HANDLING A CO_2 RESERVOIR IN MID TERM GENERATION SCHEDULING

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Abstract – This paper deals with the assessment of the impact of CO_2 emission limits on Hydro Thermal System Management. The whole thermal units emissions is modelled as a unique annual reservoir: the allowances stock. A model describes the integration of this new financial and technical asset into the global system. Some numerical experiments are given. They show how generation strategies can be modified by this new constraint. Several simulations are illustrated: impact on hydro management reservoir, marginal costs behaviour, system sensibility with respect to quota levels.

Keywords: gas emission allowances, generation scheduling, reservoir management, bundle method.

1 INTRODUCTION

The European community has established a scheme for greenhouse gas emission allowance trading [1].

The scheme requires that companies from certain sectors must limit their CO_2 emissions to allocated levels. We are interested here in their influences on energy activities. Emissions trading gives power producers the flexibility to meet annual reduction targets according to their own strategy. A generation manager might quantify and measure carbon related risks and opportunities:

- Power costs and prices;
- Long and short term unit commitment;
- Reservoir management;
- Thermal unit penalisation;
- Risk management;
- Delivered energy & investments strategies;
- …

These allowances can be represented as a unique annual reservoir that includes the whole contributing thermal units. This stock can be integrated into the global hydro-thermal generation mix. Hence, the whole system can be optimised by an adapted mid-term generation tool.

Such a model has been developed at "Electricité De France" (EDF). SOPRANO is a stochastic optimiser that:

- Handles physical and financial risks;
- Deals with random events as consumption, prices, water inflows, run of rivers and unit availabilities;
- Computes generation strategies and Bellman values.

This tool uses a scenario tree to deal with uncertainties. A Lagrangian relaxation technique and a price decomposition/coordination algorithm are applied. The demand and risk constraints are dualized. A bundle method is used to solve the master problem. Each sub-problem is linked to a generation unit and solved by a suitable method.

To take into account the CO_2 allowances, some modifications have been introduced in different stages of the methodology above. A new sub-problem is added. It represents the associated CO_2 reservoir.

In this paper, theoretical and practical aspects will be presented. The climate policy will be translated and implemented into a complete mid term scheduling model. On the one hand, the used decomposition / coordination method will be described. On the other hand, main concerns of a generation manager will be pointed out through numerical experiments. We will focus on assessing the changes that are yielded by the new law, and on the corresponding strategies facing it. The analysis will be based on a comparison with a previous situation (without carbon dioxide limit).

2 EMISSION ALLOWANCE MECHANISM

The European directive 2003/87/EC has established rules according to 2 periods [1]: a 3 years period (2005-2007) and a 5 years one (2008-2012). Allowance quotas are given each year and banking is allowed inside a period from a year to another. However, the allowance surplus is lost between 2 periods. For each tone of CO_2 emission excess, a penalty fee of 40€ and 100€ has to be paid respectively for the first and the second period. For the first period the expected schedule is as follows:

1. March the 31st 2004: National Allocation Plans are published.
2. January the 1st 2005: Launch of the emission market.
3. February the 28th 2005: allowances for 2005 are allocated.
4. December the 31st 2005: measurement of the emitted quantity for the preceding year.
5. February the 28th 2005: allowances for 2006 are allocated.

6. April the 30th 2006: each installation surrenders a number of allowances equal to the total emissions during the preceding year (compliance period).
7. And so on.

3 MODELING HYDRO THERMAL MID TERM SCHEDULING

3.1 Random trees

Several parameters in generation management are random factors: demand, natural inflows from rain and snow falls, thermal unavailability and price volatility. In order to structure theses uncertainties, we use a scenario tree [2]. Non-anticipativity constraints are implicitly represented in a tree structure: two scenarios which are undistinguishable before time t correspond to a single branch in the tree. The command at time t is then the same for both scenarios. To describe that tree, one would use the following notations:

- O is the set of nodes and n_O is the number of nodes $(n_O = cardO)$;
- $\tau(n)$ is the time step related to node n;
- π_n is the probability of being at node n, knowing that the time instant is $\tau(n)$:

$$\forall t, \sum_{\{n|\tau(n)=t\}} \pi_n = 1 \tag{1}$$

- T is the last time step and O_T is the set of nodes of this time step: $O_T = \{n \in O | \tau(n) = T\}$;
- $\rho(n)$ is the « father » of n.

