Umbrella Contingencies in Security-Constrained Optimal Power Flow

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Abstract - The notion of umbrella contingencies is rigorously defined in the context of the security-constrained optimal power flow (SCOPF) problem in both its deterministic and probabilistic forms. The set of umbrella contingencies is a subset of the set of credible contingencies that is sufficient to attain levels of security and economic performance identical or nearly identical to those found when all credible contingencies are considered. This paper shows how to identify the members of the umbrella contingency set from the norms of the Lagrange multiplier vectors associated with the post-contingency power balance relations of the SCOPF. A case study of the deterministic SCOPF investigates the range of validity of the set of umbrella contingencies as the system demand varies. In addition, we analyze the validity of the umbrella contingency set when lowly ranked contingencies are left out in a stochastic SCOPF problem.

Keywords - Umbrella contingency, security-constrained optimal power flow, Lagrange multipliers, contingency ranking.

1 Introduction

THE recent large-scale blackouts having struck Europe [1–4] and North America [5] have underscored the importance of continued close monitoring and controlling of the security of power systems. These power outages cannot be attributed to insufficient knowledge about power system security since this is an area of research that over the past three decades has received extensive attention. More likely, the underlying cause was a profit-motivated relaxation of the strict application of system security criteria.

One difficulty of including security criteria in power system operation schemes is the large number of contingencies and associated variables that have to be processed. In the past, this obstacle was approached through contingency ranking [6–11], a process based on a simplified load flow or transient stability analysis for each credible contingency relative to a given operating point. Only highly ranked contingencies were then analyzed in detail from which any necessary preventive security control actions were defined [12, 13].

A closely related notion to contingency ranking is that of umbrella contingencies, which is the focus of this paper. A contingency k_0 is said to be an umbrella contingency of contingencies $k_1, \ldots, k_n$ if the condition that the system is secure with respect to k_0 implies that the system is also secure with respect to $k_1, \ldots, k_n$. Thus, evaluating the system security with respect to k_0 only is equivalent to evaluating the system security with respect to $k_0, \ldots, k_n$. The notion of umbrella contingencies is more powerful than that of a ranked list of contingencies since the latter does not eliminate a contingency from further consideration. Umbrella contingencies are used extensively by system operators to limit the number of credible contingencies to be analyzed. The identification of umbrella contingencies is based mostly on empirical evidence and experience [13] although some related set-theoretic results based on security regions have been developed [14–17].

In this paper, we focus on the identification of umbrella contingencies in the context of electricity markets based on a security-constrained optimal power flow (SCOPF) [18]. The goal of this research is to predict in terms of the system load a reduced number of equivalent credible (umbrella) contingencies and associated variables that the SCOPF problem has to consider to obtain the same or nearly the same market-clearing solution as with the full set of contingencies. Both deterministic and stochastic SCOPF problems are considered in this paper.

We demonstrate that the members of the set of umbrella contingencies can be systematically found from the Lagrange multipliers of the underlying SCOPF optimization problem. In the deterministic case, we show how a reduced set of umbrella contingencies that yields precisely the same market-clearing solution as with the full set of contingencies can be identified. In the stochastic problem, a reduced set of umbrella contingencies can be identified such that the sensitivity of the optimum solution to the neglected contingencies is smaller than a specified threshold.

For the deterministic SCOPF, we present a numerical study on the degree to which the umbrella set varies with system demand. In the stochastic case, we analyze the heuristic threshold rule defining membership in the set of the umbrella contingencies and its impact on the market-clearing solution.

2 Security-Constrained Optimal Power Flow

Consider the security-constrained optimal power flow problem optimizing $f(\mathbf{u}, \mathbf{x})$, a measure of the social welfare over the pre- and post-contingency operating states, where the latter are defined by a set of credible contingencies $\mathcal{C}$

$$\min_{\mathbf{u},\mathbf{x}} f(\mathbf{u}, \mathbf{x}). \tag{1}$$

Here the vector $\mathbf{u}$ represents the discrete variables of the SCOPF such as on/off generator status and transformer tap settings, while $\mathbf{x}$ represents the continuous generation, demand and reserve levels, bus voltages and, possibly, load shedding. The vectors $\mathbf{u}$ and $\mathbf{x}$ include pre- and post-contingency variables.

