A NEW METHOD FOR TRANSIENT STABILITY ASSESSMENT BASED ON CRITICAL TRAJECTORY

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Abstract – This paper proposes a new method for transient stability analysis for electric power systems. Different from existing methods, a minimization problem is formulated for obtaining critical conditions for transient stability. The method is based on the computation of a trajectory on the stability boundary, which is referred to as critical trajectory in this paper. The critical trajectory is defined as the trajectory that starts from a point on a fault-on trajectory and reaches an unstable equilibrium point (UEP), in which the former point is called the exit point and the latter points, controlling UEP (CUEP). The solution of the minimization problem provides critical clearing time (CCT), exit point, and CUEP simultaneously.

Keywords: electric power system, transient stability, transient energy function, unstable equilibrium

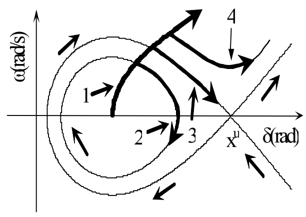
1 INTRODUCTION

Transient stability analysis plays an important role for maintaining security of power system operation. The analysis is mainly performed through numerical simulations, where numerical integration is carried out step by step from an initial value to obtain dynamic response to disturbances. In general, such a numerical simulation method is effective since it easily takes into account various dynamic models for complex power systems as well as various time sequences of events. Furthermore, the method is useful in analyzing various kinds of complex nonlinear phenomena such as in [1-3]. However, the numerical simulation is usually time consuming, and therefore, it is not necessarily suited for real time stability assessment.

An alternative approach, called transient energy function methods [4-16], assesses system stability based on the transient energy. Those methods provide fast and efficient stability assessment for a number of disturbances. Although they are practically useful, a common disadvantage is concerned with the accuracy of stability judgment. A major limitation is that they cannot deal with detailed models for power systems since the transient energy functions are available only for limited types of power system models. Another problem is that the most of the methods require the evaluation of critical energy, which affects considerably the accuracy of stability assessment. The critical energy is not necessarily easily calculated.

Among various transient energy function methods, the Boundary Controlling Unstable (BCU) equilibrium point method seems to be a promising method in the sense that it has a theoretical background for the evaluation of the critical energy [10-16]. The method evaluates the critical energy at CUEP. Improved techniques have been proposed in [12,13], while there are discussions on the underlying assumption for the BCU method [14-16].

This paper proposes a new method for transient stability analysis. In order to describe the proposed method, typical dynamic behaviors of a power system are given in figure 1, where a single machine case without damping is presented as an example. Three kinds of trajectories are given in phase plane starting at different points on a fault-on trajectory (1). Trajectory (2) is for a stable case where the fault is cleared early enough and it oscillates around a stable equilibrium point (SEP). Trajectory (4) corresponds to an unstable case, where the fault clearing is too late. Trajectory (3) corresponds to a critical case for stability and is referred to as critical trajectory in this paper. The critical trajectory is defined as the trajectory that starts from a point on a fault-on trajectory and reaches CUEP. It is generally difficult to compute the critical trajectory by means of conventional numerical simulations. In this paper, the problem is formulated as a minimization problem for computing the critical trajectory. Critical conditions for transient stability such as CCT and CUEP are computed as the solution of the minimization problem.



1: fault-on trajectory,
2: stable case after fault clearing
3: critical case, 4: unstable case
x^u: unstable equilibrium point (UEP)

Figure 1: Trajectories in a phase plane for a single machine to infinite bus system without damping.

2 PROBLEM FORMULATION

In this section, we formulate a problem for obtaining critical conditions for stability of a power system represented by the following nonlinear equation.

$$\dot{x} = f(x) \qquad (1)$$

Letting a solution of equation (1) at time t^k be denoted as x^k, the following equation holds using the trapezoidal formula.

$$x^{k+1} - x^k = \frac{1}{2}(\dot{x}^{k+1} + \dot{x}^k)(t^{k+1} - t^k) \quad (2)$$

where

$$\dot{x}^k = f(x^k)$$

In this paper, super-script k is used for state transition number with respect to time.

