A CONTINUOUS LOADING METHOD TO ESTIMATE LIMITING STATIC-STABILITY OPERATING CONDITIONS OF ELECTRIC POWER SYSTEMS IN REAL TIME

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Abstract – The paper proposes a method of continuous loading to determine the limiting static-stability operating conditions of an electric power system on a specified loading path. This method analytically specifies a loading trajectory in a space of active and reactive components of nodal voltages in the form of a convergent power series whose coefficients are determined using the curvilinear descent method. The computational cost to calculate each coefficient in this series is equivalent to the computational cost of completing one iteration of the modified Newton method. The proposed method allows to determine the limiting operating conditions along a specified loading path essentially in a single step. The method's timing characteristics satisfy the requirements of real-time environment.

Keywords: power systems, operating conditions, stability, calculation

1. INTRODUCTION

The problem of estimating the limiting static-stability operating conditions in electric power systems (EPS) is one of the core issues when estimating the reliability at EPS development and operational management stages. This problem focuses on estimation of static-stability limits in current and future normal operating conditions, and also possible critical and post-failure operating conditions. The estimation of static-stability limits is done at the system's development stage, as well as when planning and managing the EPS operating conditions at all geographical and time levels of operational dispatching control. This issue becomes especially important in a set of problems associated with: (1) estimation of EPS operational reliability in real time; (2) automated dispatching simulation under normal operating conditions; (3) automated dispatching simulation in critical and post-failure operating conditions; (4) automated dispatching simulation in EPS recovery after major system failures; (5) controlling for static-stability limits in optimization of systems' operating conditions; etc.

To determine the limiting static-stability operating conditions, the method of sequential variation (loading) of initial stable operating conditions is used with a check of the stability criterion for loaded operating conditions [1], i.e. in the general case the procedure for determining the limiting operating conditions assumes that the load iteration is specified, that the steady-state operating conditions are calculated at each iteration, and that the stability criterion is calculated at this iteration.

The amount of work involved in the whole process of determining the limiting operating conditions is determined to a considerable extent by the specification of the load iteration. For a small iteration, the number of calculations of the steady-state operating conditions and of the stability criterion increases. For a large iteration, there is a danger of passing over the stability boundary at the last iteration and there is the same problem of an increase in the number of calculations of steady-state operating conditions and of the stability criterion when splitting the value of the last load iteration.

The proposed method substantially simplifies a solution to the problem of estimating the limiting operating conditions. This is achieved because of:

- the rational choice of load iterations when estimating limiting static-stability operating conditions on the given loading path;
- combination of the three components (loading stages, computation of steady-state operating conditions, and estimation of their static stability) into a unified computational procedure based on the mathematical, algorithmic, and programming apparatus of the methods of curvilinear descent [2 - 5];
- the method's fast and reliable convergence properties.

2. ANALYSIS OF STATIC APERIODIC STABILITY IN CALCULATION OF STEADY-STATE OPERATING CONDITIONS USING METHODS OF CURVILINEAR DESCENT

It is shown in [6 - 8] that the Jacobian of the system of equations for the steady-state operating conditions of power systems containing busbars of infinite power is equal to the free term of the characteristic equation for the transients in these systems, if, when calculating the steady-state operating conditions: I) the active powers and the voltage moduli for generating nodes are given as independent variables; 2) load nodes are introduced into the calculation by the same static characteristics, as when calculating the stability; 3) busbars of infinite power are chosen as the balancing node, and such that when the Jacobian of the equations of the power system steady-state operating conditions corresponds to the free term of the characteristic equation the static aperiodic stability can be analyzed at the stage when their steady-state operating conditions are calculated on the basis of the convergence of the iteration processes of the methods, using the Jacobi matrix. The methods must not diverge when calculating stable operating conditions, and during their iterations a double change in the sign of the Jacobian must not occur. Methods of a curvilinear descent belong to this class [2 - 5].