This optimization problem is subject to the pre-contingency nodal power balances

$$\mathbf{h}_0(\mathbf{u},\mathbf{x}) = \mathbf{0} \quad (\boldsymbol{\mu}_0), \tag{2}$$

the nodal power balance conditions for each contingency $k \in \mathcal{C}$

$$\mathbf{h}_k(\mathbf{u},\mathbf{x}) = \mathbf{0}; \quad \forall\, k \in \mathcal{C} \quad (\boldsymbol{\mu}_k), \tag{3}$$

and all inequalities applying to the pre- and post-contingency states

$$\mathbf{g}(\mathbf{u},\mathbf{x}) \geq \mathbf{0} \quad (\boldsymbol{\sigma}). \tag{4}$$

Associated with the constraints (2), (3) and (4) we define, respectively, the Lagrange multiplier vectors $\boldsymbol{\mu}_0$, $\boldsymbol{\mu}_k; \forall\, k \in \mathcal{C}$, and $\boldsymbol{\sigma}$.

In the *deterministic* SCOPF, the objective function $f(\mathbf{u},\mathbf{x})$ ignores the probability of occurrence of the contingencies. Moreover, it does not optimize the associated loss of welfare due to reserve deployment following the occurrence of any credible contingency [19].

On the other hand, in the *stochastic* SCOPF, the objective function is the expected value of the social welfare considering the probabilities of occurrence of the credible contingencies and the expected associated loss of welfare due to reserve deployment. For instance, if the contingencies $k \in \mathcal{C}$ have probabilities p_k and the pre-contingency state has a probability p_0, the objective function of the stochastic SCOPF can be written as

$$f(\mathbf{u},\mathbf{x}) = p_0 f_0(\mathbf{u},\mathbf{x}) + \sum_{k \in \mathcal{C}} p_k f_k(\mathbf{u},\mathbf{x}), \tag{5}$$

where $f_0(\cdot)$ and $f_k(\cdot)$ express respectively the pre- and post-contingency social welfare functions.

Unlike its deterministic counterpart, by accounting for the post-contingency expected social welfare, the stochastic SCOPF is well suited to balance the relative benefits and costs of reserve scheduling and deployment versus those of load shedding [20, 21].

3 Identifying Umbrella Contingencies

Consider a subset of the set of credible contingencies, $\mathcal{U} \subseteq \mathcal{C}$. We define a relaxation of the full SCOPF, (1)–(4), replacing (3) by

$$\mathbf{h}_k(\mathbf{u},\mathbf{x}) = \mathbf{0}; \quad \forall\, k \in \mathcal{U}. \tag{6}$$

We denote the solution to the full SCOPF by $(f,\mathbf{u},\mathbf{x}) = (f',\mathbf{u}',\mathbf{x}')$ and that of the relaxed SCOPF by $(f,\mathbf{u},\mathbf{x}) = (f^*,\mathbf{u}^*,\mathbf{x}^*)$. Note that the solution vectors $(\mathbf{u}^*,\mathbf{x}^*)$ of the relaxed SCOPF are of lower dimension than their full SCOPF counterparts because the relaxed SCOPF does not optimize over the omitted contingencies and corresponding variables.

Definition 1 *The subset $\mathcal{U}$ is a set of* umbrella contingencies *of the full SCOPF if f^* is the same or very close to f'.*

We now propose a method to identify the set of umbrella contingencies from the $\boldsymbol{\mu}_k$ vectors, the Lagrange multipliers of the post-contingency power balance relations.

Proposition 1 *The elements of the set of umbrella contingencies of the full SCOPF correspond to those contingencies whose associated Lagrange multiplier vectors satisfy $\|\boldsymbol{\mu}_k\|_p \geq \varepsilon$, for a pre-specified threshold $\varepsilon > 0$.*

The above proposition is based on the following argument. As shown in the Appendix, the sensitivity of the full SCOPF objective function to an infinitesimal perturbation $d\mathbf{S}_k$ in the right-hand side of the power balance relation under contingency k is given by

$$\frac{\partial f}{\partial \mathbf{S}_k} = \boldsymbol{\mu}_k. \tag{7}$$

This sensitivity relation says that if $\|\boldsymbol{\mu}_k\|_p < \varepsilon$, the power balance under contingency k can be perturbed without significantly affecting the optimum objective function of the full SCOPF. This also implies that if $\|\boldsymbol{\mu}_k\|_p < \varepsilon$, then ignoring contingency k in the relaxed SCOPF will likewise not significantly affect the corresponding optimum objective function compared to that of the full SCOPF.

