DYNAMIC PROGRAMMING FOR OPTIMAL SEQUENCING OF OPERATIONS IN DISTRIBUTION NETWORKS

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Abstract – Distribution networks are frequently reconfigured to fix node voltage and branch current profiles when damaged by maintenance or fault isolation decisions. Reconfiguration is usually taken as a sequence of ON/OFF switching operations. These operations need considerable analysis effort to be found and to be sequenced in a securely and optimal manner: they must be dynamically investigated in order to minimize load interruption costs and not to violate power-flow constraints during reconfiguration. The paper formulates the reconfiguration problem in a way that tackles its practical complexities and can be solved with Dynamic Programming. The reconfiguration approach is illustrated for a simple restoration example. The example highlights the effects on the optimal solution found caused by different switching operation times and different client priorities.

Keywords: Distribution networks, operations planning, optimization, dynamic programming, quality of service

1 INTRODUCTION

THE problem of sequencing operations in distribution system is an important operations planning problem. The problem is addressed (1) online in control centers for real-time fault-restoration, and (2) offline in planning centers for settling the operations for periodic maintenance and programming the operations for selected network contingencies. For a given fault or contingency, the problem consists in *selecting* and *sequencing* a set of switching operation to restore power in a secure and prompt manner [1]-[3].

Selecting the switching operations consists in selecting a set of switches to be opened and a set of switches to be closed. That corresponds to finding an improved configuration. Many authors have been concerned with improving network configuration to minimize power losses, maximize reliability, etc. [4]-[7]. Our concern is different. Our concern is not the one of finding an improved configuration but the problem of sequencing such operations in order to minimize load interruption costs without violating power-flow constraints during reconfiguration.

In order not to interrupt loads while changing configurations, operators usually agree upon sequencing basic operations like the following: (*i*) close a switch to create a mesh and then (*ii*) open a different mesh-switch. We call that operation a *switching step*[1] (See Fig. 1) and write it as (e,d) where e is the switch to be closed and d the switch to be opened.

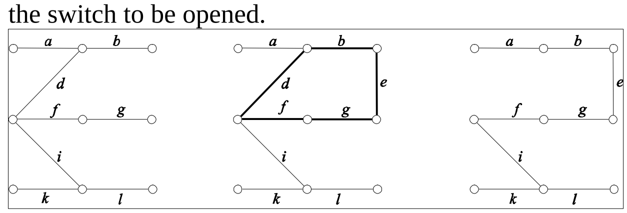

Figure 1: Example of a switching step (e,d) that changes the left most configuration into the right most one. The step consists on switching ON branch e and switching OFF branch d.

Hence, the task of reaching a particular configuration can be seen as the task of determining and sequencing switching steps. However, this task can be very complex to undertake. Suppose that there are n switches to be closed and n switches to be opened. It is easy to understand that there are $n!$ possible sets of different pairs of switches (switching step candidates) and that each of these sets can be sequenced by $n!$ different orders. That makes a space of $(n!)^2$ possible operations where just a few guarantee network connectivity along the switching process.

In this paper we formulate the sequencing problem and propose to solve it with Dynamic Programming (DP). The DP formulation is then changed to address explicitly the existence of priorities for clients' restoration. The overall solution process is illustrated for a large contingency in a real distribution network.

2 SOLUTION APPROACH

2.1 Problem Formulation

The optimal sequencing problem can be stated as the problem of defining a set S of switching steps and their sequence o, so that the pre-fault network configuration, say T_i, is changed into a post-fault configuration, say T_f, in a way that constraints are satisfied and Energy Not Supplied (ENS) is minimized.

Depending on the set and its sequence of steps, the pair (S,o), different intermediate configurations are obtained. These network configurations must satisfy the problem constraints so that the final solution of the problem is both topologically and electrically admissi-

[1] In the loop optimization heuristics context, the operation is also called a *branch exchange* operation [7].

ble. Whereas the topological constraints are rather easy to satisfy, operational constraints are not. It is not always possible to find a solution that fully respects power flow constraints. Therefore, operational constraints must be relaxed and a violation penalty function be defined to be added to the objective. Once defined the objective function and problem constraints, one may formulate the problem as in the following:

$$\begin{array}{lll} (P) & \min & g(S,o) = c(S,o) + e(S,o) \\ & s.t. & (S,o) \in F \end{array}$$

where c is the cost function associated with the ENS cost and e is the penalty function associated with the violation of the operational constraints. The set F is the set of all feasible sequences, i.e., the sequences that generate intermediate network configurations that are radial and connected. The overall cost of the network configuration results from the sum of c and e for all the intermediate configurations yielded during the process.

