LOAD SHEDDING – AN EFFICIENT USE OF LTC TRANSFORMERS

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Abstract – This paper focuses on a methodology to deal with load shedding in electric power systems such as to make its impact on the demand level as minimal as possible. The power demand is parameterized by the factor α, which is minimized subject to equality (real and reactive power mismatches) and inequality constraints (operational and equipment limits). Lower and upper limits on voltage magnitudes and taps of the load tap change (LTC) transformers are considered. Aiming at evaluating their influence on the load shedding, the taps of LTC transformers are included in the set of optimization variables. It is well known that as a consequence of this inclusion the iterative process is prone to convergence problems. This paper presents a simple and efficient methodology to deal with LTC transformer taps, which consists in a slight modification of the Interior Point methodology used to solve the load shedding optimization problem. Numerical results for IEEE-test systems and a realistic power system will be presented in order to assess the performance of the proposed method.

Keywords: Load shedding, Parameterized load, Interior Point algorithms, LTC transformers

1 INTRODUCTION

Steady state power system analysis frequently requires the assessment of the feasibility of the power supply for several loading levels. Under certain circumstances, there is a well-known difficulty to determine even a simple solution for the conventional power flow problem. For some load levels, it is not possible to obtain operational solutions, that is, those for which the power demand is supplied and the operational constraints (power generation limits, voltage magnitude bounds, transformer tap limits, power flow limits, etc) are satisfied. In these cases, the power system controls are fully monitored though eventually the load curtailment becomes mandatory.

Figure 1(a) shows a scheduled demand that can not be integrally supplied by the electric system. In order to restore the solution of the electric network equations, we can perform the following actions:

- a load shedding, Figure 1(b), being this case treated as the worst situation in electrical energy industries and this action is performed only when all control adjustments have already been set;
- new adjustments in the set of the system control variables, Figure 1(c);
- joining the two ones, Figure 1(d).

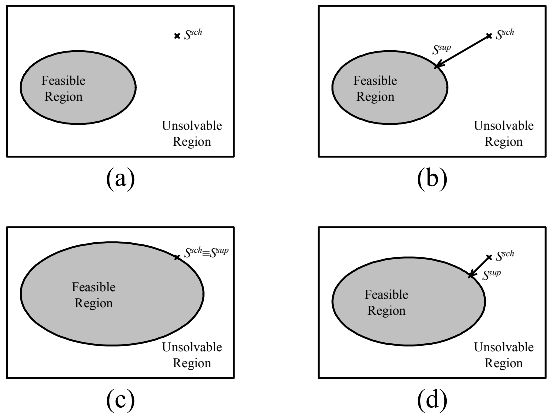

Figure 1: Figure 1. Restoring the solution of the electric network equations.

Critical loading conditions are a consequence of a number of factors. Frequently the lack of a suitable reactive power support implies in limitations with respect to the power load supply, mainly if the reactive power sources are far from the load centers. This situation becomes worse if the power system controls are not fully explored.

Determining solutions for the electric network equations in steady-state, under high loading level, can be performed basically through two kinds of methodology:

- based on conventional solution of the load flow problem;
- based on optimization algorithms.

The solutions provided by strategies from the first type have been frequently used in voltage collapse studies. In this case, the operational conditions are usually adverse, such that a simple power flow solution is extremely important as a base point to determine corrective adjustments. Some of the methodologies used to this aim minimize the load shedding [1][2]. In spite of its simplicity, this type of formulation has the following restrictions:

- the control adjustment used for reducing the load shedding is performed through a trial-error process which does not guarantee that these adjustments are suitable;
- it is not possible to include other operational constraints, further than those, which have already modeled in the conventional power flow problem.

Methodologies based on optimization algorithms [3][4] formulate a problem with dimension considerably larger. The solutions of the electric network equations are obtained such that all inequality constraints included in the analytical model are satisfied. Besides, it is possible to select the variables to be adjusted in order to get the desired optimality. This characteristic makes these techniques suitable to real operation studies.

This work focuses on the use of the taps of LTC transformers in the solution of the load shedding problem. Two aspects are emphasized: the strategy to handle these variables and the decrease in the load shedding. It is shown that, in spite of the unfavorable effects of the LTC's referred to in [5], the inclusion of the transformer tap control provides an additional resource to reduce the amount of non-supplied demand under the critical loading conditions. The determination of the load shedding is modeled as an optimization problem with equality and inequality constraints. The former ensures that the power balance at each bus is satisfied whereas the latter guarantees operational solutions. The proposed formulation is based on the inclusion of a heuristic strategy to handle the taps of the LTC transformers (modeled according to [6]) in the load shedding optimization problem. Numerical results obtained with power systems of different sizes illustrate the proposed analysis.