Whereas the above conclusion is theoretically sound when comparing the objective functions of the full and relaxed SCOPF's, additional numerical tests are needed to assess the validity of the optimal optimization variables obtained with the relaxed SCOPF. The results of this paper present a number of simulation tests that suggest that the identification of umbrella contingencies based on Lagrange multipliers is valid.

4 Discussion

The vector norm of the Lagrange multipliers can be interpreted as a severity index which can be used to rank the contingencies. An alternative to the specification of a rejection threshold ε is to define the set of the umbrella contingencies by retaining a pre-specified number of the most highly ranked contingencies.

Likewise, the nature of the vector norm used ($p = 1, 2, \infty$, etc.) has an effect on the selection of the umbrella contingencies. For instance, the one and two norms average out the Lagrange multiplier vectors over the entire network while the infinity norm puts the emphasis on extreme values. Consider, for instance, a contingency k leading to line congestion with large differences in the elements of the vector $\boldsymbol{\mu}_k$, that is, in the nodal marginal values of the objective function. It is readily seen that the infinity norm would rank this contingency higher than under the one and two norms.

The set of umbrella contingencies for a given SCOPF is also strongly dependent on the parameters of the SCOPF (loading, network configuration, unit commitment, etc.).

Hence, as the parameters of the problem change, the membership in the set of umbrella contingencies also varies. Nonetheless, in most practical situations, the set of umbrella contingencies is generally stable over a neighborhood of a given set of parameters. This property will be investigated in the next section with respect to changes in loading. In addition, one could also investigate whether there exist contingencies which remain umbrella contingencies independently of the value of a given system parameter. Identifying these "super-umbrella" contingencies is clearly of interest because of their intrinsic insensitivity to parameter uncertainties.

The principal use of the proposed identification scheme should be as an offline market operations planning tool whose role would be to specify a set of umbrella contingencies associated with particular operating conditions. For instance, simulation runs of the SCOPF using past and/or predicted operating conditions could be used to pre-specify a relaxed set of contingencies to constrain the market-clearing SCOPF in a day-ahead market for which the computational complexity must be kept within reasonable levels.

We remark that the dimensions of the SCOPF to be solved are important for large systems because of the explicit modeling of both the pre- and the post-contingency network conditions. Also, depending on the refinement of the SCOPF model used, the computational complexity of the solution process will vary. For example, if the SCOPF is formulated as a linear program, its time complexity is polynomial [22], while the more general instances considering discrete variables and/or a nonlinear network model are definitively harder to solve. Nonetheless, since the proposed identification scheme is an offline planning tool, the size of the SCOPF that need to be solved may not be as critical as if it were to be used for day-ahead scheduling purposes. Likewise, it is understood that the set of contingencies $\mathcal{C}$ should be pre-specified carefully based on operational experience so as not to let it grow unnecessarily large from the consideration of contingencies already known to have very marginal impacts or probabilities.

5 Numerical Examples

In this section, we study numerically the proposed umbrella contingency identification rule on the power system shown in Figure 1. We assume that the three lines have identical reactances of 0.13 per unit, on bases of 41 MVA and 120 kV, and maximum power-carrying capacities of 55 MVA. A linearized (dc) power flow model is used. In the studies with the stochastic SCOPF, we assume that all three lines ($\ell = 1, 2, 3$) have a forced-outage rate U_ℓ equal to 5×10^{-4}.

The generator data ($i = 1, 2, 3$) are given in Table 1. We assume that generator i: *a*) produces energy at the rate of g_i MW in the range $[\,0, g_i^{\max}\,]$ for an incremental cost of a_i €/MWh; *b*) provides up-spinning reserve, r_i^{up}, and down-spinning reserve, r_i^{dn}, at the rates of q_i^{up} and q_i^{dn} €/MWh respectively; and, *c*) possesses a forced-outage rate given by U_i.