As this is a sequential multi-stage problem, DP can be use to solve it [8]. Some effort is yet necessary to build the feasible state space and accommodate several problem practicalities that emerge from industrial operation: (*i*) some mesh operations are prohibited, e.g., meshes between injections should not be closed; (*ii*) operations are different in what concerns to the time required to make them (some are remotely controlled); and (*iii*) clients are very much different in what concerns to their sensitivity to interruption frequency and duration (typically, there are priorities for load shedding, e.g., hospitals, emergency centers, etc).

2.2 Dynamic Programming Solution

The problem of finding the pair (S,o) is the one of changing a pre-fault configuration into a post-fault configuration. These configurations define the initial and final states of the problem. Once defined the initial and final states, a DP classic formulation that guarantees topology constraint satisfaction can be given by (D):

$$\begin{array}{lll} (D) & \min & J_{(S,o)}(T_0, T_N) = \sum_{n=0}^{N-1} g_n(T_n, u_n) \\ & s.t. & T_{n+1} = f_n(T_n, u_n) \\ & & u_n \in M_n \\ & & T_0 \equiv T_i,\, T_N \equiv T_f \\ & & n = 0, \ldots, N-1 \end{array}$$

where N is the number of steps necessary to reach the final state. Each state that can be reached is represented by a spanning tree T_n. The possible switching steps for each state T_n are given by the set M_n of feasible steps — feasible in the sense that the steps produce a state transition f to a different spanning tree. The set of all feasible states is called the State Space (SS).

2.3 Feasible State Space

The feasible transitions from a state are its possible cycle reconfiguration steps. The set of cycle reconfiguration steps is the set of switching operations that may be undertaken without customer interruption (see Fig. 1). The feasibility of a step (e,d) can be inspected by verifying if the mesh closed by branch e includes branch d. Feasible steps are just a few when compared to the n! different combinations of 2n spanning-tree differences (between the actual configuration and the final one).

2.4 Priority Clients

Practicalities that emerge from industrial operation are easy to accommodate by the DP formulation.

i) The mesh operations that are prohibited can be represented by prohibited state transitions or by transitions that involve an additional interruption as one has to open the switch before closing the other.

ii) Different operation times lead to different ENS costs for the same outage power.

iii) Different sensitivity to interruption frequency and duration could be represented by different ENS unitary costs.

However, different ENS unitary costs cannot guarantee Priority Clients (PCs) to be restored first unless a very high ENS unitary cost is defined. Because power flow constraints are relaxed, such a high cost would tradeoff with branch ampacity and node voltage violations. Thus, an explicit priority approach is necessary.

An explicit priority DP formulation of the sequencing problem relies upon the decomposition of the original problem (D) into two sequential problems: (D.1) the optimal sequencing of all PCs and (D.2) the optimal sequencing of the ordinary clients. The first problem produces the initial configuration for the second problem. We can formulate these two problems as in the following:

$$\begin{array}{lll} (D.1) & \min & J_{(S_1,o_1)}(T_0) = \sum_{n=0}^{N'-1} p_n(T_n, u_n) \\ & s.t. & T_{n+1} = f_n(T_n, u_n) \\ & & u_n \in M_n \\ & & T_0 \equiv T_i \\ & & n = 0, \ldots, N'-1 \end{array}$$

$$\begin{array}{lll} (D.2) & \min & J_{(S_2,o_2)}(T_{PC}, T_N) = \sum_{n=0}^{N-N'-1} g_n(T_n, u_n) \\ & s.t. & T_{n+1} = f_n(T_n, u_n) \\ & & u_n \in M_n \\ & & T_0 \equiv T_{PC} \\ & & n = 0, \ldots, N - N'-1 \end{array}$$

where, the objective function p depends solely on the number of outage PCs and penalty function associated with the violation of the operational constraints; N' is the horizon of (D.1); and T_{PC} is its final state. N' is not known when one starts solving (D.1). One has to solve (D.1) until each and every valid state transition yields null transition cost p. When that happens, we find T_{PC} as the state of minimum J.