2 THE PROPOSED METHODOLOGY

The load shedding problem can be state as [4]

$$\begin{array}{ll} Min & \alpha \\ s.t. & \boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{0} \\ & \boldsymbol{h}^m \le \boldsymbol{h}(\boldsymbol{x}) \le \boldsymbol{h}^M \end{array} \tag{1}$$

where α is the factor that parameterizes the real and reactive power demands;
$\boldsymbol{x}$ is the set of optimization variables;
$\boldsymbol{g}$ is the vector composed by equality constraints (real and reactive power mismatches at all load buses);
$\boldsymbol{h}$ is the vector of inequality constraints, with its minimum and maximum limits on voltage magnitudes, taps of LTC transformers and power generations.

The optimization variables vector is composed by

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{V} & \boldsymbol{\delta} & \boldsymbol{a} & \alpha \end{bmatrix}^T \tag{2}$$

where $\boldsymbol{V}$ is the set with the complex voltage magnitudes at all buses, with its minimum and maximum limits;
$\boldsymbol{\delta}$ is the vector with complex voltage phase angles of all buses, except for the slack bus;
$\boldsymbol{a}$ is the set of LTC transformers taps, with its minimum and maximum limits.

The load flow equations parameterized by the factor α, vector $\boldsymbol{g}(\boldsymbol{x})$, are mathematically equal to

$$\boldsymbol{g}(\boldsymbol{x}) = \begin{bmatrix} \boldsymbol{P}(\boldsymbol{x}) + \boldsymbol{P}_{d0}(\boldsymbol{x}) - \alpha\Delta\boldsymbol{P}_d \\ \boldsymbol{Q}(\boldsymbol{x}) + \boldsymbol{Q}_{d0}(\boldsymbol{x}) - \alpha\Delta\boldsymbol{Q}_d \end{bmatrix} \tag{3}$$

where $\boldsymbol{P}(\boldsymbol{x})$ and $\boldsymbol{Q}(\boldsymbol{x})$ are the vectors with evaluated real and reactive power injections at all load buses, respectively;
$\boldsymbol{P}_{d0}$ and $\boldsymbol{Q}_{d0}$ are the vectors with scheduled real and reactive power demands, respectively;
$\Delta\boldsymbol{P}_d$ and $\Delta\boldsymbol{Q}_d$ are respectively the decreasing direction of real and reactive power demands. These directions can be determined from previous conditions or from real time operation.

The vector $\boldsymbol{h}(\boldsymbol{x})$ which represents the set of inequality constraints can be expressed as

$$\boldsymbol{h}(\boldsymbol{x}) = \begin{bmatrix} \boldsymbol{V} & \boldsymbol{a} & \boldsymbol{P}_G(\boldsymbol{x}) & \boldsymbol{Q}_G(\boldsymbol{x}) \end{bmatrix}^T \tag{4}$$

where $\boldsymbol{P}_G(\boldsymbol{x})$ and $\boldsymbol{Q}_G(\boldsymbol{x})$ are the vectors with real and reactive power generation at all system generation buses, respectively.

It is supposed that the power demand at generation buses is effectively supplied, that is, there is no load shedding at these buses. Thus, the power balance equations at the generation buses are

$$\begin{array}{l} \boldsymbol{P}_G(\boldsymbol{x}) = \boldsymbol{P}(\boldsymbol{x}) + \boldsymbol{P}_{d0} \\ \boldsymbol{Q}_G(\boldsymbol{x}) = \boldsymbol{Q}(\boldsymbol{x}) + \boldsymbol{Q}_{d0} \end{array} \tag{5}$$