3.2 Generation management problem

The global problem of optimising generation schedules consists in minimizing generation costs over the random tree, while taking into account:

- the dynamic operating constraints of each generation unit (reservoir levels, start-up ...);
- the demand constraint at each node of the scenario tree.

This problem may be written as follows:

$$\left.\begin{array}{l} \min\limits_u \sum\limits_{l\in L}\left(\sum\limits_{n\in O}\pi_n C_{l,n}\left(u_{l,n},x_{l,n}\right)\right) \\ \forall n \in O, \sum\limits_{l\in L} g_{l,n}\left(u_{l,n}\right) = d_n \\ \forall l \in L, \left(u_l, x_l\right) \in D_l \end{array}\right\} \tag{2}$$

where:

- L is the set of indices l of the generation units;
- $u_{l,n}$ and $x_{l,n}$ are the control and state variables of unit l at node n; $u_l = (u_{l,n})_{n\in O}$ and $x_l = (x_{l,n})_{n\in O}$;
- $C_{l,n}$ is the generation cost of unit l at node n;
- d_n is the demand corresponding to node n;
- $g_{l,n}$ is the generation of unit l at node n;
- D_l is the operating domain of unit l.

4 SOLVING THE PROBLEM

4.1 A decomposition approach

Because of the three dimension characteristics of the problem (multi reservoir, stochastic, dynamic), one needs some approximations. For instance a SDDP (Stochastic Dual Dynamic Programming) method can be used [3], [4]. In our approach, we apply a decomposition technique in order to separate the global problem into many small size sub-problems. We use a spatial decomposition method. It consists in solving one sub-problem related to each generation unit and one problem related to the coordination system.

The Lagrangian of problem (2) is written by dualizing the demand constraints:

$$L(u,\lambda) = \sum_{n\in O}\pi_n\left[\sum_{l\in L} C_{l,n}\left(u_{l,n},x_{l,n}\right) + \lambda_n\left(d_n - \sum_{l\in L} g_{l,n}\left(u_{l,n},x_{l,n}\right)\right)\right]$$

Problem (1) can then be formulated as follows:

$$\max_\lambda \psi(\lambda) \tag{3}$$

where $\psi(\lambda) = \min\limits_u L(u,\lambda)$ is the dual function.

The following iterative algorithm is applied in order to solve problem (3) ; at iteration k+1:

1. for each generation unit l, compute command and state variables u_l^{k+1} and x_l^{k+1} by solving:

$$\min_u \sum_{n\in O}\pi_n\left[C_{l,n}\left(u_l,x_l\right) - \lambda_n^k g_{l,n}\left(u_l,x_l\right)\right] \tag{4}$$

2. compute updated prices λ^{k+1}

Fig. 1 is an illustration of this algorithm.

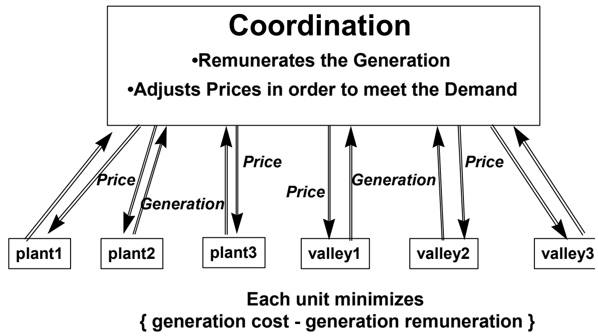

Figure 1: Spatial decomposition

4.2 Solving sub-problems

Sub-problems are generally solved by linear programming or by stochastic dynamic programming algorithm: Bellman values and optimal commands are computed at each node of the scenario tree for each feasible state.

