalent problem by means of Benders' decomposition.

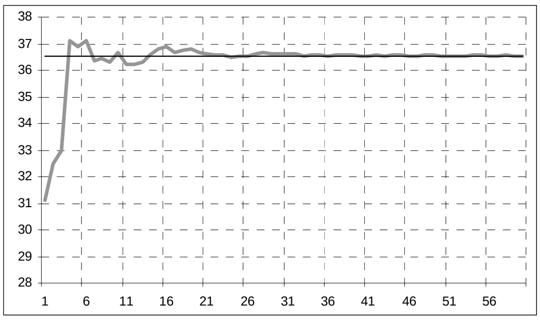

Fig. 8. On-peak January working-days price of electricity in €/MWh

Fig. 8 and Fig. 9 show the evolution of the spot price of electricity for two different load levels of January during the execution of the algorithm (in light grey). They also show as a reference the values obtained from the direct resolution of the problem with PATH (in black). In this example, the stopping rule for Benders algorithm has been set to a tolerance of 1.E-4. The algorithm requires 88 iterations to reach this tolerance -although we only represent 60 iterations-. Computation time is about 13 minutes in a Pentium IV, 2.80 GHz (512 MB RAM).

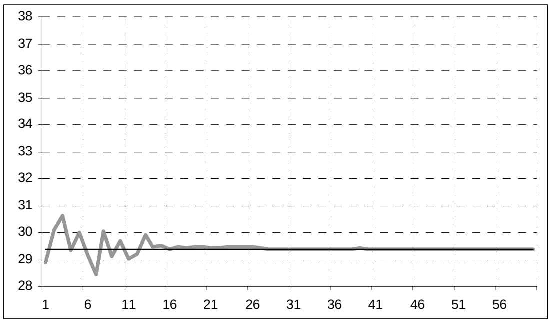

Fig. 9. Off-peak January non-working–days price of electricity in €/MWh

Fig. 10 and Fig. 11 show the evolution of the power production for two of the companies in the working-days on-peak load level of January during the execution of the algorithm (in light grey), and the production provided by the direct resolution with PATH (in black).

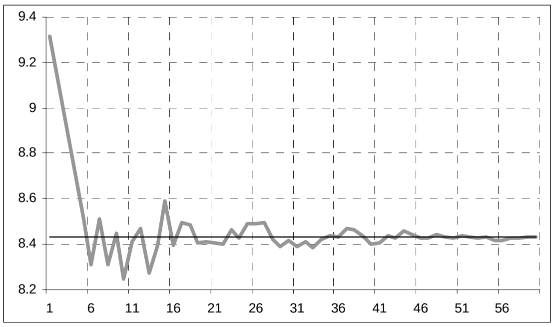

Fig. 10. Firm 1 power production in the working-days on-peak January load levels, in GW

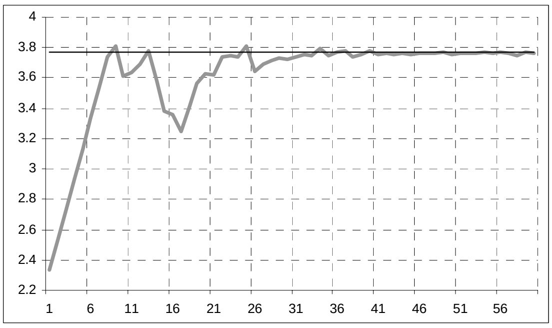

Fig. 11. Firm 6 power production in the working-days on-peak January load levels, in GW

Finally, Fig. 12 shows the evolution of the stopping-rule value –in logarithmic scale– during the execution of the algorithm.