If the Interior Point algorithm is applied to solve the optimization problem expressed in (1), the augmented Lagrangean function is given by

$$\begin{array}{r} \pounds(\boldsymbol{x},\boldsymbol{s}_\ell,\boldsymbol{s}_u,\boldsymbol{\lambda},\boldsymbol{\pi}_\ell,\boldsymbol{\pi}_u) = \alpha - \mu\sum(\ln \boldsymbol{s}_\ell + \ln \boldsymbol{s}_u) - \boldsymbol{\lambda}^T\boldsymbol{g}(\boldsymbol{x}) - \\ -\boldsymbol{\pi}_\ell^T\left[\boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{s}_\ell - \boldsymbol{h}^m\right] - \boldsymbol{\pi}_u^T\left[\boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{s}_u - \boldsymbol{h}^M\right] \end{array} \tag{6}$$

where μ is the barrier parameter;
$\boldsymbol{s}$ is the vector with slack variables;
$\boldsymbol{\lambda}$ is the vector with the Lagrange multipliers associated to the equality constraints;
$\boldsymbol{\pi}$ is the vector with the Lagrange multipliers associated to the inequality constraints.

The first order optimality conditions (Karush-Kuhn-Tucker conditions) are expressed as

$$\nabla_{\boldsymbol{x}}\pounds = \begin{bmatrix} 0 & 0 & \dots & 1 \end{bmatrix}^T - \left[\mathbf{G}(\boldsymbol{x})\right]^T\boldsymbol{\lambda} - \left[\mathbf{H}(\boldsymbol{x})\right]^T\left(\boldsymbol{\pi}_\ell + \boldsymbol{\pi}_u\right) = \boldsymbol{0} \tag{7}$$

$$\nabla_{\boldsymbol{s}_\ell}\pounds = \mu\boldsymbol{e} - \mathbf{S}_\ell\boldsymbol{\pi}_\ell = \boldsymbol{0} \tag{8}$$

$$\nabla_{\boldsymbol{s}_u}\pounds = \mu\boldsymbol{e} + \mathbf{S}_u\boldsymbol{\pi}_u = \boldsymbol{0} \tag{9}$$

$$\nabla_{\boldsymbol{\lambda}}\pounds = -\boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{0} \tag{10}$$

$$\nabla_{\pi_\ell} \pounds = \boldsymbol{h}(\boldsymbol{x}) - \boldsymbol{s}_\ell - \boldsymbol{h}^m = \boldsymbol{0} \quad (11)$$

$$\nabla_{\pi_u} \pounds = \boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{s}_u - \boldsymbol{h}^M = \boldsymbol{0} \quad (12)$$

where $\boldsymbol{e}$ is a unity column vector;

G is the Jacobian matrix of the equality constraints;

H is the Jacobian matrix of the inequality constraints;

S is a diagonal matrix with the slack variables.

The set of nonlinear equations (7), (8), (9), (10), (11) e (12) is solved through the perturbed Newton's method. The barrier parameter μ is updated at each iteration and must be zero at the optimal solution. The convergence of the iterative process is reached if the norm-∞ of the gradient of the Lagrangean function, equations (7) to (12), is smaller than a pre-specified tolerance and the barrier parameter value tends to zero.

Two alternative versions of the Interior Point method [7][8][9] can be used to solve the optimization problem expressed by equation (1). Figure 2 shows the algorithm for Minimum Load Shedding in a Pre-Specified Direction (MLSSD) based on the Primal Dual Interior Point method.

1. Initialize primal and dual variables and the barrier parameter.
2. Evaluate the norm-∞ of the gradient of the augmented Lagrangean function.
3. Convergence test:
   - if the norm-∞ of the gradient of the augmented Lagrangean function and the barrier parameter are smaller than the pre-specified tolerances, STOP.
   - otherwise, go to step 4.
4. Calculate the right side of the linear system of Newton's method (negative of the gradient of the augmented Lagrangean function).
5. Calculate the Hessian matrix of the augmented Lagrangean function.
6. Solve the linear system of Newton's method.
7. Evaluate the step lengths in both primal and dual spaces.
8. Update the optimization variables, the slack variables and the Lagrange multipliers.
9. Evaluate the new estimate for the barrier parameter.
10. Go to step 2.

Figure 2: Solution algorithm for MLSSD problem using Primal Dual Interior Point method.

Small modifications in the Primal Dual algorithm are used to obtain the Predictor Corrector Primal Dual Interior Point version [7][9]. This approach has shown to be more efficient on solving the optimization problems using Interior Point methodology.

3 THE TAP CONTROL HEURISTIC STRATEGY

If the voltage magnitude and the tap of the LTC transformers are simultaneously used as optimization variables, the iterative process is prone to convergence problems. This can be attributed to the strong coupling between these variables. Thus, the larger the number of LTC transformers the more difficult the convergence of the iterative process. As a consequence, during subsequent iterations the norm of the gradient of the Lagrangean function can increase significantly.