4.3 Solving the master problem (bundle method)

It can be easily proved that:

- the dual function ψ is non-differentiable but concave,

- $s_n = \sum_{l \in 1} g_{l,n} - d_n$ is an element of $\partial\psi(\lambda_n)$, sub-differential of ψ at λ_n.

To maximize this type of function, we use a Bundle method [2], [5], [6], [7], which can be seen as a stabilized version of the classical cutting planes algorithm.

The cutting planes algorithm consists in replacing the maximization of ψ by the maximization of a polyhedral approximation $\breve{\psi}_k$ of ψ, which is refined at each iteration k of the algorithm :

$$\breve{\psi}_k(\lambda) = \min_{i=1,\ldots,k} \left(\psi(\lambda_k) + \langle s_k, \lambda - \lambda_k \rangle\right) \quad (5)$$

In some cases, the cutting planes algorithm may exhibit an oscillatory behaviour. This is the reason why the Bundle algorithm introduces a stabilizing centre, $\hat{\mu}$, which is the last iterate associated with a « sufficiently serious » increase in ψ. The « seriousness » of an increase in ψ is measured by comparing the increase obtained at each iteration, $\psi(\lambda_{k+1}) - \psi(\lambda_k)$, with a fraction of a nominal increase δ_k, which, for instance, can be the increase in the approximation model $\breve{\psi}_k$.

The idea is to prevent the instability of the cutting planes algorithm, by penalizing the distance to the stabilizing centre $\hat{\mu}$ and replacing the polyhedral approximation by the quadratic approximation:

$$\tilde{\psi}_k(\lambda) = \breve{\psi}_k(\lambda) - \frac{1}{2t}\|\hat{\mu} - \lambda\|^2 \quad (6)$$

The basic scheme of the Bundle algorithm can be described as follows:

1. Initialisation : Choose $\delta \geq 0, m \in]0,1[, \lambda_0$. Compute $\psi_0 = \psi(\lambda_0)$ and $s_0 \in \partial\psi(\lambda_0)$. $\delta_0 = +\infty$. $\hat{\mu} = \lambda_0$.
2. If $\delta_k \leq \delta$, then STOP.
3. Solve the quadratic problem: $\lambda_{k+1} \in \arg\max \tilde{\psi}_k(\lambda)$.
4. Compute $\psi_{k+1} = \psi(\lambda_{k+1})$ and $s_{k+1} \in \partial\psi(\lambda_{k+1})$ (by solving problem (4)).
5. Compute $\delta_k = \tilde{\psi}_k(\lambda_{k+1}) - \tilde{\psi}_k(\hat{\mu})$
6. Ascent test : if $\psi(\lambda_{k+1}) - \psi(\lambda_k) \geq m\,\delta_k$ then $\hat{\mu} = \lambda_{k+1}$.
7. Compute the approximation model $\tilde{\psi}_{k+1}$.
8. $k = k+1$; Return to 2.

5 INTEGRATING THE CO_2 RESERVOIR

5.1 The methodology

At the surrenders stage (compliance period, every April the 30th) a statement of emissions is established. If the deviation between the measured emissions and the allocated quantity is negative, a penalty is paid. Otherwise, the exceeded quantity is added to the next allocation.

The quantity allowances can be represented as a unique annual reservoir that includes the whole contributing thermal units. The state of the current emission stock, at each time step t is given by :

Stock$_{CO2}$ (t) = Allowance (period) + trading exchanges (t-1) – emitted gas (t-1),

where trading exchanges can be positive or negative depending on the transaction: bought or sold volume.
We will focus on the management of this stock by calculating its Bellman values. The management cost of this reservoir is afterwards added to the global cost. To deal with the CO_2 constraint, one could choose between two methods:

1. Dualizing the CO_2 constraint, using the same framework as for the demand, and solving one sub-problem per thermal unit,
2. Or building a unique sub-problem (called CO_2 sub-problem) that includes the whole thermal units emitting gas.