The solution of the problem of calculating the steady-state operating conditions of power systems using methods of curvilinear descent and the solution of the problem of evaluating the static stability of power system operating conditions using the sign of the free term of the characteristic equation are very similar. The methods of curvilinear descent proposed in [2 - 5] realize the idea of a descent using a curve given in a special manner

$$\mathbf{F}(\mathbf{X}) = \mathbf{S}^{\mathrm{T}} \cdot \mathbf{W}(\mathbf{X}) = \mathbf{0} \qquad (1)$$

in the space of dependent variables connecting the initial approximation $\mathbf{X}^{(0)}$ with the solution $\mathbf{X}^*$ of the equations of the steady-state operating conditions investigated. Here $\mathbf{W(X)}$ is a vector-function of the equations of the steady-state operating conditions of the power system

$$\mathbf{W(X)=0} \qquad (2)$$

and $\mathbf{S}$ is a square matrix of order N, constructed from the column vectors $\mathbf{S}_1, \ldots, \mathbf{S}_N$ such that

$$\left(\mathbf{S}_i, \mathbf{W}\left(\mathbf{X}^{(0)}\right)\right) = 0, \quad i = 1,\ldots,N;$$

$$\mathbf{S}_i = \mathbf{e}^{\mathrm{T}}{}_i - \alpha_i \mathbf{W}\left(\mathbf{X}^{(0)}\right); \ \mathbf{e}_i = \left\{e_i = 1, e_j = 0 \middle| i \neq j\right\};$$

$$\alpha_i = \frac{\left(\mathbf{e}_{i,} \mathbf{W}\left(\mathbf{X}^{(0)}\right)\right)}{\left\|\mathbf{W}\left(\mathbf{X}^{(0)}\right)\right\|^2},$$

where $\mathbf{(a,b)=a^t b}$ is the scalar product of the vectors.

The existence of curve (1) and its accepted parameterization [2-4]

$$\mathbf{X}(\lambda) = \mathbf{X}^{(0)} + \sum_{k=1}^{\infty} \lambda^k \cdot \Delta\mathbf{X}_k; \ \lambda \in [0, \lambda_0]; \ \mathbf{X}(\lambda_0) = \mathbf{X}^*, \quad (3)$$

where $\Delta\mathbf{X}_k$ is a certain vector of coefficients of the series in λ, follows from the nondegeneracy of the Jacobi matrix in a certain region containing this curve. In other words, if the point of the initial approximation $\mathbf{X}^{(0)}$ and the point of the solution $\mathbf{X}^*$ of the equations of the steady-state operating conditions investigated belong to a connected region of one sign of the Jacobian $\mathbf{G}_x$, then parameterization (3) of curve (1), connecting the points $\mathbf{X}^{(0)}$ and $\mathbf{X}^*$ of the equations and not intersecting the surface of degeneration of the Jacobi matrix, will exist in this region.

Let us assume that the initial approximation $\mathbf{X}^{(0)}$ corresponds to the solution of the equations of the static stable operating conditions and the Jacobian of the equations of the operating conditions investigated corresponds to the free term of the characteristic equation of the transients. For $\mathbf{X^{(0)}} \in \mathbf{G}_X$ the region $\mathbf{G}_X$ is the region of static stability. If the system of equations of the steady-state operating conditions has the solution $\mathbf{X}^*$ and this solution belongs to the region $\mathbf{G}_X$, then in the region $\mathbf{G}_X$ the curve (1), given by the parameterization (3), exists and motion along it from the point $\mathbf{X}^{(0)}$ will lead to the determination of the solution $\mathbf{X}^* \in \mathbf{G}_X$, corresponding to the statically aperiodically stable operating conditions. Thus, motion along curve (1), specified by parameterization (3), in this case guarantees the determination of the solution of the equations of the operating conditions in question and is the sufficient condition for its aperiodic stability. In a certain sense curve (1) may be considered as an analytically defined trajectory of stable transition within the region of static stability from the known operating conditions, characterized by the parameters $\mathbf{X}^{(0)}$ to the operating conditions investigated, defined by the parameters $\mathbf{X}^*$ with a smooth variation of the defining parameters of the operating conditions.

When curve (1) is represented accurately by an appropriate approximation, the solution of the equations of the operating conditions investigated is obtained in a single step and, if the sign of the Jacobian at the point of the solution obtained is the same as the sign of the Jacobian at the point of the initial approximation and the point of the initial approximation corresponds to the stable operating conditions, then the calculated operating conditions are statically stable. Note that a double change in the sign of the Jacobian in a step is impossible in view of the special features of the proposed methods of curvilinear descent [3] and the assumed calculation conditions, which define, by the specified initial approximation, the region of possible existence of the solution of the equations of the operating conditions and consequently the region of convergence of the methods of curvilinear descent.