However, as shown in [10], transformer taps have a second order effect on the power injections obtained in load shedding and maximum loadability studies. These variables are used basically to control the voltage magnitude in order to satisfy the operational constraints. For this reason, close to the convergence of the iterative process the components of the gradient of the Lagrangean function related to the optimization variables, equation (7), become very small and the taps of the LTC transformers practically do not change until the end of the iterative process.

Based on these considerations, the following heuristic can be established. If the norm-∞ of the gradient of the Lagrangean function related to the optimization variables, that is,

$$\left\|\nabla_{x}\pounds\right\|_\infty = \left\|\left[0\ 0\ \ldots\ 1\right]^T - \left[\mathbf{G}(\boldsymbol{x})\right]^T \boldsymbol{\lambda} - \left[\mathbf{H}(\boldsymbol{x})\right]^T \left(\boldsymbol{\pi}_\ell + \boldsymbol{\pi}_u\right)\right\|_\infty \quad (13)$$

is smaller than the pre-specified tolerance, the taps of LTC transformers can be excluded from the set of the optimization variables $\boldsymbol{x}$. In other word, from this point until the convergence of the iterative process the optimization variable set is reduced to

$$\boldsymbol{x} = \left[\boldsymbol{V} \quad \boldsymbol{\delta} \quad \boldsymbol{a}\right]^T \quad (14)$$

This modification in the algorithm contributes significantly to the convergence of the iterative process because:

- the coupling between the remaining optimization variables is not so strong;
- the reduction in the set of optimization variables results in a decrease in the dimension of the linear system solved at each iteration of the Newton's method.

Considerable improvements in the iterative process are achieved with these changes. It can be easily noted that the proposed approach is simply an extension of the conventional Interior Point method, the addition consisting in the test involving the partial derivatives of the Lagrangean function with respect to the optimization variables and the eventual modification of the optimization variables set. Figure 3 summarizes the proposed algorithm.

1. Initialize primal and dual variables and the barrier parameter.
2. Set variable *flag* = *True*.
3. Evaluate the norm-∞ of the gradient of the augmented Lagrangean function.
4. Convergence test:
   - if the norm-∞ of the gradient of the augmented Lagrangean function and the barrier parameter are smaller than the pre-specified tolerances, STOP.
   - otherwise, go to step 5.
5. If the variable *flag* is *True*:
   - convergence test in the vector $\nabla_x £$:
     - if the norm-∞ of $\nabla_x £$ is smaller than the pre-specified tolerance, exclude variables $\boldsymbol{a}$ from the set of optimization variables. Set *flag* = *False* and go to step 6
     - otherwise, go to step 6.
6. Calculate the right side of the linear system of Newton's method.
7. Calculate the Hessian matrix of the augmented Lagrangean function.
8. Solve the linear system of Newton's method.
9. Evaluate the step lengths in both primal and dual spaces.
10. Update the optimization variables, the slack variables and the Lagrange multipliers.
11. Evaluate the new estimate for the barrier parameter.
12. Go to step 3.

Figure 3: Solution algorithm for MLSSD problem using Primal Dual Interior Point method with the proposed scheme to deal with the taps.

Comparing Figures 2 and 3, we perceive that both conventional and proposed algorithms are slightly different. Carrying out this modification in the conventional approach is relatively simple.

Next section will present the numerical results obtained from a wide range of test systems.

4 NUMERICAL RESULTS

Aiming at assessing the performance of the proposed methodology, tests with several systems were performed. A pre-specified tolerance of $1.0{\times}10^{-3}$ was used to verify the convergence of the iterative process. The demand is modeled as constant power injection and load decrease is determined under the assumption of constant power factor.

Two main aspects were observed:

- the influence of the taps of LTC transformers on the amount of load shedding obtained through the use of optimization techniques;
- the reduction in the number of iterations for convergence achieved with the proposed heuristic strategy.

All tests were performed in a 1 GHZ Pentium III computer with 256 Mb of RAM memory.

The main characteristics of the test systems are presented in Table 1. The data for IEEE test systems can be found in [11][12]. The power network with 1,916 buses is a Brazilian realistic equivalent system. In this table, *NB* is the number of buses, *NC* is the number of circuits, *NG* is the number of generators, and *LTC* is the number of LTC transformers.