Because in our data the allowances are global (not per unit), we make here the second choice. It can be illustrated as one of the plants in the previous scheme (figure 1).

5.2 Solving the CO_2 sub-problem

After the coordination stage, a price is dispatched to every sub-problem. The CO_2 sub-problem would optimise its generation planning in order to maximize its revenue, accounting for its own constraints. The result is sent back to the coordinator until convergence. We solve:

$$\max_{u,S,q,\Omega,\Gamma} (R_G + R_S + R_V + R_F) \quad (7)$$

where:

$R_M = \sum_{n \in O} \pi_n \sum_{p \in P_n} \sum_{l \in T_e} \left(\lambda_p - C_l\right)\ u_{l,p} d_p$ is the generation revenue;

$R_S = \sum_{n/\tau(n) \in Mc} \pi_n \Omega_n$ is the penalty fee at the compliance period; it can be 0 or negative;

$R_V = \sum_{n \in O_T} \pi_n \Gamma_n$ is the final stock value;

$R_F = \sum_{f \in Forw} q_f p_f$ is the trading exchanges revenue;

and where, for the study period [0,T] and at each node n:

- P_n is the set of intra day consumption levels (for instance : base, peak, off-peak).
- T_e is the set of thermal units that emit CO_2.
- Forw is the set of CO_2 forward exchanges.
- *Mc* is the set of time steps that belong to the surrenders stage period.
- λ_p is the generation price at intra day level p (in euro/MWh).
- C_l is the proportional generation cost of the thermal unit *l* (in euro/MWh).

- $u_{l,p}$ is the supplied power by the thermal unit *l* at the intra day level p (in MW).
- d_p is the duration of intra day level p (in hour).
- q_f is the CO_2 quantity of the forward *f* (in tone).
- p_f is the CO_2 price of the forward *f* (in euro/tone).
- Ω_n is the penalty cost (in euro).
- Γ_n is the CO_2 stock value (in euro).
- S_n is the stock level at the end of node n.
- $Ap(\tau(n))$ is the available contract year.
- $Cont(\tau(n))$ is the allowance contribution.
- α_l is the specific rate related to the thermal unit $l \in T_e$ (tone/MWh).
- $\delta(v^{LT})$ is the convex bellman set.

Problem (7) is solved, while the following constraints are satisfied:

<u>CO_2 stock dynamic system</u>:

At the initial time: $\forall\, n \,/\, \tau(n) = 0$,

$$S_0 = S_n + \sum_{p\in P_n} \sum_{l\in T_e} \alpha_l\, u_{l,p}\, d_p - Cont((\tau(n))$$

At a surrenders time stage: $\forall\, n \,/\, \tau(n) \in Mc$,

$$S_{\rho(n)} = S_n + \sum_{p\in P_n} \sum_{l\in T_e} \alpha_l\, u_{l,p}\, d_p + \sum_{f\in Ap(\tau(n))} q_f - Cont((\tau(n))$$

Elsewhere: $\forall\, n \,/\, \tau(n) \notin Mc$,

$$S_{\rho(n)} = S_n + \sum_{p\in P_n} \sum_{l\in T_e} \alpha_l\, u_{l,p}\, d_p - Cont((\tau(n))$$

<u>Surrenders stage and penalty operation</u>:

$$\forall\, n \,/\, \tau(n) \in Mc\ ,\ \Omega_n \le penalty \cdot S_n$$

<u>Finite time value:</u>

$$\forall\, n \in O_T\ ,\ \forall\, i \in \delta\left(v^{LT}\right),\ \Gamma_n \le a_i^{LT} S_n + b_i^{LT}$$

<u>Stock value limits</u>:

$$\Gamma_n \ge a_0^{LT} S_{\max} + b_0^{LT} \ \text{ and } \ \Gamma_n \le a_{state\, n-1}^{LT} S_{\max} + b_{state\, n-1}^{LT}$$

<u>Generation limits:</u>

$$0 \le u_{l,n,p} \le u_{l,p}^{\max}$$

5.3 Getting CO_2 Bellman values

At the convergence of the decomposition / coordination iterative process, we compute CO_2 Bellman values, by classical dynamic programming method.