Computational investigations have shown [2-4] that the best results from the viewpoint of the effectiveness and the time complexity of the descent from the point $\mathbf{X}^{(0)}$ of the initial approximation to the solution $\mathbf{X}^*$ of Equations (2) are achieved when using a quadratic approximation of curve (1), specified by parameterization (3):

$$\bar{\mathbf{X}}(\lambda) = \mathbf{X} + \lambda \cdot \Delta\mathbf{X}_1 + \lambda^2 \cdot \Delta\mathbf{X}_2 \qquad (4)$$

and a descent along this approximation according to the following formula:

$$\mathbf{X}^{(p+1)} = \mathbf{X}^{(p)} + \lambda_p \cdot \Delta\mathbf{X}_1^{(p)} + \lambda_p{}^2 \cdot \Delta\mathbf{X}_2^{(p)} \qquad (5)$$

where the vector $\Delta\mathbf{X}_1$ is determined at each iteration from the solution of the system of equations

$$\mathbf{J}\left(\mathbf{X}^{(p)}\right) \cdot \Delta\mathbf{X}_1^{(p)} = -\mathbf{W}\left(\mathbf{X}^{(p)}\right);$$

the vector $\Delta\mathbf{X}_2$ is determined from the expression [2, 3]:

$$\Delta\mathbf{X}_2^{(p)} = -\bar{\mu}_2^{(p)} \cdot \Delta\mathbf{X}_1^{(p)} + \Delta\bar{\mathbf{X}}^{(p)}, \qquad (6)$$

and the vector $\Delta\bar{\mathbf{X}}^{(p)}$ is determined from the solution of the system

$$\mathbf{J}\left(\mathbf{X}^{(p)}\right) \cdot \Delta\bar{\mathbf{X}}^{(p)} = -\frac{1}{2}\left(\mathbf{W}'' \cdot \Delta\mathbf{X}_1^{(p)}, \Delta\mathbf{X}_1^{(p)}\right);$$

$\mathbf{J}(\mathbf{X}^{(p)})$ is the Jacobi matrix of equations (2) at the point $(\mathbf{X}^{(p)})$; $\mathbf{W}''$ is the matrix of the second partial derivatives of equations (2) at the point $(\mathbf{X}^{(p)})$ which is constant in the case of quadratic equations of steady-state operating conditions (2); and $\bar{\mu}_2$ is a parameter determined from the condition for minimum deviation of the quadratic approximation (4) from curve (1).

The iteration process of the method of the quadratic descent (5) realizes motion to the solution of the equations of the operating conditions not along curve (1), but along its approximation, which is minimally «remote» from it. The deviation of quadratic approximation (4) from curve (1) is regulated by the value of the parameter $\bar{\mu}_2$ in (6), which is chosen from the condition for minimum deviation of approximation (4) from curve (1) [2-5]. In the case when approximation (4) coincides with curve (1), iteration process (5) converges in a single iteration to the solution of the equations of the operating conditions (2). When the signs of the Jacobian at the point of the initial approximation $\mathbf{X}^{(0)}$ and at the point of the solution obtained are the same, the calculated operating conditions are statically stable. In the general case, approximation (4) of curve (1) is constructed at each step of the iteration process (5), and if the sign of the Jacobian at the points where approximation (4) is constructed remains unchanged, the calculated operating conditions are statically stable. Cases of a double change in the sign of the Jacobian in a step are excluded in view of the above.

3. DETERMINATION OF LIMITING STATIC STABILITY OPERATING CONDITIONS ALONG A SPECIFIED LOADING PATH

We will consider the quadratic equations of the steady-state operating conditions of a power system (2) in the form of the power balance at the nodes in a rectangular system of coordinates of the variables

$$w_i(\mathbf{X}) = \left\{ w_{P_j};\ w_{Q_l};\ w_{U_k} \,\middle|\, j = 1,\dots,n;\ l = 1,\dots,M;\ k = 1,\dots,K \right\} = 0 \qquad (7)$$

where

$$w_{P_i} = P_{G_i} - P_{l_i} - P_{N_i} = P_{G_i} - P_{l_i} - U_i^2 \cdot y_{iia} + U_{ia} \cdot \sum_{j \in S_i} (U_{ja} \cdot y_{ija} + U_{jr} \cdot y_{ijr}) + U_{ia} \cdot \sum_{j \in S_i} (U_{jr} \cdot y_{ija} - U_{ja} \cdot y_{ijr}), \quad i = 1,\dots n;$$