As is stated in the Introduction, we pay attention to the critical trajectory, where a system fault is cleared at CCT and then the state variables converge to CUEP with infinite time. Figure 2 shows the critical trajectory, where two boundary points, x^0 and x^u, represent the initial point at CCT and CUEP. A difficulty in obtaining the critical trajectory is that infinite time is taken to reach CUEP, implying that infinite time steps are required in the conventional numerical integration with respect to time. To avoid the problem, the distance between the two points in (2) is defined as:

$$\varepsilon = \left|x^{k+1} - x^k\right| = \frac{1}{2}\left|\dot{x}^{k+1} + \dot{x}^k\right|(t^{k+1} - t^k) \quad (3)$$

Thus, the time duration is replaced with the distance as follows:

$$t^{k+1} - t^k = \frac{2}{\left|\dot{x}^{k+1} + \dot{x}^k\right|}\varepsilon \quad (4)$$

Equation (4) is substituted into (2) to obtain the following form.

$$x^{k+1} - x^k - \frac{\dot{x}^{k+1} + \dot{x}^k}{\left|\dot{x}^{k+1} + \dot{x}^k\right|}\varepsilon = 0 \quad (5)$$

By the above equation, the numerical integration with respect to time is transformed into that with distance. This transformation makes it possible to represent the critical trajectory by finite points with a same distance as shown in figure 2. Thus, the problem for obtaining the critical condition for transient stability for system (1) is formulated as follows:

$$\min_{x,\varepsilon,\tau} \sum_{k=0}^{m} |\mu^k|^2 \quad (6)$$

where

$$\mu^k = x^{k+1} - x^k - \frac{\dot{x}^{k+1} + \dot{x}^k}{\left|\dot{x}^{k+1} + \dot{x}^k\right|}\varepsilon \quad (7)$$

$$\dot{x}^k = f(x^k)$$

with boundary conditions:

$$x^0 = x^0(\tau) \quad (8)$$

$$x^{n+1} = x^u \quad \text{with} \quad f(x^u) = 0 \quad (9)$$

In the above equations, μ_k is ideally zero, implying a numerical error due to the trapezoidal approximation of (5). Equation (8), a boundary condition for the initial point, expresses a fault-on trajectory as a function of fault clearing time, τ. Equation (9) is the other boundary condition, where x^u is UEP that satisfies equilibrium equation $f(x^u) = 0$, which is equivalent to the power flow equations. It is noted that the latter boundary condition is additional compared with the conventional numerical integration formulated as an initial value problem. In such a conventional method, numerical error μ^k is accumulated as k increases so that a final point in general has a considerable error. On the other hand, the proposed method specifies the final point additionally as in (9), then solves the redundant equations as a minimization problem so that the individual errors μ^k are properly distributed.

The solution of the problem, (6)-(9), is interpreted as follows. The set of points, x^k, k=0 to m+1, represent the critical trajectory, where ε is automatically determined when the number of integration steps, m, is specified; CCT and CUEP are respectively obtained as τ and x^{m+1} at the solution. Note that m is an important parameter that affects the accuracy and computation time for the proposed method.

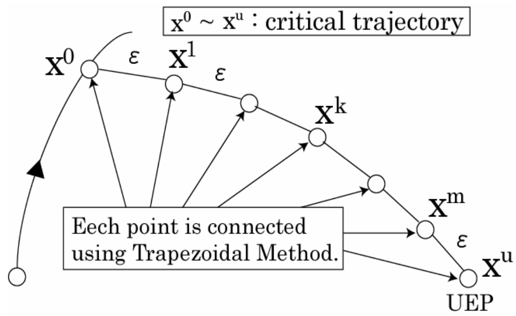


Figure 2: Concept of the proposed method.

3 APPLICATION TO ONE MACHINE SYSTEM

3.1 A Single Machine System

In this section, the proposed method is applied to a simple system in figure 3. The system has a synchronous machine with AVR, governor, as well as damping, represented by the following equations.

$$\begin{aligned}
\dot{\theta} &= \omega - \omega_s \\
M\dot{\omega} &= P_M - \frac{EV_b}{X}\sin\theta - \frac{d}{\omega_s}(\omega - \omega_s) \\
\dot{E} &= \frac{1}{T_{AVR}}\left\{-(E - E_0) + K_{AVR}(V_{ref} - V_t)\right\} \\
\dot{P}_M &= \frac{1}{T_{GOV}}\left\{-(P_M - P_{Mref}) + K_{GOV}\frac{(\omega_s - \omega)}{\omega_s}\right\}
\end{aligned} \quad (10)$$

Parameters for this system are given in the appendix. The state variable vector for equation (1) is given as follows:

$$x = [\theta\ \omega\ E\ P_M]^T \quad (11)$$

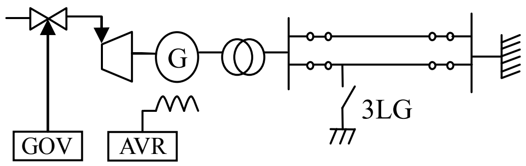


Figure 3: A single machine to infinite bus system

In this examination, a 3-L-G fault occurs at one of the double transmission lines at t=0; the fault is cleared by tripping the line at t=τ. Then, CCT will be evaluated by analyzing the post fault system. The analysis will be carried out by the proposed method as well as by the conventional simulation method.