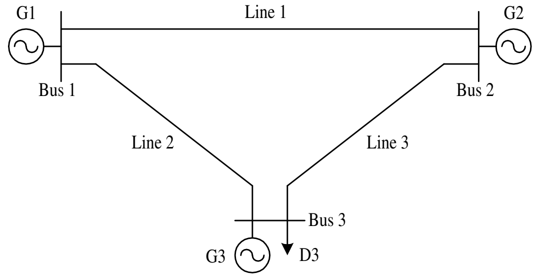

Figure 1: Three-bus, three-line, three-generator power system.

		Generator i		
		1	2	3
$g_i^{\max}$	(MW)	100	100	50
a_i	(€/MWh)	30.00	40.00	20.00
q_i^{up}	(€/MWh)	5.00	7.00	8.00
q_i^{dn}	(€/MWh)	5.00	7.00	8.00
U_i		0.05	0.02	0.07

Table 1: Generator data.

As seen in Figure 1, the load center is located at bus 3. We assume that the consumer at bus 3 is inelastic but offers to reduce or increase its consumption in the form of up- ($\hat{r}^{up}$) or down- ($\hat{r}^{dn}$) spinning reserve up to 10% of its scheduled consumption at rates $\hat{q}^{up} = €20$/MWh and $\hat{q}^{dn} = €20$/MWh respectively. The inelasticity assumption on the demand-side implies that the objective function of the SCOPF minimizes the cost of scheduling generation and reserve services only.

Lastly, the set of credible contingencies $\mathcal{C}$ comprises all single generator and line outages. Thus, contingencies $k = \{1, 2, 3\}$ correspond respectively to the individual failure of generators 1, 2 and 3, while contingencies $k = \{4, 5, 6\}$ correspond respectively to the failure of lines 1, 2 and 3.

5.1 *Coverage Provided by the Umbrella Set for Deterministic SCOPF*

This first numerical study examines the workings of Proposition 1 for the deterministic SCOPF. Specifically, we assess the membership of the set of umbrella contingencies subject to demand variations.

Here, the SCOPF is formulated for the following linear objective function

$$\min[\mathbf{a}^T\mathbf{g} + (\mathbf{q}^{up})^T\mathbf{r}^{up} + (\mathbf{q}^{dn})^T\mathbf{r}^{dn} + (\hat{\mathbf{q}}^{up})^T\hat{\mathbf{r}}^{up} + (\hat{\mathbf{q}}^{dn})^T\hat{\mathbf{r}}^{dn}]. \quad (8)$$

Table 2 shows the Lagrange multipliers of the post-contingency power balance relations as a function of the load. For each load range, we assume that the umbrella contingencies are those for which $\|\boldsymbol{\mu}_k\|_p > 0$, regardless of the norm type used.

Load range (MW)	Bus	$\boldsymbol{\mu}_k$ (€/MWh) 1	2	3	4	5	6
	1	0	0	5	0	0	0
$[0, 50]$	2	0	0	5	0	0	0
	3	0	0	5	0	0	0
	1	2	0	5	0	0	0
$(50, 100]$	2	2	0	5	0	0	0
	3	2	0	5	0	0	0
	1	7	5	0	0	0	0
$(100, 105]$	2	7	5	0	0	0	0
	3	7	5	0	0	0	0
	1	7	0	0	–3	0	–2
$(105, 116.6)$	2	7	0	0	0	0	–2
	3	7	0	0	0	0	13

Table 2: Lagrange multipliers of the post-contingency power balance relations, $\boldsymbol{\mu}_k$, as a function of the load.

In the range of demand from 0 to 50 MW, we observe that the vector $\boldsymbol{\mu}_k$ is nonzero only for $k = 3$, meaning that the set of umbrella contingencies is composed of the contingency corresponding to the failure of generator 3 only. In this range, all the energy is produced by generator 3, which is the cheapest incrementally as seen in Table 1. In order to meet the loss of that generator, an equal amount of up-spinning reserve is produced by generator 1, which is the cheapest provider of up-spinning reserve. The Lagrange multipliers at the three buses are all equal to €5/MWh, the marginal cost of up-spinning reserve provided by generator 1.