$$w_{Q_i} = Q_{G_i} - Q_{l_i} - Q_{N_i} = Q_{G_i} - Q_{l_i} - U_i^2 \cdot y_{iir} + U_{ia} \cdot \sum_{j \in S_i} (U_{jr} \cdot y_{ija} + U_{ja} \cdot y_{ijr}) + U_{ir} \times \sum_{j \in S_i} (U_{ja} \cdot y_{ija} - U_{jr} \cdot y_{ijr}), \quad i = 1,\dots M;$$

$$w_{U_i} = U_{i\,giv}^2 - U_{ia}^2 - U_{ir}^2;\ i = 1,\dots,K;$$

$$U_i^2 = U_{ia}^2 - U_{ir}^2; \qquad M + K = n.$$

Here P_{G_i}, Q_{G_i}, P_{l_i}, Q_{l_i} and P_{N_i}, Q_{N_i} are the active and reactive power of the generation and the load, and the active and reactive network power of node *i* respectively; y_{iia}, y_{iir}, y_{ija}, y_{ijr} resistive and reactive components of the self admittance of node *i* and the mutual admittance of the branch between nodes *i* and *j* respectively; U_i, U_{ai}, and U_{rj} are the modulus, active and reactive components of the voltage U of node *i*; U_{igiv} is the given constant modulus of the voltage of node *i*; S_i is the set of subscripts of nodes adjacent to node *i*; n is the number of nodes in the equivalent circuit of the power system, ignoring the basic node; *M* is the number of nodes with the active and reactive powers of the generators and loads given as independent variables; and *K is* the number of nodes with the active powers and the voltage moduli given as independent variables.

We recall that these equations are defined everywhere in the space of the dependant variables **X** and are twice continuously differentiable. We will represent (7)

$$\mathbf{W(X)=f(X)\ -Y=0},$$

where $\mathbf{X}=\left\{U_{ai},\ U_{r(n+i)}\middle|\ i=1,...,n\right\}$ is the vector of the dependent variables and

$$\mathbf{Y}=\left\{P_i=P_{G_i}-P_{l_i};\ Q_j=Q_{G_j}-Q_{l_j};\ U^2_{k\ giv}\middle|\ i=1,...,n;\ j=1,...,M;\ k=1,...,K\right\}$$

is the vector of the independent variables.

We will assume that the Jacobian of equations (7) corresponds to the free term of the characteristic equation. Suppose the point $\mathbf{X}_0$ of the space of the dependent variables **X** corresponds to the initial statically stable operating conditions. Let us consider a simply connected region $\mathbf{G}_X$ of the space X containing point the $\mathbf{X}_0$ and bounded by the surface of degeneration of the Jacobi matrix $|\mathbf{J(X)}|=0$. We will denote the boundary of the region $\mathbf{G}_X$ by $\partial\mathbf{G}_X$. For all $\mathbf{X}\in\partial\mathbf{G_x}$ $|\mathbf{J(X)}|=0$. Note that according to the conditions assumed the region $\mathbf{G}_X$ is the region of static stability. It is obvious that for each point $\tilde{\mathbf{X}}\in\mathbf{G}_X$ the following conditions are satisfied:

1) $\text{sign}\left|\mathbf{J}(\tilde{\mathbf{X}})\right|=\text{sign}\left|\mathbf{J}(\mathbf{X}_0)\right|$;

2) a curve exists lying in the region $\mathbf{G}_X$ not crossing its boundary and connecting the points $\tilde{\mathbf{X}}$ and $\mathbf{X_0}$.

In the space of independent variables **Y**, we will consider the region $\mathbf{G_Y}$ which is the image of the region $\mathbf{G}_X$ for the action of the operator **Y=f(X)**. We will separate in the region $\mathbf{G_Y}$ a region $\tilde{\mathbf{G}}_Y\subseteq\mathbf{G}_Y$ which is star convex relative to the point $\mathbf{Y_0=f(X_0)}$. We take an arbitrary point $\tilde{\mathbf{Y}}$ in the region $\tilde{\mathbf{G}}_Y$. By the definition of star convexity, section

$$\mathbf{Y}=\mathbf{Y}_0+\lambda\cdot(\tilde{\mathbf{Y}}-\mathbf{Y}_0),\ 0\le\lambda\le1$$

belongs to the region $\tilde{\mathbf{G}}_Y$.