System	*NB*	*NC*	*NG*	*LTC*
IEEE	6	7	2	2
	30	41	6	4
	57	80	7	15
	118	179	34	9
BR	1,916	2,788	153	401

Table 1: Main characteristics of test systems.

Table 2 shows the load level considered in the tests. Note that there is no operational solution for these load levels and therefore a load curtailment is need.

System	Total Real Demand (MW)
IEEE-6	202.50
IEEE-30	510.12
IEEE-57	1,876.20
IEEE-118	7,012.50
BR-1916	38,191.75

Table 2: Loading levels of test systems.

Aiming at assessing the load shedding quantitatively, it was used the index *lsi*, which is defined as

$$lsi = \frac{P_d^{tot} - P_{d_{\sup}}^{tot}}{P_d^{tot}} \cdot 100\% \qquad (15)$$

where P_d^{tot} is the scheduled total real power demand;
$P_{d_{\sup}}^{tot}$ is the total real power demand effectively supplied by the system.

4.1 Minimum Load Shedding Using the Taps of LTC Transformers

Two tests were performed:

- Test A: the taps of LTC transformers were not included in the optimization variable set;
- Test B: the taps of LTC transformers were included in the set of optimization variables.

Table 3 presents the results obtained in tests A and B. In this table, *Iterations* refers to the number of iterations required to the convergence of the iterative process and *lsi* is the minimum value in percentage for the load shedding determined by the application of the proposed approach.

	Iterations		*lsi* (%)	
System	Test A	Test B	Test A	Test B
IEEE-6	5	5	27.66	12.63
IEEE-30	5	7	22.58	5.65
IEEE-57	9	9	13.56	8.45
IEEE-118	14	10	8.78	6.87
BR-1916	14	47	2.21	2.03

Table 3: Results for tests A and B.

From Table 3, it is observed that:

- a suitable tap adjustment reduces the load shedding necessary for determining operational solutions for the electric power system;
- the inclusion of the transformer taps in the set of optimization variables increases the number of iterations of the iterative process, mainly in the case of the realistic power system due to its large number of LTC transformers.

4.2 Applying the Proposed Heuristic Strategy

This section shows the results of the application of the heuristic strategy to the conventional optimization algorithm using the control of the LTC transformers taps.

Tables 4, 5 e 6 present the results obtained for the IEEE-6, IEEE-30 and IEEE-57 bus systems, respectively. In these tables, the first column (*Iter*) is the number of the iteration, the second one (*NLG*) is the norm-∞ of the gradient of the augmented Lagrangean function with respect to all (primal and dual) variables and the third one (*NLFOV*) is the norm-∞ of the vector of the partial derivatives of the Lagrangean function with respect to the optimization variables only.

Iter	*NLG*	*NLFOV*
1	1.5596×10^{1}	1.1308×10^{1}
2	2.6467×10^{1}	2.6467×10^{1}
3	2.3300×10^{0}	2.3300×10^{0}
4	1.0719×10^{-1}	1.0719×10^{-1}
5	1.3776×10^{-4}	1.3776×10^{-4}

Table 4: Convergence of conventional iterative process – IEEE-6 bus system.

Iter	*NLG*	*NLFOV*
4	4.5295×10^{-1}	4.5295×10^{-1}
5	8.3845×10^{-2}	8.3845×10^{-2}
6	6.1560×10^{-3}	6.1560×10^{-3}
7	1.3716×10^{-5}	1.3716×10^{-5}

Table 5: Convergence of conventional iterative process – IEEE-30 bus system.

Iter	*NLG*	*NLFOV*
6	1.4242×10^{-1}	1.4242×10^{-1}
7	1.6918×10^{-1}	1.6918×10^{-1}
8	1.6825×10^{-2}	1.6825×10^{-2}
9	3.9574×10^{-5}	3.9574×10^{-5}

Table 6: Convergence of conventional iterative process – IEEE-57 bus system.

From these tables, we notice that these norms are similar, the components of latter, corresponding to equation (7), being predominant in terms of the numerical value of the former. Thus, the addition of the proposed strategy to the algorithm does not deteriorate the convergence path.

The load shedding index corresponding to the optimal solution resumed in Table 4 (IEEE-6) is $lsi = 12.63\%$. Similar results are shown in Tables 5 (IEEE-30) and 6 (IEEE-57), with load shedding indexes of $lsi = 5.65\%$ and $lsi = 8.45\%$, respectively, as shown in Table 3.