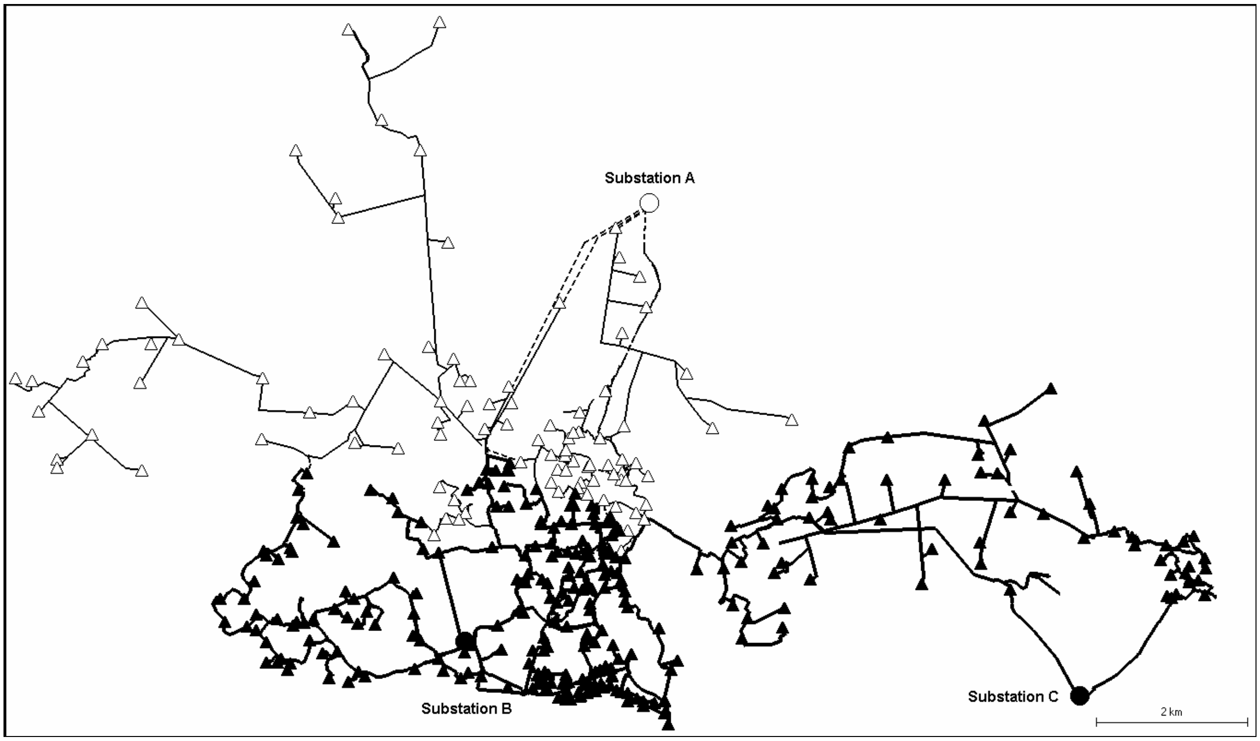

Figure 2: 15kV overhead and underground distribution network geography. The distribution transformers are identified with triangles: blank triangles identify transformers under substation outage.

After restoring all PCs, we restore ordinary clients. This problem is similar to the original one. Both initial and final states are known. The difference is that fewer steps are necessary to reach the final network configuration. The final solution is given by both partial solutions of (D.1) and (D.2), $(S,o)=(S_1,o_1)\cup(S_2,o_2)$.

3 APPLICATION RESULTS

To illustrate the application of DP to the optimal sequencing problem, we use the real 15kV distribution network shown in Fig. 2. In the figure, the triangles represent loads and circles represent substations. Suppose that an outage occurred at Substation A and the clients supplied by its feeders are to be restored from the nearby feeders of the other two substations (B and C). The branches that are to be closed and the branches that are to be opened to achieve the post-fault configuration have different switching operation tomes. The branch operation times are given in Table 1.

Substation A has four feeders, so, a substation outage would lead to at least four switching operations (one operation for each outage feeder). Network optimization [9]-[10] yields a configuration T_f that requires six switching steps to be achieved from T_i: these are the minimum number of steps necessary to change the initial configuration into an electrically admissible one. The four operations are unable to restore all clients without branch ampacity violations.