Suppose $\mathbf{Y_1}$ is an internal point of the region $\mathbf{G_Y}$. Since the Jacobian of Equations (7) is nonzero in $\mathbf{G}_X$ and equations (7) are continuously differentiable everywhere in the space **X**, by the theorem on an implicit function [9], neighbourhoods $\mathbf{D}_X$ and $\mathbf{D}_Y$ of the points $\mathbf{X}_0$ and $\mathbf{Y}_0$ respectively in the spaces **X** and **Y** can be found such that for each $\mathbf{Y}\in\mathbf{D_Y}$ a single solution of Equations (7) exists in $\mathbf{D}_X$

$$\mathbf{X}=\mathbf{f(Y)}\in\mathbf{D}_X, \tag{8}$$

where the components of the function **f(Y)** are continuously differentiable in $\mathbf{D_Y}$ and $\mathbf{X_0=f(Y_0)}$.

From the continuous differentiability of **f(Y)** it follows that **Y1** the regions $\mathbf{G}_Y$ and $\mathbf{G}_X$ respectively. Hence a value λ_1 ($0<\lambda_1\le1$) can be found such that the section $\mathbf{Y=Y_0}+\lambda_1\ (\mathbf{Y_1}\text{-}Y_0)\in\mathbf{D_Y}$ for all $0\le\lambda\le\lambda_1$.

Using (8) we determine $\mathbf{X_1=f(Y}_1)$ where $\mathbf{Y}_1=\mathbf{Y}_0+\lambda_1\cdot(\tilde{\mathbf{Y}}-Y_0)$. Since $\mathbf{Y_1}\notin\partial\mathbf{G_Y}$, then $|\mathbf{J(X_1)}|\neq0$, $\mathbf{X_1}\in\mathbf{D}_1\subset\mathbf{G}_Y$. In addition in the region $\mathbf{G}_X$ we uniquely obtain the curve connecting the points $\mathbf{X_0}$ and $\mathbf{X}_1$, the equation of which **X=f(Y)** is the image of the section $[\mathbf{Y_0},\mathbf{Y_1}]$.

Passing similarly further from the point $\mathbf{X_1}$ to the point $\mathbf{X_2}$, then from the point $\mathbf{X}_2$ to the point $\mathbf{X}_3$ etc., we obtain in $\mathbf{G}_X$ a curve as the inverse image of the section $\mathbf{Y_0}+\lambda\ (\tilde{\mathbf{Y}}\ \text{-}\mathbf{Y_0})$. Consequently, for any section $\left[\mathbf{Y}_0,\tilde{\mathbf{Y}}\right]\in\tilde{\mathbf{G}}_Y$ it is possible to indicate uniquely a curve connecting $\mathbf{X_0}$ and $\tilde{\mathbf{X}}$, where $\tilde{\mathbf{X}}=\mathbf{f}\left(\tilde{\mathbf{Y}}\right)$, belonging to the region $\mathbf{G_X}$, such that its image when the operator **Y=f(X)** acts coincides with the section $\left[\mathbf{Y}_0,\tilde{\mathbf{Y}}\right]$. Such a curve defines the function $\mathbf{X}(\lambda)$, which maps the section [0, 1] into the region $\mathbf{G_X}$.

Consider the formally written sequence

$$\mathbf{X}_0+\sum_{n=1}^{\infty}\lambda^n\cdot\Delta\mathbf{X}_n, \tag{9}$$

where

$$\left.\begin{aligned}&\Delta\mathbf{X}_1 = -\mathbf{J}^{-1}(\mathbf{X}_0)\cdot(\tilde{\mathbf{Y}}-\mathbf{Y}_0),\\&\dots\dots\dots\dots\dots\dots\dots\dots\\&\Delta\mathbf{X}_n = -\mathbf{J}^{-1}(\mathbf{X}_0)\cdot\left[\sum_{k=1}^{n-1}(\mathbf{W}''\cdot\Delta\mathbf{X}_k,\Delta\mathbf{X}_{n-k})\right],\\&n\geq 2.\end{aligned}\right\}\quad(10)$$