When the demand is between 50 and 100 MW, $\boldsymbol{\mu}_k$ is nonzero for $k = 1$ and 3, which are the contingencies corresponding to the failure of generators 1 and 3 respectively. Here, generator 3 produces its maximum of 50 MW while generator 1 supplies the residual demand. To cover the loss of generator 3, generators 1 and 2 provide together at least 50 MW of up-spinning reserve with generator 2 providing enough up-spinning reserve to cover the loss of generator 1. For example, if the load is 60 MW, then generator 1 produces 10 MW while providing 50 MW of reserve. At the same time, generator 2 provides the 10 MW of reserve needed to cover the loss of generator 1.

In the following demand range running between 100 and 105 MW, generator 3 exits the set of umbrella contingencies while generator 2 enters. In this case, generator 3 keeps on producing at its maximum of 50 MW while generator 1 picks up the residual demand. Generator 2 provides all the reserve required to cover the failure of generators 1 or 3 while producing no energy. Since the pre-contingency production level of generator 1 exceeds that of generator 3, the reserve production of generator 2 is now set by generator 1, explaining why generator 3 has exited the umbrella set. The nonzero Lagrange multiplier vector associated with the failure of generator 2 signals that if generator 2 had to produce some nonzero level of energy (if $g_2^{\min} > 0$), then generator 1 would need to provide that corresponding amount as up-spinning reserve at the rate of €5/MWh.

In the last range of load for which there exists a feasible schedule (for a demand in excess of 116.6 MW there exists no feasible solution), generator 1 is still in the umbrella set while the failures of line 1 ($k = 4$) and line 3 ($k = 6$) enter the umbrella set. Here again generators 1 and 3 are the sole producers of energy in the pre-contingency state, while generator 2 provides 55 MW of up-spinning reserve to counter the loss of generators 1 and 3, regardless of the loading. Here demand-side up-spinning reserve must be scheduled to counter the loss of generator 1 and the losses of lines 1 and 3. Moreover, in this case, generator 1 has to provide down-spinning reserve to make it possible to meet the line flow limitations after one of the lines is lost. It should be noted, in addition, that the umbrella set here is not unique since the loss of line 2 ($k = 5$) produces the same effect as the loss of line 3. Thus, as long as one of them is included in the umbrella set, the optimal solution will be identical to the full SCOPF. We remark also here that in the case of the line failures, the associated Lagrange multipliers are not all equal over all buses. This situation signals the presence of line congestion. We notice also the negative signs of some of the multipliers associated with the failures of the lines. Such values reflect the fact that to relieve the line congestion in the post-contingency states, it would be economically favorable to have demand located at buses 1 and 2.

It is also seen from Table 2 that there is no "super-umbrella" contingency immune to load variations in this example.

Lastly, we remark that there are generally few umbrella contingencies. In the above example, when the network can be reduced to a single node for the pre- and all the credible post-contingency states (that is, within the load range $[\,0, 105\,]$ MW), there are at most two umbrella contingencies corresponding to the failure of the two generators producing the largest combined amounts of power and up-spinning reserve. This observation is not particular to the current example only, wherein two generators are producing reserve dedicated to cover the failure of one another. This observation forms the basis for setting the up-spinning reserve requirement equal to the capacity of the largest generator in network-free reserve-constrained generation scheduling problems.

5.2 Contingency Identification for Stochastic SCOPF

The second numerical example studies the impact on the membership of the set of umbrella contingencies of varying the parameters of the ranking and cut-off rule in the stochastic SCOPF.

Here, the formulation of the SCOPF is identical to that of the previous subsection except for: *a*) the objective function now measures the expected cost in both the pre- and post-contingency states

$$\min p_0[\mathbf{a}^T\mathbf{g} + (\mathbf{q}^{up})^T\mathbf{r}^{up} + (\mathbf{q}^{dn})^T\mathbf{r}^{dn} + (\hat{\mathbf{q}}^{up})^T\hat{\mathbf{r}}^{up} + (\hat{\mathbf{q}}^{dn})^T\hat{\mathbf{r}}^{dn}] + \sum_{k\in\mathcal{C}} p_k[\mathbf{a}^T\mathbf{g}_k + \mathbf{v}^T\mathbf{L}_k], \quad (9)$$