The recurrent equation is given by:

For each node, $\forall\, n \,/\, \tau(n) \notin Mc$, we have:

$$V_n(S_i) = \sum_{m/\rho(m)=n} \pi_m \max_u \begin{bmatrix} \sum_{p\in P_n} \sum_{l\in T_e} (\lambda_p - C_l) u_{l,p} \\ + Vm\,(S_i - \sum_{p\in P_n} \sum_{l\in T_e} \alpha_l u_{l,p} + Cont(\tau(m))) \end{bmatrix}$$

Otherwise, the penalty cost is added, $\forall\, n \,/\, \tau(n) \in Mc$,

$$V_n(S_i) = penalty\ S_i + \sum_{m/\rho(m)=n} \pi_m \max_u \begin{bmatrix} \sum_{p\in Pn} \sum_{l\in Te} (\lambda_p - C_l) u_{l,p} \\ + Vm\,(S_i - \sum_{p\in Pn} \sum_{l\in Te} \alpha_l\, u_{l,p} + Cont(\tau(m))) \end{bmatrix}$$

where:

$$\begin{cases} \forall\, m \in \rho(n), \quad S_{\min} \le S_i - \sum_{p\in P_n} \sum_{l\in T_e} \alpha_l u_{l,p} + Cont(\tau(m)) \le S_{\max} \\ \forall l \in T_e, \forall p \in P_n, \quad 0 \le u_{l,p} \le u_{l,p}^{\max} \end{cases}$$

The maximization term above must be solved for each son of node n. It is done using a linear programming method.

6 CASE STUDY

This part is devoted to numerical experiment of the model described above. We use a simulator in order to analyse the output of our model. This tool manages generation units mix for each scenario and each time step using Bellman Values for stocks and thermal units costs to meet demand constraint.

In the case of CO_2, stock values are considered constant within a day. These values are added to each thermal plant cost taking into account its specific emitting efficiency (Tm of CO_2 per generated MWh).

Since data sets are built with real values and costs, only relative figures will be given in this section.

6.1 Data

EDF operates a highly integrated generation system in France. Its powerful installed base of nuclear and hydropower plants, which do not emit greenhouse gases, accounted for more than 90% of generation in 2004:

- Nuclear : 427.1 TWh,
- Hydropower : 20.9 TWh,
- Fossil-fired : 45.3 TWh.

However, growing consumption in Europe put pressure on fossil energy markets, triggering a sharp increase in hydrocarbon and coal prices. Thus, handling a CO_2 reservoir will become a key element of EDF's mid-term generation management.

In order to test method and implementation described above four data sets have been prepared: a reference one with no CO_2 management, and three other with different emission allowances. We study the impact of CO_2 for the period 01/01/2005 to 12/31/2007 (representing 1099 time steps and 3297 infra-day periods), where penalty is of 40 €/Tm of CO_2.

Data sets stand for the French generation mix with 3 aggregated hydro valleys, 58 nuclear power plants, about 40 CO_2 emitting thermal units, an aggregated demand side management contract and a representation of the spot market. We do not represent here any emission market.

The three CO_2 allowance cases are based upon the reference case average emission amount: for the first data set – Case A – allowance exceeds emission by 20%; Case B represents a balanced emission-allowance case. At last, Case C is a tense situation (allowances short of 20% with respect to CO_2 consumption reference).