We put

$$\alpha = \left\|\mathbf{J}^{-1}(\mathbf{X}_0)\right\|\cdot\left\|\mathbf{W}''\right\|\cdot\left\|\Delta\mathbf{X}_1\right\|. \qquad (11)$$

It is shown in [4] that for all $n \geq 1$ the following relation holds:

$$\left\|\Delta\mathbf{X}_n\right\| \leq \frac{(16\cdot\alpha)^{n-1}}{n^2}\cdot\left\|\Delta\mathbf{X}_1\right\|. \qquad (12)$$

Consider the power series

$$\sum_{n=1}^{\infty}\lambda^n\cdot\frac{(16\alpha)^{n-1}}{n^2}\cdot\left\|\Delta\mathbf{X}_1\right\|.$$

Its radius of convergence is

$$\lambda_0 = \lim_{n\to\infty}\frac{\frac{(16\alpha)^{n-1}}{n^2}\cdot\left\|\Delta\mathbf{X}_1\right\|}{\frac{(16\alpha)^{n}}{(n+1)^2}\cdot\left\|\Delta\mathbf{X}_1\right\|} = \frac{1}{16\alpha}.$$

Consequently the radius of convergence of series (9) is

$$\lambda_1 \geq 1/16a, \qquad (13)$$

where α is defined by (11).

We write equations (7) at the point of the solution $\mathbf{X}_0$ as

$$\mathbf{W}(\mathbf{X}_0) = \mathbf{Y}_0.$$

Suppose the direction of loading of the initial operating conditions is specified by the vector $\Delta\mathbf{Y}$. Then any operating conditions on the specified loading trajectory $\mathbf{Y}_0 + \Delta\mathbf{Y}$ can be found from the solution of the equations

$$\mathbf{W}(\mathbf{X}) = \mathbf{Y}_0 + \lambda\cdot\Delta\mathbf{Y}.$$

We will denote power series (9) with radius of convergence (13) by

$$\tilde{\mathbf{X}}(\lambda) = \mathbf{X}_0 + \sum_{n=1}^{\infty}\lambda^n\cdot\Delta\mathbf{X}_n\,. \qquad (14)$$

We substitute (14) into equations (7), assuming that $\Delta\mathbf{X}_n$ are found from (10)

$$\mathbf{W}(\mathbf{X}) = \mathbf{W}(\tilde{\mathbf{X}}(\lambda)) = \mathbf{W}\left(\mathbf{X}_0 + \sum_{n=1}^{\infty}\lambda^n\cdot\Delta\mathbf{X}_n\right) = 0.$$

As a result we obtain that

$$\begin{aligned}&\mathbf{W}(\tilde{\mathbf{X}}(\lambda)) = \mathbf{W}(\mathbf{X}^{(0)}) + \mathbf{J}(\mathbf{X}_0)\cdot\sum_{n=1}^{\infty}\lambda^n\cdot\Delta\mathbf{X}_n +\\&+\frac{1}{2}(\mathbf{W}''\cdot\sum_{n=1}^{\infty}\lambda^n\cdot\Delta\mathbf{X}_n,\sum_{n=1}^{\infty}\lambda^n\cdot\Delta\mathbf{X}_n) = \mathbf{W}(\mathbf{X}_0) +\\&+\lambda\cdot\mathbf{J}(\mathbf{X}^{(0)})\cdot\Delta\mathbf{X}_1 + \sum_{n=2}^{\infty}\lambda^n\cdot(\mathbf{J}(\mathbf{X}_0)\cdot\Delta\mathbf{X}_n +\\&+\frac{1}{2}\sum_{k=1}^{n-1}(\mathbf{W}''\cdot\Delta\mathbf{X}_k,\Delta\mathbf{X}_{n-k})) = \mathbf{Y}_0 + \lambda(\tilde{\mathbf{Y}}-\mathbf{Y}_0),\end{aligned}$$

i.e. when $0 \leq \lambda \leq \lambda_1$

$$\tilde{\mathbf{X}}(\lambda) = \mathbf{X}(\lambda)\,.$$

Thus we have shown that when $0 \leq \lambda \leq \lambda_1$ series (9) converges and curve (14) in this case is the image of the section of the straight line $\mathbf{Y}_0 + \lambda\cdot\Delta\mathbf{Y}$ in the space of the dependent variables.