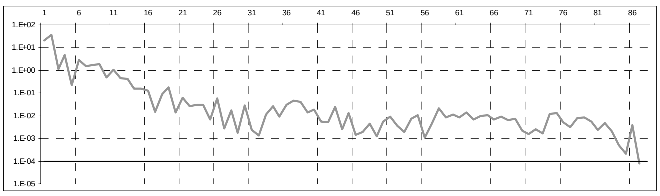

Fig. 12. Evolution of the stopping-rule value in logarithmic scale

In Table 2 we present the execution time and iterations of several problems solved directly with PATH and by means of our Benders decomposition algorithm. As can be seen, as the number of scenarios increases, the computational effort required to solve the problem directly with PATH also increases. In contrast, our Benders decomposition algorithm provides a solution with a reasonable tolerance in an acceptable execution time.

TABLE 2
COMPARISON OF EXECUTION TIMES

		4 scenarios	8 scenarios	16 scenarios
Direct resolution	Time	2 min	35 min	—
Benders	Time	13 min	17 min	40 min
Decomposition	Iterations	88	60	62

IV. CONCLUSION

This paper has presented a natural approach to solve a certain type of large-scale market equilibrium problems by means of Benders decomposition algorithm. This approach exploits the fact that in this type of problems two types of decision levels can be identified: a strategic level in which the agents decide the market share they want to cover and an operational level in which each agent decides how to best cover this market share. This structure suggests resorting to the classical Benders decomposition algorithm for two-stage problems. In the first-stage (master problem) we have an equilibrium problem with an approximate representation of the agents' production costs and constraints. In the second-stage we have as many subproblems as agents are being considered. Each subproblem represents the cost-minimization decision process that each agent carries out in order to cover the market share he or she has decided in the corresponding strategic decision level.

The proposed algorithm has been successfully applied to a middle-size stochastic problem in the context of a wholesale electricity market with a time scope of one year and six generation companies. Several stochastic deterministic equivalent problems have been formulated and solved with uncertainty in the fuel costs.

For practical purposes, the traditional Benders decomposition algorithm has been enhanced by incorporating into the master problem limits in the movement of primal variables. These limits are obtained at each iteration after the subproblem resolution of each company by applying sensitivity analysis to the right-hand-side values of the coupling constraints. These limits force the primal solutions to smoothly move from a Benders optimality cut to an adjacent one. The combination of this approach with a good initial starting solution (the result of an overall cost minimization problem) leads to an acceptable performance of the method towards the optimal solution. This might be understood as a transition from a centralized cost-minimization solution to an equilibrium solution.

We would like to emphasize that it has not been the goal of this research to improve the resolution time of commercial complementarity problem solvers. However, the results obtained prove that it is worth using a decomposition algorithm when the complementarity problem that has to be solved is large and has an adequate structure. This broadens the field of applications of equilibrium models to a number of situations in which the resulting complementarity problem sizes are not affordable for a commercial solver.

REFERENCES

[1] E. P. Kahn, "Numerical Techniques for Analyzing Market Power in Electricity," *The Electricity Journal*, vol. 11, pp. 34-43, 1998.

[2] M. Ventosa, Á. Baíllo, M. Rivier, and A. Ramos, "Electricity market modeling trends," *Energy Policy*, 2004.

[3] EMF, "An International Comparison of Models for Measuring Market Power in Electricity." Energy Modeling Forum Working Paper 17.1. Stanford University, 1999.

[4] S. S. Oren, "Economic Inefficiency of Passive Transmission Rights in Congested Electricity Systems with Competitive Generation," *The Energy Journal*, vol. 18, pp. 63-83, 1997.

[5] S. Borenstein and J. B. Bushnell, "An Empirical Analysis of the Potential for Market Power in California's Electricity Industry." Program on Workable Energy Regulation (POWER) PWP-044r. University of California Energy Institute. Berkeley, California, 1998.

[6] R. W. Cottle, J.-S. Pang, and R. E. Stone, *The Linear Complementarity Problem*: Kluwer Academic Press, 1992.

[7] B. F. Hobbs, "Linear Complementarity Models of Nash-Cournot Competition in Bilateral and POOLCO Power Markets," *IEEE Transactions on Power Systems*, vol. 16, pp. 194-202, 2001.