3.2 Conventional Simulation Method

The 4-th order Runge-Kutta method is used for numerical integration with step time of 0.001 [s]. First, the fault-on trajectory is obtained numerically, which is stored as $x^0(\tau)$ as a function of time, τ. Then, $x^0(\tau)$ with a specified τ is selected as an initial condition to simulate the dynamic behavior to judge the stability of the system. This process is repeated by setting different value of τ. The binary search method is used to estimate a critical value of τ, that is CCT. The obtained results are given in Table 1, where the time intervals in which CCT exists are listed as well as computation time for different step size of the Runge-Kutta method. Although the CCT values obtained in Table 1 could contain truncation errors of the Runge-Kutta method as well as round-off errors, the result will be used as a benchmark.

Table 1: CCT Obtained by Conventional Method

step size	CCT [s]	CPU time [s]
0.01	1.37-1.38	2.7
0.005	1.375-1.380	6.0
0.002	1.378-1.380	15.4
0.001	1.379-1.380	50.4
0.0005	1.3795-1.3800	200.7

3.3 The Proposed Method

The proposed method is performed as in the following manner. First, the fault-on trajectory obtained by the conventional method is approximated as a quadratic function corresponding to (8) as follows.

$$x^0 = a\tau^2 + b\tau + c \tag{12}$$

Concerned with the other boundary condition, (9), we solve $f(x^u) = 0$ to obtain x^u in advance. Namely, equations (10) with setting the left-hand sides to null are solved to obtain an UEP, which is listed in Table 2. Such a separate computation for CUEP should be effective in the proposed method if CUEP is reliably obtained. A method of obtaining the CUEP will be examined in section 4.

Table 2: SEP and UEP for single machine system

x	SEP	UEP
ω	376.9911	376.9911
δ	0.2339	2.8189
E	1.1	1.0722
P_M	0.85	0.85

Now that equations (6) - (9) are defined, the Newton's method is used to solve the least square minimization problem. Table 3 lists CCT, the number of iterations for convergence, CPU time for various cases where m is changed as a parameter from 1 to 100. As is observed, enough accuracy is obtained in CCT value even for a small number for m. For example, even a case with m=1 provides CCT with error around 0.01 [s] only.

Table 3: Performance of the proposed method

m	CCT[s]	iteration	CPU[s]
1	1.374	19	0.390
2	1.380	15	0.291
3	1.382	18	0.300
4	1.383	16	0.291
5	1.384	18	0.310
6	1.384	17	0.300
7	1.384	20	0.331
8	1.384	19	0.320
9	1.385	17	0.321
10	1.385	17	0.310
20	1.385	19	0.461
30	1.385	17	0.591
40	1.385	20	0.951
50	1.385	19	1.242
60	1.385	22	1.752
70	1.385	21	2.333
80	1.385	27	3.766
90	1.385	22	3.725
100	1.385	54	10.896

4 APPLICATION TO BCU METHOD

4.1 Introduction

In the previous section, it has been confirmed that the critical trajectory is successfully obtained when CUEP is directly specified in a single machine to infinite bus system. Then, we are interested in an extension of the method to a multi-machine system. However, since a direct extension of the method may cause complexity, we should start with a case with a simple power system model for (1). In this sense, a simple but meaningful approach can be an application to the BCU method [10,11]. Although there are discussions on the underlying assumption for the BCU method [14-16], the method utilizes a simplified system model called the gradient system in order to obtain CUEP. This implies that, if the gradient system is used as (1), the proposed formulation provides an alternative solution method for the same problem.

As is known, the problem of finding CUEP is a tough problem, but is an important subject for the transient energy function methods. Among various approaches, an effective approach seems to be the shadowing method [12] based on the BUC method. In the following, the proposed formulation is applied to the BCU method to develop a new approach for obtaining CUEP for multi-machine systems.