4. THE ALGORITHM OF THE METHOD OF CONTINUOUS LOADING

Using the mathematical and program apparatus of the methods of curvilinear descent it is possible to construct the following effective algorithm for determining the limiting operating conditions on a specified loading path.

Suppose the point $(\mathbf{X_0}, \mathbf{Y}_0)$ corresponds to the initial statically stable operating conditions and the loading direction is specified by the vector $\Delta\mathbf{Y}_0$.

1. p:=0.
2. Using (10), n vectors $\Delta\mathbf{X}_i^{(p)}$, are determined, where the number n is determined by the convergence of the series

$$\lambda_i^{(p)} = \frac{\left\|\Delta\mathbf{X}_{i-1}^{(p)}\right\|}{\left\|\Delta\mathbf{X}_i^{(p)}\right\|},\quad i=1,\dots,n;\quad \lambda^{(p)} := \lambda_n^{(p)},$$

and the vector $\Delta\mathbf{X}_i^{(p)}$ is determined from the expression

$$\Delta\mathbf{X}_1^{(p)} = -\mathbf{J}^{-1}(\mathbf{X}_0)\cdot\Delta\mathbf{Y}_0.$$

3. The point $(\mathbf{X}^{(p+1)}, \mathbf{Y}^{(p+1)})$ with coordinates

$$\mathbf{X}^{(p+1)} = \mathbf{X}_0 + \textstyle\sum_{i=1}^{n}\lambda^{i}_{(p)}\cdot\Delta\mathbf{X}_i^{(p)}$$

$$\mathbf{Y}^{(p+1)} = \mathbf{Y}_0 + \lambda^{(p)}\cdot\Delta\mathbf{Y}_0$$

is determined.

4. If at the point $\mathbf{X}^{(p+1)}$ the coefficient $\beta_{(p)} > 1$, where from [3, 4]

$$\beta_{(p)} = \frac{1}{2}\frac{\left\|\left(\mathbf{W}'' \cdot \Delta\mathbf{X}_1^{(p+1)}, \Delta\mathbf{X}_1^{(p+1)}\right)\right\|}{\left\|\mathbf{W}(\mathbf{X}^{(p+1)})\right\|} =$$

$$= \frac{\left\|\mathbf{W}(\mathbf{X}^{(p+1)} + \Delta\mathbf{X}_1^{(p+1)})\right\|}{\left\|\mathbf{W}(\mathbf{X}^{(p+1)})\right\|},$$

then the calculation is finished and the point ($\mathbf{X}^{(p+1)}$, $\mathbf{Y}^{(p+1)}$) corresponds to the limiting operating conditions; otherwise p:=p+1; $\mathbf{X}_0$:=$\mathbf{X}^{(p)}$; $\mathbf{Y}_0$:= $\mathbf{Y}^{(p)}$, and we return to step 2.

5. TESTING FOR CONVERGENCE PROPERTIES OF THE CONTINUOUS LOADING METHOD IN ESTIMATION OF LIMITING OPERATING CONDITIONS

Numerous computational simulations on a large number of different power systems showed that this method enables one to determine the limiting operating conditions with a very high accuracy in one or two iterations.

Let's illustrate the convergence properties of the proposed method with an example of estimating the limiting operating conditions in three different electric power systems (see Table 1), where the respective three schemes contain: 33 nodes and 47 branches in System 1; 107 nodes and 122 branches in System 2; and 192 nodes and 215 branches in System 3. The initial operating conditions were loaded by: increasing the generator power of two stationary nodes (System 1); increasing the loading power in six nodes (System 2); and unloading generators in four nodes (System 3). Along this, generators of the node, set as the balancing node, were loaded. The precision in calculating steady-state operating conditions at each loading iteration step in all three cases was given by 0.1 MWA. The parameters of calculated steady-state operating conditions obtained at each previous load iteration were used as the initial approximation in calculation of each limiting operating condition by the method of sequential loading. The estimation of operating-condition stability at each load iteration was done by using the Jacobian of equations describing steady-state operating conditions.

When using the algorithm of sequential loading, the limiting operating conditions were determined in seventeen iterations for System 1, in seven iterations for System 2, and in sixteen iterations for System 3. The proposed method of continuous loading allowed us to determine limiting operating conditions within just one iteration in all three cases. Please see Table 1 for the results.