CO_2 stock management satisfies functional constraints mentioned in § 2. At first time step of the cases investigated, CO_2 stock is equal to zero till the quota is given, which occurs on week 9.

6.2 Implementation issues

For the model run, a 12000 nodes scenario tree was computed out of 484 scenarios (demand, inflows, run of river production and availability). Each node is representative of one time step with its corresponding uncertainties.

The used computer is a Sun SPARC, Sun-Fire-280R. In table 1, the label #iteration represents the number of bundle algorithm iterations to reach the optimum. Maximum number of bundle algorithm iterations is limited to 50. CPU time is given in seconds.

	Ref	Case A	Case B	Case C
#iteration	47	50	50	50
CPU time	80	2078	2120	2251
Time per iteration	1.7	41.6	42.0	45.0

Table 1: Computational time for emitting thermal units and CO_2 reservoir optimization during bundle iterative algorithm.

6.3 Results Analysis

For each run we will be focusing in water stock and CO_2 reservoir trajectories, global system cost, weekly marginal cost, and generated volumes of all kinds of units: CO_2 emitting thermal units, market exchanges and hydro reservoirs.

One has to bear in mind that French mix is mainly composed of nuclear power plants and the expected effect of CO_2 emissions to be marginal. Table 2 shows the impact may be significant.

	Ref	A	B	C
CO_2 emitting plants	1	.83	.76	.67
Nuclear plants	1	1	1	1
Yearly water reservoir	1	1.01	1.01	1.01
Seasonal pumping	1	1.15	.68	.68
Weekly pumping	1	1.04	1.03	1.06
Demand Side	1	1.02	1.02	1.04
Spot purchases	1	1.36	1.54	1.72
Spot sales	1	.95	.93	.91
Curtailment	1	1.92	1.93	2.17

Table 2: Relative generation balance for the three allowance hypothesis per unit type

As expected, the lower the CO_2 allowance is, the lower the emitting plants production is. Nuclear power is baseload and thus its management inchanged, as well as water yearly reservoir, whose energy balance does not seem to be significantly impacted. The drop is mostly compensated by market exchanges (more purchases and less sales). However, even if the yearly water energy produced is not modified, figure 2 shows that its use is different.

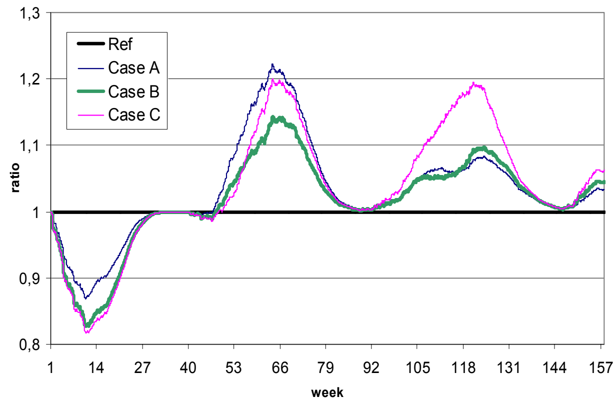

Figure 2: Average yearly water reservoir stock evolution relative to reference.

We could expect a more intensive use of our reservoirs mainly during winter when thermal plants are typically marginal. This effectively occurs after week 9, but reservoir behaviour before may be explained by the fact that allowances are not yet allocated and the CO_2 stock is negative (as shown in figure 3). For all our cases, the reservoirs strategies computed in that period of time turn out to be more cautious than the reference. Figure 3 shows the stock evolution relatively to the average amount of CO_2 produced in the reference case and a value of 1 stands for the average yearly emission without CO_2 constraint.