[8] O. Daxhelet and Y. Smeers, "Variational Inequality Models of Restructured Electricity Systems," in *Complementarity: Applications, Algorithms and Extensions*, M. C. Ferris, O. L. Mangasarian, and J.-S. Pang, Eds.: Kluwer Academic Publishers, 2001.

[9] Y. Smeers, "Market Incompleteness in Regional Electricity Transmission." Department of Mathematical Engineering and Center for Operations Research and Econometrics. Université Catholique de Louvain, Louvain-la-Neuve, Belgium, 2001.

[10] S. C. Billups, S. P. Dirkse, and M. C. Ferris, "A Comparison of Large Scale Mixed Complementarity Problem Solvers," *Computational Optimization and Applications*, vol. 7, pp. 3-25, 1997.

[11] T. F. Rutherford, *Extension of GAMS for Complementarity Problems Arising in Applied Economic Analysis*, vol. 19, 1995.

[12] J.-Y. Wei and Y. Smeers, "Spatial Oligopolistic Electricity Models with Cournot Players and Regulated Transmission Prices," *Operations Research*, vol. 47, pp. 102-112, 1999.

[13] J. B. Bushnell, "Water and Power: Hydroelectric Resources in the Era of Competition in the Western US." Program on Workable Energy Regulation (POWER) PWP-056. University of California Energy Institute. Berkeley, California, 1998.

[14] M. Ventosa, M. Rivier, A. Ramos, and A. García-Alcalde, "An MCP Approach for Hydrothermal Coordination in Deregulated Power Markets," presented at 2000 IEEE - PES Summer Meeting, Seattle, WA, 2000.

[15] M. Rivier, M. Ventosa, A. Ramos, F. Martínez-Córcoles, and Á. Chiarri, "A Generation Operation Planning Model in Deregulated Electricity Markets based on the Complementarity Problem," in *Complementarity: Applications, Algorithms and Extensions*, M. C. Ferris, O. L. Mangasarian, and J.-S. Pang, Eds.: Kluwer Academic Publishers, 2001.

[16] C. J. Day, B. F. Hobbs, and J.-S. Pang, "Oligopolistic Competition in Power Networks: A Conjectured Supply Function Approach," *IEEE Transactions on Power Systems*, vol. 17, pp. 597-607, 2002.

[17] A. García-Alcalde, M. Ventosa, M. Rivier, A. Ramos, and G. Relaño, "Fitting Electricity Market Models. A Conjectural Variations Approach," presented at 14th PSCC, Sevilla, Spain, 2002.

[18] J. Barquín, E. Centeno, and J. Reneses, "Medium-term generation programming in competitive environments: A new optimization approach for market equilibrium computing," *IEE Proceedings on Generation, Transmission and Distribution*, vol. 151, pp. 119-126, 2004.

[19] M. Ventosa, A. García-Alcalde, A. Mencía, M. Rivier, and A. Ramos, "Modeling Inflow Uncertainty in Electricity Markets: A Stochastic MCP Approach," presented at 6th International Conference on Probabilistic Methods Applied to Power Systems, Funchal, Madeira (Portugal), 2000.

[20] R. M. Van Slyke and R. Wets, "L-Shaped Linear Progams with Applications to Optimal Control and Stochastic Programming," *SIAM Journal on Applied Mathematics*, vol. 17, pp. 638-663, 1969.

[21] C. Batlle and J. Barquín, "A strategic production cost model for electricity market price analysis," *IEEE Transactions on Power Systems*, 2003.

[22] L. Clewlow and C. Strickland, *Energy derivatives: Pricing and risk management*. London: Lacima Publications, 2000.

[23] S. Cerisola, J. M. Latorre, Á. Baíllo, and A. Ramos, "Scenario Tree Generation through the Neural Gas Algorithm," *Working Paper IIT-04-052A*, 2004.