For comparability of computational characteristics of these methods, computations of steady-

Table 1: Examples of convergence in methods of sequential and continuous loading in the problem of estimating limiting static-stability operating conditions of electric power systems.

EPS	Values of loaded parameters in the initial operating condition (MW, MWA)	Load iteration $\Delta\Pi$ and given precision in estimation of peak loads ξ (MW, MWA)	Values of loaded parameters in the limiting operating condition (MW, MWA)	The number of load iterations ***n*** and timing in estimation of peak loads (in relative units $t^*=t/t_{sl}$)	
				By the sequential loading method	By the proposed method
EPS #1	P_{G1} = 1380 P_{G2} = 1150	ΔP_{G1} = 50, ΔP_{G2} = 50, ξ=5.	1838.48 1608.48	17, t_{sl}=1	1, t^*=0.12
EPS #2	$S_{\ell 1}$ = 13.78+j6.36; $S_{\ell 2}$ = 30.74+j8.48; $S_{\ell 3}$ = 9.54+j2.23; $S_{\ell 4}$ = 15.90+j7.24; $S_{\ell 5}$ = 25.44+j12.72; $S_{\ell 6}$ = 13.25+j4.24.	ΔS_i = 0.1 $S_{\ell i}$, i= 1,…, 6, ξ=0.1.	$S_{\ell 1}$ = 18.58+j8.54; $S_{\ell 2}$ = 41.82+j11.44; $S_{\ell 3}$ = 12.86+j2.96; $S_{\ell 4}$ = 21.44+j10.01; $S_{\ell 5}$ = 34.67+j17.15; $S_{\ell 6}$ = 18.05+j5.72.	7, t_{sl}=1	1, t^*=0.35
EPS #3	P_{G1} = 53.9 P_{G2} = 104.5 P_{G3} = 115.5 P_{G4} = 27.5 P_{G5} = 77.5	ΔP_{Gi} =0.1 P_{Gi} , i= 1,…, 4, ξ=1.	P_{G1} = 2.31 P_{G2} = 1.32 P_{G3} = 2.00 P_{G4} = 0 P_{G5} = 4.77	16, t_{sl}=1	1, t^*=0.19

state conditions and estimation of their static stability at each continuous loading step were performed using the quadratic descent method. Therefore, the relative computational times, presented in Table 1, characterize the relative effectiveness of the loading algorithms. It is worth noting that the Newton method and are not effective in computation of steady-state operating conditions for EPS #2 and #3 from Table 1 because the Newton method diverges and the Newton-based minimization method converges very slowly. Meanwhile, the method of quadratic descent converges at each loading step in only one or two iterations.

The time, which is required to solve the problem of estimating limiting static-stability operating conditions for an electric power system of any size, does not exceed 200-300 percent of the time, which is necessary to estimate the single steady-state operating condition for this system by using the quadratic descent method. If one takes into account the fact that the quadratic descent method converges as a rule in one or two iterations [2-5], then the costs of estimating limiting static-stability operating conditions by the continuous loading method is equivalent to the costs of 2-6 iterations of the quadratic descent method. For example the time, which is necessary to perform one iteration of the quadratic descent method for an EPS scheme of a standard size (200-500 nodes) on a personal computer with a 200 MHz Pentium processor by using the program based on this method, does not exceed 2-4 ms. These timing characteristics satisfy the requirements of the real-time operational control environment; therefore, the proposed method for estimation of limiting static-stability operating conditions can be effectively used in many problems associated with real-time decisions.

6. CONCLUSION

The method proposed in this paper allows to considerably increase the effectiveness and efficiency in the procedure of estimating the limiting static-aperiodic-stability operating conditions done by using the method of sequential loading of initial steady-state operating conditions. This is done by: (1) combining the stages in selecting the load iteration; (2) estimating steady-state operating conditions at each load iteration; and (3) calculating the stability criterion at each iteration. This method is based on the mathematical and programming framework of the earlier developed methods of curvilinear descent [2-5].

The proposed method guarantees estimation of the limiting static stability operating conditions in a basically single step, and it greatly increases the speed of obtaining the problem's solution in comparison with that of the traditional sequential loading method.

Thanks to its superior timing characteristics, the proposed method can be effectively applied in many problems associated with the real-time environment, including: estimation of operational reliability of current and future operating conditions, different automated dispatching simulations, controlling for static-stability limits in optimization of operating conditions, etc.

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