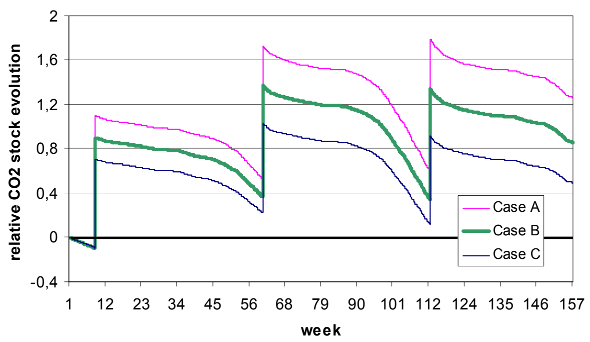

Figure 3: Mean CO_2 stock evolution relative to the average amount of CO_2 produced in the reference case.

As per strategies found, no penalty is paid in any case, since average emission permits stock is always positive at compliance date.

Figure 4 illustrates the difference induced by CO_2 allowances on the marginal cost.

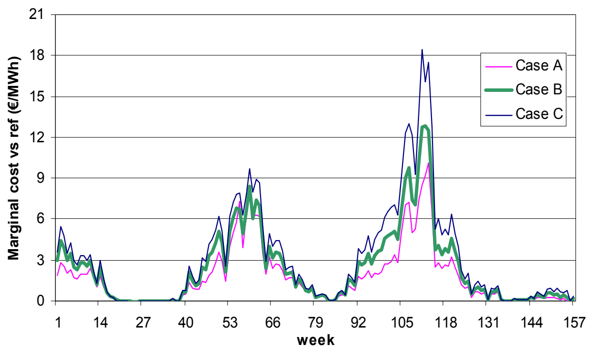

Figure 4: Average marginal cost difference between cases A, B, C and the reference one.

As we could expect, introducing a CO_2 quota leads to higher thermal plant costs scale and so are marginal costs during winter. On the contrary, when thermal units are not used - in summer time -, marginal costs remain the same as the reference ones.

These results highlight the interest of optimising the management of a CO_2 emission reservoir even in the case of small emitting installed capacity such as the French generation mix. Even with only one percent overhead, the optimal stock management is considerably modified. Hence, market exchanges are altered.
Moreover, risk profile changes. The global cost for our cases is hardly higher than the reference cost by one percent but the cost at risk (CaR: 95% quantile of cost distribution) increases up to seventeen percent and conditional cost at risk (CCaR: average cost in 5% worst scenarios) increases up to fifteen percent as shown in table 3.

	Ref	Case A	Case B	Case C
Average cost (%)	1.00	1.01	1.01	1.01
CaR (%)	1.00	1.14	1.15	1.17
CCaR (%)	1.00	1.14	1.14	1.15

Table 3: Cost and risk ratios relative to reference.

6.4 Quantile examples for case C

In this section, we illustrate the model behaviour for different quantiles for case C.

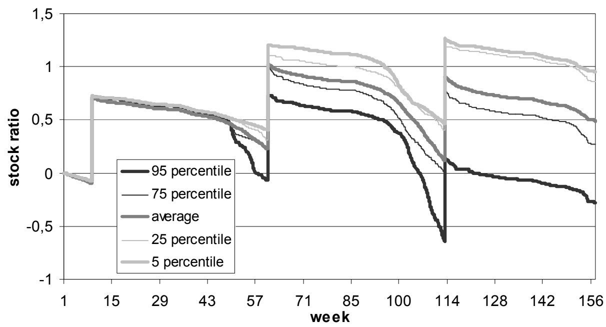

Figure 5: Quantiles 5, 25, 75, 95% and average CO_2 stock evolution for case C relative to the average amount of CO_2 produced in the reference case.

The compliance mechanism does not prevent the generation manager to have negative stocks before 31/12/2007 as for instance before week 9 when CO_2 quotas are not allocated, or for quantiles superior to 75%. In extreme scenarios, where demand is rather high, final stock may be negative at the compliance date, inducing penalty fee. Nevertheless, these kinds of scenarios may produce high profits even at the cost of a penalty.