4.2 Power System Model

We consider n-machine power system model as follows:

$$M_i\dot{\omega}_i = P_{m_i} - P_{e_i}(\theta)$$
$$\dot{\theta}_i = \omega_i - \omega_s\,,\quad \text{i=1..n} \qquad (13)$$

where

$$P_{e_i}(\theta) = \sum_{j=1}^{n} Y_{ij}E_iE_j\sin(\theta_i - \theta_j + \alpha_{ij})$$

The energy function corresponding to this system is given as:

$$V = \frac{1}{2}\sum_{i=1}^{n} M_i\tilde{\omega}_i^2 + \sum_{i=1}^{n}\int_{\delta_i}^{\delta_i^s}(P_{e_i}(\delta) - P_{m_i} + \frac{M_i}{M_T}P_{coa}(\delta))\,d\delta_i$$
$$= V_K(\tilde{\omega}) + V_P(\delta) \qquad (14)$$

where

$$\delta_i = \theta_i - \frac{1}{M_T}\sum_{i=1}^{n}M_i\theta_i\,,\quad \tilde{\omega}_i = \omega_i - \frac{1}{M_T}\sum_{i=1}^{n}M_i\omega_i$$
$$P_{coa}(\delta) = \sum_{i=1}^{n}(P_{m_i} - P_{e_i}(\delta))\,,\quad M_T = \sum_{i=1}^{n}M_i$$

In order to compute CUEP, the following system called the gradient system has been proposed to be used in the BCU method [10,11].

$$\dot{\delta} = \frac{\partial V_P(\delta)}{\partial \delta} \qquad (15)$$

4.3 A Method for Obtaining CUEP

The proposed method is applied to the BCU method to develop an alternative method for the computation of a CUEP. The system (1) to be analyzed is now given by the gradient system, (15), which is equivalently expressed as follows.

$$\dot{\delta}_i = P_{m_i} - P_{e_i}(\delta) - \frac{M_i}{M_T}P_{coa}(\delta)\,,\quad \text{i=1...n} \qquad (16)$$

Namely, the state vector in (1) is:

$$x = \delta = [\delta_1, \delta_2, \cdots, \delta_n]^T \qquad (17)$$

For an initial point corresponding to conditions (8), we use a fixed point as follows.

$$\delta^0 = \delta^{exit} \qquad (18)$$

where δ^{exit} is the exit point detected as the first local maxima of the potential energy, V_P, on the fault-on trajectory. For the condition for UEP, we use the following form of the power flow equations, corresponding to (9).

$$P_{m_i} - P_{e_i}(\delta^u) - \frac{M_i}{M_T}Pcoa = 0\,,\quad \text{i=1...n} \qquad (19)$$

Thus, the problem (6)-(9) is defined. Then, we will use the Newton's method to solve the condition of minimizing the least square error for the set of equations.

4.4 Numerical Examinations

Numerical examinations are carried out using 3-machine 9-bus system [18] and IEEE 6-machine 30-bus system as shown in figures 4 and 5, respectively.

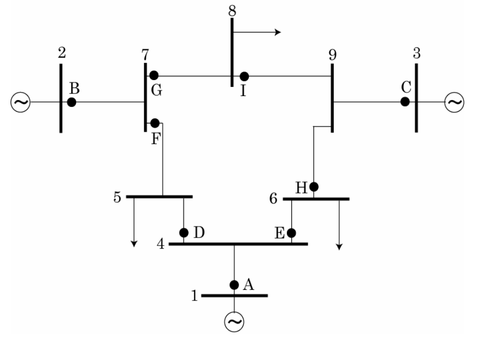


Figure 4: Anderson & Fouad 3-machine 9-bus system.

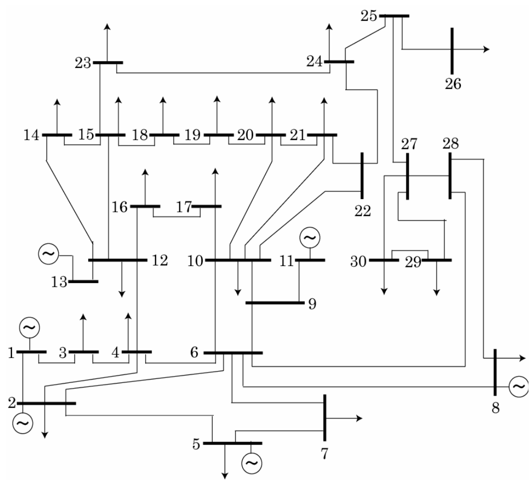


Figure 5: IEEE 6-machine 30-bus system

It is assumed that every transmission line consists of double parallel circuits, and that a 3-L-G fault occurs at a point very close to a bus on one of parallel lines; after a while the fault is cleared by opening the faulted line. For this condition, CUEP will be obtained to evaluate CCT using the transient energy function (14). For this condition for examinations, the proposed method is compared with the Shadowing method [12].

The obtained CCTs are listed in Tables 4 and 5 for various cases for different fault locations. The tables also show the fault location and the number of iterations required for the proposed method, where max $|dx_i|<10^{-3}$ is used as a convergence criterion for the Newton's method. It is observed that both the methods provide the same CCTs except for case G in Table 4 and cases A and D in Table 5. In these cases, the Shadowing method cannot provide the CUEPs, while the proposed method yields the right solutions confirmed by the numerical simulation for the original system of (13).