where $\mathbf{v}$ is the vector of the nodal value of lost load—equal to €500/MWh for all buses in this example; *b*) the post-contingency nodal power balance that now take into account possible load shedding actions $\mathbf{L}_k$

$$\mathbf{h}_k(\mathbf{g}_k, \mathbf{d}_k, \boldsymbol{\delta}_k, \mathbf{L}_k) = \mathbf{0}; \quad \forall k \in \mathcal{C}, \quad (10)$$

where $\mathbf{g}_k$ and $\mathbf{d}_k$ represent the re-dispatch of generation and demand through reserve deployment, respectively, and $\boldsymbol{\delta}_k$ are the post-contingency voltage angles; and, *c*) associated bounds on load shedding variables

$$\mathbf{0} \leq \mathbf{L}_k \leq \mathbf{d}_k; \quad \forall k \in \mathcal{C}. \quad (11)$$

Moreover, the state probabilities p_0 and p_k are calculated from the forced-outage rates [20] given before, assuming that random contingencies occur independently. The probabilities are given in Table 3. We point out that the state probabilities shown here in Table 3 do not sum up to one because we did not consider all possible failure modes involving multiple-equipment contingencies.

k	0	1	2	3	4	5	6
p_k (10^{-4})	8645	455	176	651	4	4	4

Table 3: Contingency state probabilities.

Assuming a load of 110 MW at bus 3, the stochastic SCOPF schedule was computed and is reported in Table 4, while the expected value of the involuntary load shedding associated with each of the contingencies is given in Table 5. Here the 60 MW of up-spinning reserve provided by generator 2 is used to cover the loss of generators 1 and 3, while the 5 MW of down-spinning reserve provided by generator 1 is required to meet the steady-state flow limits of the transmission lines in the event of the loss of any of the lines.

Generator i	g_i (MW)	r_i^{up} (MW)	r_i^{dn} (MW)
1	60	0	5
2	0	60	0
3	50	0	0
Demand	d_3 (MW)	$\hat{r}_3^{up}$ (MW)	$\hat{r}_3^{dn}$ (MW)
	110	0	0

Table 4: Optimal pre-contingency generation, demand and reserve schedule.

k	1	2	3	4	5	6
$p_k L_k$ (kWh)	0	0	0	0	2	2

Table 5: Expected load shed at bus 3 following contingency k.

The optimal value of the expected cost here equals €3265.73. We point out that involuntary load shedding actions are used here rather than demand-side reserve in the event of the loss of lines 2 and 3. This is so because the corresponding expected cost of load shedding is small (4×10^{-3}MWh $\times$ 500€/MWh $=$ €2) compared to the expected cost of scheduling demand-side reserve, which would eliminate the need for involuntary load shedding (5MWh $\times$ 20€/MWh $\times$ $0.8645 =$ €86.45).

The first three rows of Table 6 show the values of the Lagrange multiplier vectors associated with the six credible contingencies, while the last three correspond to respectively the one, two and infinity norms of the vectors.

	$\boldsymbol{\mu}_k$ (€/MWh)					
Bus	1	2	3	4	5	6
1	7.87	4.85	1.95	–4.31	0.01	0.01
2	7.87	4.85	2.60	0.02	0.02	0.01
3	7.87	4.85	3.25	0.02	0.22	0.22
$\|\cdot\|_1$	23.61	14.55	7.80	4.35	0.25	0.24
$\|\cdot\|_2$	13.63	8.40	4.60	4.31	0.22	0.22
$\|\cdot\|_\infty$	7.87	4.85	3.25	4.31	0.22	0.22

Table 6: Lagrange multipliers of the post-contingency power balance relations, $\boldsymbol{\mu}_k$, and their corresponding norms.

As seen in Table 6, the various norms of the six vectors of Lagrange multipliers lead to different rankings of the contingencies. In the cases of the one and two norms, the ordering is $\{1,2,3,4,5,6\}$ whereas it is $\{1,2,4,3,5,6\}$ in the case of the infinity norm. For all three norms considered, the line outage contingencies $k = 5$ and 6 are seen to have very little impact on the solution of the full SCOPF. Indeed, if we omit these two contingencies and solve the corresponding relaxation of the SCOPF, the optimal pre-contingency schedule is identical to that of Table 4. As one would expect, the measure of the expected cost has decreased slightly to €3264.54—a difference of €1.19 or 0.04%.