Results are shown to compare the optimal solutions found when CP are not considered with the optimum solution found when CP are considered. Fig. 3 shows a partial view of the SS that includes optimal solutions. Optimal solutions are depicted as trajectories in the SS. The bold solid line shows

- the optimum trajectory for problem (D.1)

The dashed lines show

- the optimum trajectory for problem (D.2) and
- two optimal trajectories for problem (D) when solved without considering client priorities.

Branch	Operation time [min]
a	2
b	32
c	2
d	5
e	20
f	25
g	2
h	2

Table 1: Operation time of each branch.

3.1 Neglecting Priorities: Solving (D)

The optimum solution of (D) is not the one of sequencing operations by an order of merit that reflects the amount of restored power. Operations require different time to be undertaken and may need to be preceded by other switching operations in order not to violate power flow constraints.

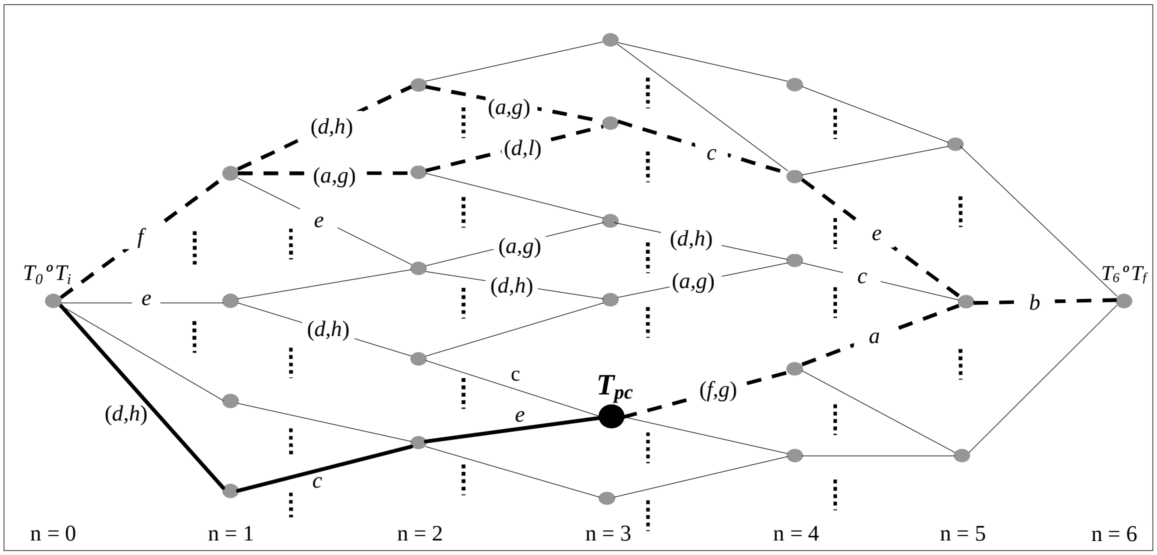

Figure 3: Partial representaion of the problem's state space.

For this particular outage and for the operation times shown in Table 1, there are two different optimal sequences as given bellow:

$$(S_{a1},o_{a1})=\{\ f\rightarrow(d,h)\rightarrow(a,g)\rightarrow c\rightarrow e\rightarrow b\ \}$$
$$(S_{a2},o_{a2})=\{\ f\rightarrow(a,g)\rightarrow(d,h)\rightarrow c\rightarrow e\rightarrow b\ \}$$

where the symbol "→" means "followed by".

The existence of two optimal solutions results from the fact that steps (*d*,*h*) and (*a*,*g*) are preparation steps for step *c*. They do not restore power to isolated clients. They only transfer load from one feeder to another. These two possible solutions are represented in Fig. 3 in dashed line. The evolution of the outage power along the sequencing process is represented in Fig. 4.

Note that the first steps are not necessarily those which restore more power. The steps (*d*,*h*), (*a*,*g*) and *c* are taken prior to *e* despite restoring less power than *e*. The sum of the operation times of the three first steps is shorter than the operation time of *e*, which results in a smaller value of ENS.