Table 4: CCT [s] obtained for 3-machie system

falut point	open line	BCU-Shadowing	proposed method	iteration
A	1-4	0.33	0.33	6
B	2-7	0.21	0.21	3
C	3-9	0.27	0.27	4
D	4-5	0.32	0.32	7
E	4-6	0.32	0.32	7
F	5-7	0.21	0.21	18
G	7-8	×	0.23	20
H	6-9	0.45	0.45	8
I	8-9	0.33	0.33	2

Table 5: CCT obtained for 6-machie system

falut point	open line	BCU-Shadowing	proposed method	iteration
A	1-2	×	0.82	2
B	2-4	0.75	0.75	4
C	5-7	1.15	1.15	2
D	8-6	×	0.84	3
E	11-9	0.93	0.93	3
F	13-12	0.98	0.98	3

It is noted that m=2 is used in the proposed method for all the cases except for case G for 3-machine system, where m=18 is set. Under the setting for m, the iteration numbers in Tables 4 and 5 were obtained. We should mention that case G is an unusual case, where convergence is only obtained for m ≥ 18 and a larger number of iterations are required. In order to examine the above results, case G is studied in the following.

For the 3-machine system, selecting case B as a representative normal case, cases B and G are compared in figure 6 and 7, respectively, where equipotential contours are described around CUEP and the exit point. It is observed in figure 6 that the potential energy surface shows a simple shape around the exit point and CUEP. This is a quite typical shape where the Shadowing method successfully works as well as the proposed method. On the other hand, a more complicated shape for potential energy is observed in figure 7, where the critical trajectory obtained by the proposed method shows relatively a long and winding curve. It is understood the reason why the proposed method requires a larger number for m. It is observed that the Shadowing method followed a ridge for wrong direction and could not find the UEP.

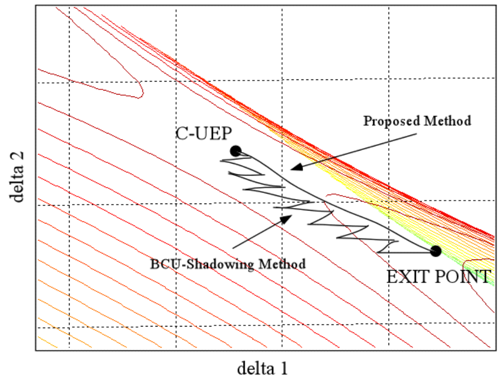


Figure 6: A Normal Case B for 3-Machine System

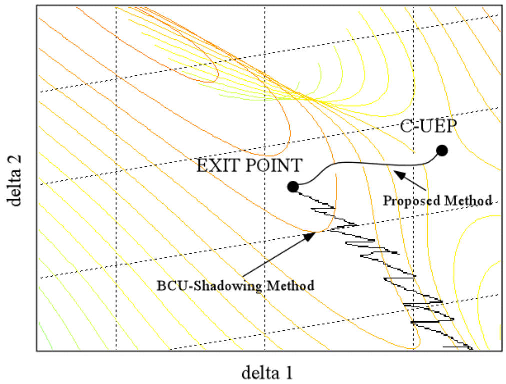


Figure 7: Unusual Case G for 3-Machine System

5 CONCLUSION

This paper proposes a new formulation for transient stability analysis for electric power systems. Different from conventional simulation methods, a critical condition for stability such as a critical clearing time (CCT) is directly obtained as a solution of a minimization problem. The method is based on the computation of critical trajectory that represents a critical case for stability. It is demonstrated that CCT is computed without major approximations for a single machine to infinite bus system, where damping effect, AVR and governor are taken into account.

As another application of the proposed formulation, the paper presents a method for computing a controlling UEP (CUEP) based on the BCU method. The CUEP is useful for transient stability assessment by the energy function method as well as for the future extension of the proposed method to obtain exact CCT. The effectiveness of CUEP computation is demonstrated in 3-machine 9-bus system and 6-machine 30-bus system. It has been confirmed that the computation time is reasonable as well as the accuracy of the solution.

APPENDIX

The system parameters for single machine system are given as follows:
M =0.5, $X = x_d^{'} + x_l$, $x_d^{'}$ =0.2, x_l =0.1 (pre-fault), V_b =1.0, d =0.05, K_{AVR} =10, T_{AVR} =2, K_{GOV} =20, T_{GOV} =3, E =1.1, ω=100π, P_M =0.85

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