Table 7 presents the convergence features of the IEEE-118 bus system. In this table, column 4 ($NLF\lambda$) refers to the norm-∞ of the vector of the partial derivatives of the Lagrangean function with respect to the Lagrange multipliers ($\nabla_\lambda £$), equation (10). This set of partial derivatives corresponds to the real and reactive power mismatches at each load bus.

Iter	*NLG*	*NLFOV*	$NLF\lambda$
5	1.6696×10^{-1}	1.6696×10^{-1}	1.8488×10^{-2}
6	1.1508×10^{-1}	1.1508×10^{-1}	1.5402×10^{-2}
7	1.0938×10^{-1}	1.0938×10^{-1}	6.1064×10^{-2}
8	6.6791×10^{-1}	3.9618×10^{-2}	6.6791×10^{-1}
9	1.0558×10^{-2}	6.5671×10^{-4}	1.0558×10^{-2}
10	7.5305×10^{-4}	3.8922×10^{-5}	7.5305×10^{-4}

Table 7: Convergence of conventional iterative process – IEEE-118 bus system.

In Table 7, it is observed that the convergence of the iterative process is strongly dependent on the power mismatches, equation (10). At iteration 9, the norm-∞ of the vector of the partial derivatives of the Lagrangean function with respect to the optimization variables (*NLFOV*) has already satisfied the pre-specified tolerance (see column 3). Nevertheless, an additional iteration is necessary until the norm-∞ of the vector of power mismatches $\nabla_\lambda £$ satisfies the tolerance (see column 4). In this case, the index *lsi* is 6.87%, as presented in Table 3.

Table 8 summarizes the convergence of the iterative process for the IEEE-118 bus system resulting from the application of the proposed technique. From iteration 9 on, the taps of LTC transformers are excluded from the optimization variables set.

Iter	*NLG*	*NLFOV*	$NLF\lambda$
5	1.6696×10^{-1}	1.6696×10^{-1}	1.8488×10^{-2}
6	1.1508×10^{-1}	1.1508×10^{-1}	1.5402×10^{-2}
7	1.0938×10^{-1}	1.0938×10^{-1}	6.1064×10^{-2}
8	6.6791×10^{-1}	3.9618×10^{-2}	6.6791×10^{-1}
9	1.0558×10^{-2}	6.5671×10^{-4}	1.0558×10^{-2}
10	4.9064×10^{-4}	5.3302×10^{-5}	1.0965×10^{-4}

Table 8: Convergence of iterative process – IEEE-118 bus system with the proposed methodology.

The introduction of the proposed scheme in the conventional algorithm does not increase the number of iterations required to the convergence. In this case, the load shedding index *lsi* is 6.87%. The main difference between the conventional and the proposed approaches is concerned with the transformer taps. Table 9 presents the tap values of the LTC transformers. In this table, the first column corresponds to the circuit in which the LTC transformer is inserted.

LTC	Conventional Approach	Proposed Approach
5–8	1.0596	1.0655
25–26	0.9181	0.9150
17–30	0.9902	0.9864
37–38	1.0724	1.0671
59–63	1.0132	1.0097
61–64	1.0425	1.0398
65–66	0.9092	0.9088
68–69	1.0945	1.0930
80–81	0.9995	1.0016

Table 9: Final tap values of LTC transformers in IEEE-118 bus system.

Figure 3 shows the voltage profiles for the IEEE-118 bus system in the tests previously mentioned, that is, applying the conventional (o) and the proposed (×) methodologies.

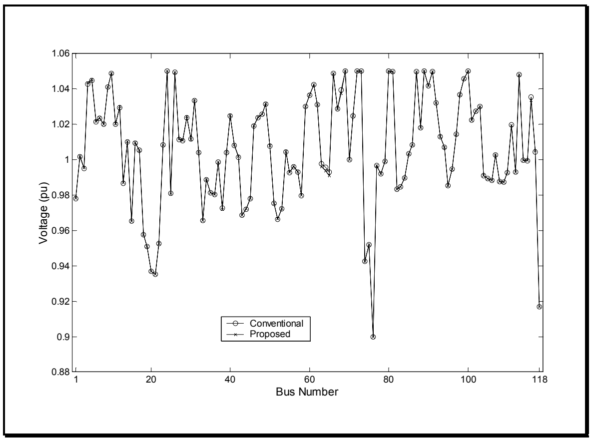

Figure 4: Voltage profiles in IEEE-118 bus system.