Next, we assess the scheduling results obtained if the stochastic SCOPF is further relaxed based on different norms and cut-off thresholds. If, for example, the cut-off rule uses the one norm and an $\varepsilon >$ €4.35/MWh, the umbrella set would be $\mathcal{U} = \{1,2,3\}$. If, on the other hand, the infinity norm with some $\varepsilon >$ €3.25/MWh is used, the umbrella set would then be $\mathcal{U} = \{1,2,4\}$.

In the case when the set of umbrella contingencies is $\mathcal{U} = \{1,2,4\}$, the pre-contingency schedule obtained by the relaxed SCOPF is identical to the one found with the full set of contingencies (shown in Table 4). We note that in this relaxed SCOPF there is no scheduled involuntary load shedding. However, we recall from Table 5 that load shedding was expected in the full SCOPF schedule for contingencies $k = 5$ (loss of line 2) and 6 (loss of line 3), which were omitted in the relaxed problem. The expected cost here has gone down to €3240.60, a difference of €25.13 or 0.77% from the expected cost of the full SCOPF.

In the case where the alternate set $\mathcal{U} = \{1,2,3\}$ is used to constrain the relaxed stochastic SCOPF, we obtain a different pre-contingency schedule as shown in Table 7.

As in the previous case with $\mathcal{U} = \{1,2,4\}$, there is no scheduled load shedding under the umbrella set. Here, even though the pre-contingency schedule differs from that of the full SCOPF (unlike with $\mathcal{U} = \{1,2,4\}$), the recorded change in the expected cost is lower; the expected cost being equal to €3263.90 here, corresponding to a decrease of €1.83 or 0.06% from the expected cost found with the full SCOPF.

Generator i	g_i (MW)	r_i^{up} (MW)	r_i^{dn} (MW)
1	57.5	0	0
2	2.5	55	0
3	50.0	0	0
Demand	d_3 (MW)	$\hat{r}_3^{up}$ (MW)	$\hat{r}_3^{dn}$ (MW)
	110	2.5	0

Table 7: Optimal generation, demand and reserve schedule with $\mathcal{U} = \{1,2,3\}$.

One could argue that the umbrella set $\mathcal{U} = \{1,2,4\}$ is superior to $\mathcal{U} = \{1,2,3\}$ because it can cover a wider range of credible contingencies, offering the coverage of the full set of contingencies. By this we mean that the inclusion of contingency $k = 4$ schedules the necessary down-spinning reserve at bus 1 to counter the post-contingency congestion resulting from any of the line losses. Nonetheless, given the forced-outage rates of the equipments, we observe that generator failures are clearly more likely than line failures. Therefore, one can also well argue in favor of $\mathcal{U} = \{1,2,3\}$ since it is a set of contingencies whose members have a much higher probability of occurrence.

If the contingency ranking is based on the infinity norm, the contingency $k = 4$ is deemed to have a high impact on the SCOPF even though its impact is significant at bus 1 only. On the other hand, contingency $k = 3$ is deemed more important by the one and two norms. In this case, the network-wide rather than the local effects of the contingency are underlined. This different ranking behavior due to different norms thus requires that the user of the proposed identification scheme be aware of which type of contingency the umbrella set consists of: those imposing network-wide effects or those with more significant localized effects.

6 Conclusion

In this paper, we have defined rigorously the notion of the set of umbrella contingencies in the context of the security-constrained optimal power flow problem (SCOPF) in both its deterministic and probabilistic forms. We have demonstrated how to identify the members of the umbrella contingency set from the Lagrange multipliers of the original full SCOPF. We have then proposed a heuristic rule which first ranks the full set of credible contingencies according to their impact on the solution of SCOPF, that is, according to the norms of the associated Lagrange multiplier vectors, and then cuts off the lowly-ranked contingencies.

The first case study has investigated numerically the range of validity of the set of umbrella contingencies as the system demand is varied for a deterministic SCOPF. This study has shown that there is generally few umbrella contingencies especially when the network can be reduced to a single node in which case there are at most two. Also, this study has shown that the set of umbrella contingencies may not be uniquely determined when contingencies are umbrella to one another.