In the end, the CO_2 stock evolution profiles are similar and linked to scenario difficulty. Figure 6 shows, on the contrary, that the strategy decided for water reservoir leads to drastical different profiles depending on the scenario.

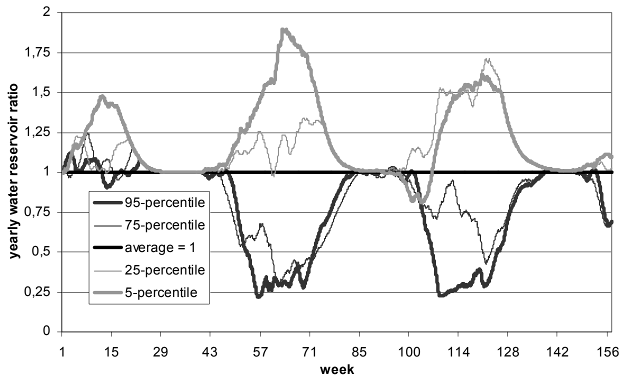

Figure 6: Quantiles 5, 25, 75, 95 and average yearly water reservoir stock evolution for case C relative to reference.

Several trends appear and stock trajectories cross each other. This phenomenon could be explained by the complex dynamics of the water reservoir and its interactions with other stocks and generation plants management as well as with market opportunities.

7 CONCLUSIONS

This paper describes how generation scheduling methodology and tools can be adapted to gas emission rules. Price decomposition/coordination model used in SOPRANO allows an easy implementation of new stocks and constraints. This is a key advantage in the changing power system context.

The numerical experiment handling a CO_2 reservoir with real data, leads to a more complex coordination problem, and thus to a longer resolution time. Nevertheless, the results of our study cases seem to prove that a generation manager cannot ignore these changes and constraints. It is necessary to meet optimal scheduling and manage both financial and physical risks at a minimum of cost.

To avoid CO_2 reservoir handling, a simple solution could be an artificial increase of emitting thermal units cost. This would penalize their use in order to meet CO_2 allowances at the compliance stage. However, this technique might disregard the CO_2 reservoir dynamic behaviour (see difficult scenarios). Moreover, some parameters such as allowances amount or penalty fee may be uncertain.

The next investigation is to analyse emission trading mechanisms in our model, accounting for CO_2 emission sales and purchases. Hence, we will be able to assess the market impact on hydro valleys and thermal plants management as well as on expected costs and risk exposure.

REFERENCES

[1] "Establishing a scheme for greenhouse gas emission allowance trading within the community and amending Council Directive 96/61/EC", DIRECTIVE 2003/87/EC of the European Parliament and of the Council.

[2] S. Brignol, H. Nguyen and L. Bacaud, "A new set of tools for Risk Management of Electricity Generation: applying Bundle Methods to large Stochastic Problems", 13th PSCC Proceedings, June 1999.

[3] M.V.F Pereira, L.M.V.G. Pinto, "Multi-Stage stochastic Optimization Applied to Energy Planning", Mathematical Programming, 52 :359-375, 1991.

[4] M.M. Belsnes, A. Haugstad, B. Mo, P. Markussen. "Quota Modeling in Hydrothermal Systems", Power Tech Conference Proceedings, 2003 IEEE Bologna.

[5] C. Lemaréchal, F. Pellegrino, A. Renaud, C. Sagastizabal, "Bundle Methods Applied to the Unit Commitment problem", in: J. Dolezal and J. Fidler (eds) System Modelling and Optimization, Chapman & Hall, pp. 395-402, 1995 (ref 9).

[6] J.F. Bonnans, J.Ch. Gilbert, C. Lemaréchal, C. Sagastizabal, "Numerical Optimization. Theoretical and Practical Aspects", Springer – Verlag, Berlin, 2002.

[7] A. Belloni, C. Sagastizabal, "Dynamic Bundle Methods. Application to Combinatorial Optimization", IMPA Preprint A2004/296.