Except, for some small differences, similar voltage profile are observed in Figure 3. For instance, the voltage magnitude at bus 63 is 0.9979 pu (conventional approach) and 1.0012 pu (proposed approach). These small variations are due to the changes in the tap values of LTC transformers determined as a solution of the optimization process (Table 9). Nevertheless, the index *lsi* (6.87%) is the same in both tests. From Table 9 and Figure 3, we see that the tap changes and small corrections in the voltage magnitude do not affect considerably the index *lsi*.

Table 10 shows the numerical results obtained for BR-1916 bus system. In this case, $NLF\pi$ refers to the norm-∞ of the partial derivatives of the Lagrangean function with respect to the Lagrange multipliers associated to the inequality constraints (vectors ∇_{π_ℓ} £ and ∇_{π_u} £), equations (11) and (12).

Iter	*NLG*	*NLFOV*	$NLF\pi$
27	5.7636×10^{-2}	5.4490×10^{-3}	5.7636×10^{-2}
28	3.6721×10^{-1}	3.2625×10^{-4}	3.6721×10^{-1}
29	1.9423×10^{-1}	6.8887×10^{-4}	1.9423×10^{-1}
⋮	⋮	⋮	⋮
43	1.0828×10^{0}	1.0302×10^{-2}	1.0828×10^{0}
44	1.8878×10^{-1}	2.2688×10^{-4}	1.8878×10^{-1}
45	3.9073×10^{-3}	5.7216×10^{-5}	3.9073×10^{-3}
46	2.7190×10^{-3}	2.6897×10^{-6}	2.7190×10^{-3}
47	5.5196×10^{-4}	1.9499×10^{-7}	5.5196×10^{-4}

Table 10: Convergence of conventional iterative process – BR-1916 bus system.

From Table 10, we observe that the convergence of the iterative process is dependent from the components of the vectors ∇_{π_ℓ} £ and ∇_{π_u} £ . At the iteration 28, the norm-∞ of the gradient vector of the Lagrangean function with respect to the optimization variables has already satisfied the pre-specified tolerance. However, additional iterations are required to obtain the convergence. The index *lsi* in this case is 2.03%.

Table 11 shows the main features of the convergence process for the BR-1916 bus system, resulting from the application of the proposed heuristic. In this case, the main benefit of the application of the proposed approach was to prevent the iterative process form zigzagging near the optimal solution. The taps of LTC transformers are removed from the optimization variables set at iteration 28.

Iter	*NLG*	*NLFOV*	$NLF\pi$
27	5.7636×10^{-2}	5.4490×10^{-3}	5.7636×10^{-2}
28	3.6721×10^{-1}	3.2625×10^{-4}	3.6721×10^{-1}
29	4.1790×10^{-3}	3.4917×10^{-3}	4.1790×10^{-3}
30	1.4551×10^{-3}	2.7647×10^{-5}	1.4551×10^{-3}
31	5.4126×10^{-4}	1.3111×10^{-5}	5.4126×10^{-4}

Table 11: Convergence of iterative process – BR-1916 bus system with the proposed methodology.

In Table 11, we see that the number of iterations to the convergence decreases considerably (from 47 to 31 iterations) if the heuristic strategy is used. Load shedding index obtained in this case is $lsi = 2.03\%$, showing that although the number of iterations has decreased considerably the quality of the result has not been deteriorated.

On the other hand, it is important to point out that the number of LTC transformers in this system is relatively high (401 transformers). Incorporating the heuristic methodology into the conventional algorithm, the resultant algorithm has shown quite efficient in dealing with taps of LTC transformers in large dimension optimization problems.

5 CONCLUSIONS

The use of transformers taps as optimization variables decreases the load shedding necessary for determining operational solutions. The voltage profile becomes flatter and the power losses in the transmission network also decrease.

The inclusion of a heuristic approach in the conventional algorithm has been shown to be an efficient way of dealing with taps of the LTC transformers in the iterative process. If the power system has a large number of LTC transformers the proposed heuristic is very effective in the convergence process, decreasing considerably the number of iterations. In cases, where the heuristic is not necessary, the inclusion of this strategy does not influence on the convergence of the iterative process.

The test performed with a wide range of power systems show that the use of LTC transformers can be a suitable method to reduce additionally the load shedding in electric power networks.

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