The second study has examined the effects of various ranking and cut-off rules used to heuristically determine the umbrella set for a stochastic SCOPF. The study showed that the optimal schedules of the SCOPF solved subject to the reduced set of umbrella contingencies are identical or very close to the schedules found with the full SCOPF. This study also illustrated the impact of the vector norm used in ranking the contingencies by showing that the one and two norms favor contingencies with network-wide effects while, on the other hand, the infinity norm favors those with important but more localized effects.

Appendix

Under smoothness assumptions on the functions $f(\cdot)$, $\mathbf{h}_0(\cdot)$, $\mathbf{h}_k(\cdot)$ and $\mathbf{g}(\cdot)$, an infinitesimal perturbation $d\mathbf{S}_k$ of the right-hand side of the post-contingency power balance vector equality corresponding to contingency k causes an incremental change in the optimal value of the continuous variables in the full SCOPF in the amount $d\mathbf{x}$. On the other hand, for the same perturbation, the discrete variables are assumed to remain constant. Following the argument of [19], this implies that

$$df = \left(\frac{\partial f}{\partial \mathbf{x}}\right)^T d\mathbf{x}, \tag{12}$$

$$d\mathbf{h}_0 = \frac{\partial \mathbf{h}_0}{\partial \mathbf{x}}\, d\mathbf{x} = \mathbf{0}, \tag{13}$$

$$d\mathbf{h}_j = \frac{\partial \mathbf{h}_j}{\partial \mathbf{x}}\, d\mathbf{x} = \begin{cases} d\mathbf{S}_k; & j \in \mathcal{C},\, j = k, \\ \mathbf{0}; & j \in \mathcal{C},\, j \neq k, \end{cases} \tag{14}$$

$$d\mathbf{g} = \frac{\partial \mathbf{g}}{\partial \mathbf{x}}\, d\mathbf{x}. \tag{15}$$

The Lagrangian function of the full SCOPF evaluated at the optimal vector of discrete variables, $\mathbf{u}'$, is given by

$$\mathcal{L}(\mathbf{u}', \mathbf{x}, \boldsymbol{\mu}, \boldsymbol{\sigma}) = f(\mathbf{u}', \mathbf{x}) - \boldsymbol{\mu}_0^T \mathbf{h}_0(\mathbf{u}', \mathbf{x}) - \sum_{j \in \mathcal{C}} \boldsymbol{\mu}_j^T \mathbf{h}_j(\mathbf{u}', \mathbf{x}) - \boldsymbol{\sigma}^T \mathbf{g}(\mathbf{u}', \mathbf{x}). \tag{16}$$

where $\boldsymbol{\mu} = (\boldsymbol{\mu}_0, \boldsymbol{\mu}_j, \forall j \in \mathcal{C})$. The associated first-order Karush-Kuhn-Tucker necessary optimality condition is

$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}} = \frac{\partial f}{\partial \mathbf{x}} - \left(\frac{\partial \mathbf{h}_0}{\partial \mathbf{x}}\right)^T \boldsymbol{\mu}_0 - \sum_{j \in \mathcal{C}} \left(\frac{\partial \mathbf{h}_j}{\partial \mathbf{x}}\right)^T \boldsymbol{\mu}_j - \left(\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right)^T \boldsymbol{\sigma} = \mathbf{0}. \tag{17}$$

Using (13)–(15) and (17), we re-express df, the corresponding infinitesimal change in the social welfare caused by the perturbation $d\mathbf{S}_k$, as

$$df = \boldsymbol{\mu}_k^T d\mathbf{h}_k + \boldsymbol{\sigma}^T d\mathbf{g}. \tag{18}$$

Moreover, we recall that complementary slackness requires that

$$\boldsymbol{\sigma}^T \mathbf{g}(\mathbf{u}', \mathbf{x}) = 0, \tag{19}$$

meaning that $\boldsymbol{\sigma}^T d\mathbf{g} = 0$. Using the above results, (18) can be rearranged to give

$$\frac{\partial f}{\partial \mathbf{S}_k} = \boldsymbol{\mu}_k. \tag{20}$$

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