3.2 Considering Priorities: Solving (D.1) and (D.2)

To illustrate the sequencing if considering priorities we classified two clients, say C_1 and C_2, as PCs and solve (*D*.1) and (*D*.2). The objective is now to first minimize the restoration time of C_1 and C_2 and then minimize ENS. The optimum solution when considering priorities is shown in the following:

$$(S_b,o_b)=\{\,(d,h)\rightarrow c\rightarrow e\ \}\rightarrow\{\,(\,f,g)\rightarrow a\rightarrow b\ \}$$

where

$\{\,(d,h)\rightarrow c\rightarrow e\ \}$ is the optimum solution of (*D*.1), and

$\{\,(\,f,g)\rightarrow a\rightarrow b\}$ is the optimum solution of (*D*.2).

The evolution of the outage power along the sequencing process when PCs are considered is represented in Fig. 5.

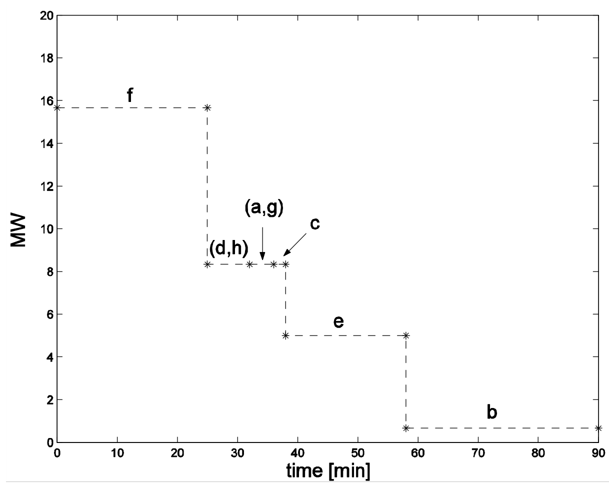

Figure 4: Evolution of the outage power for the optimum solution of (*D*) . PCs are neglected.

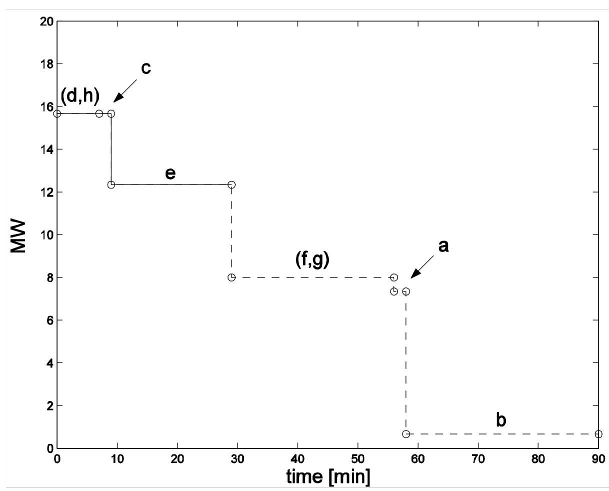

Figure 5: Evolution of the outage power for the optimum solution of (*D*.1) — solid line —, followed by the evolution of the outage power for the optimum solution of (*D*.2) — dashed line. PCs are considered.

When considering PCs, the first two steps (*d*,*h*) and *c* restore client C_1 (the step (*d*,*h*) is a preparation step in order to avoid an ampacity violation), and step *e* restores client C_2. Client C_1 is restored after 9 minutes and client C_2 is restored after 29 minutes (state T_{PC} of Fig. 3 is achieved). Note that, when PCs are neglected, client C_1 is restored after 38 minutes and client C_2 is restored after 58 minutes. By considering priorities we reduced outage time of PCs in 50%.

Once both PCs are restored, the ENS of the remaining outage clients is to be minimized. The optimal sequence involves making the steps (*f*,*g*), *a* and *b*, where the step (*f*,*g*) is a preparation step to avoid ampacity violations. The sequence to minimize ENS is shown in Fig. 3 as a dashed path from T_{PC} to T_f.

4 CONCLUSION

The sequencing of switching operations in distributions networks is an important problem as networks are frequently reconfigured to fix node voltage and branch current profiles when damaged by maintenance or post-fault isolation decisions. The paper formulates the optimal sequencing problem in a way that operation practicalities can be addressed and the problem be solved with Dynamic Programming.

To address client restoration priorities, the problem is decomposed into two sequential optimization problems. The proposed solution approach is illustrated for a real distribution network. Results are presented to illustrate the consequences of setting priorities on optimal switching sequence.

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