A CONIC OPTIMAL POWER FLOW WITH UNCERTAIN PRICES

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Abstract – The interconnection of modern electric power systems allows utilities to obtain significant savings in operational cost by scheduling power purchases. In a deregulated environment, utilities have only partial knowledge of the price of power available in the market. This paper presents a solution to the optimal power purchase problem in a utility by considering ellipsoidal cost coefficient uncertainty and a second-order network model. It is shown that the resulting risk-averted optimal power flow model is a particular instance of a convex second-order cone program for which polynomial interior-point methods and efficient implementations exist. The proposed method is used to compute the efficient frontier of the optimal purchase problem in two sample test systems. The results indicate that in the presence of fluctuating market prices, the efficient frontier gives valuable information for determining trade-off between risk and expected profit. The validity of the numerical results is confirmed using a Monte-Carlo simulation.

Keywords: ellipsoidal uncertainty, conic optimization, efficient frontier.

1 INTRODUCTION

In a competitive environment, decision making under uncertainty is crucial to maximize the profitability of electric power utilities. Even in the regulated era, uncertainties associated with load forecast errors, line faults, and unit unavailability have always existed. With the introduction of deregulation in the power industry, the price of many power purchase transactions can change at any time. As a result of this increased volatility, power utilities have to rely on risk management methods to reduce their overall operational cost. This paper focuses on the risks presented by fluctuating prices of imported power purchases [1].

In previous work considering power purchases [2], a fixed price was assumed and the uncertainty was disregarded. Yan and Luh [3], and Ferrero and Shahidehpour [1] later on modeled the uncertain prices by employing concepts from fuzzy set theory. Their formulation corresponded to a fuzzified version of the traditional economic dispatch problem. Yong and Lasseter [4] presented a formulation of the stochastic optimal power flow problem considering uncertainty in load, network structure and power supply. The uncertain cost functions were replaced by their expected value without accounting for volatility. Bjorgan et al. [5] addressed the volatility of market prices by using the efficient frontier as a tool to identify the preferred portfolio of contracts. The focus was on the relation between contract evaluation and scheduling of physical resources. However, the configuration of the interconnected power systems was not considered. Conejo et al. [6], and Yamin and Shahidehpour [7] analyzed a self-scheduling model that accounts for profit and risk simultaneously.

This paper presents a solution to the optimal power purchase problem in a utility. Within the utility, the generators have quadratic cost functions which are treated as proprietary information [8]. The price of power purchased from neighboring utilities is in practice forecasted by the utility for market analysis [1]. In this research, the forecasted cost coefficients of power import are specified by their average and covariance matrix thus yielding an ellipsoidal model of uncertainty [9]. A second-order network model is used to represent the power transmission network [10]. This network model is based on the DC load flow but accounts for line losses locally. In a recent paper, Rau [11] indicates that the deregulation of the power industry has prompted the rebirth of the DC load flow method since it offers a simple and elegant way of administering market rules based on economic theory.

In a recent research [12], the author has shown that the optimal power flow (OPF) problem based on the second-order network model can be formulated as a second-order cone program using rotated quadratic cones. In this work, it is shown that the ellipsoidal cost coefficient uncertainty model can still be accommodated in the second-order cone program via quadratic cones. A major attribute of the cone program is that it is solvable in polynomial time using interior point methods. In addition, efficient implementations of these methods exist, e.g., the software system MOSEK [13].

The rest of this paper is organized as follows. Section 2 introduces the problem of optimal power purchase. The modeling of power purchase cost uncertainty using an ellipsoidal set is presented in section 3. Section 4 presents the formulation of the risk-averse optimal power flow problem as a second-order cone program. In section 5, numerical results are reported on a 5-bus and a 30-bus test system. The efficient frontier [5] (a pareto optimal set between goals of high expected profit and low standard deviation) is obtained for both systems. The use of the efficient frontier to trade-off between expected return and risk (as measured by the standard deviation of the return) is discussed. The numerical values for expected return and standard deviation are verified by using a Monte-Carlo simulation. The paper is concluded in section 6.

2 PROBLEM DEFINITION

It is often mutually beneficial for neighboring utilities to exchange power due to dissimilarities in the utilities' hourly loads and generation resources. The problem discussed in this paper is illustrated in Figure 1.

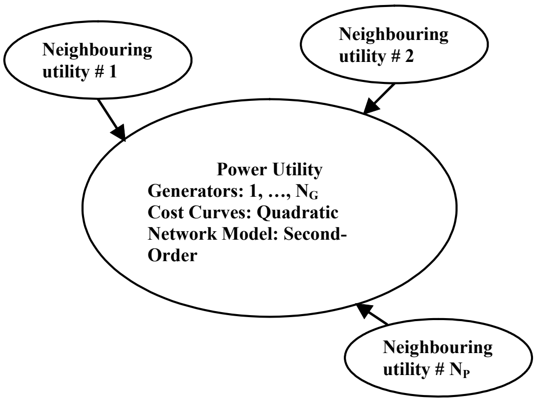

Figure 1: Power purchase in a utility

In Figure 1, the utility under study has sufficient generation capacity to serve its demand. Each generator in this utility has a quadratic cost function

$$C_{Gi} = a_i + b_i P_i + c_i P_i^2 \qquad [\$/\text{h}] \tag{1}$$

However, the utility has the option to purchase power from neighboring utilities in order to reduce its overall operational cost and thus increase profit. According to [8], the price of purchasing power is realistically represented as a linear function of the purchased power. Consequently, the expenditure for purchasing power by the utility becomes a non-linear function. In this research, this cost of purchase is approximated by a convex piece-wise linear function (Figure 2) and represented using separable programming (λ_{ik} is an interpolatory variable) [10]

$$C_{Pi} = \sum_k c_{ik} \lambda_{ik} \tag{2}$$

$$P_i = \sum_k p_{ik} \lambda_{ik} \tag{3}$$

$$\sum_k \lambda_{ik} = 1,\ \lambda_{ik} \geq 0 \tag{4}$$

The optimal purchase problem in the utility is therefore formulated as:

minimize t subject to (5)

total cost function

$$t \geq \sum_{i=1}^{N_G} C_{Gi} + \sum_{i=1}^{N_P} C_{Pi} \tag{6}$$

interpolatory variable constraints

$$P_i = \sum_k p_{ik} \lambda_{ik} ,\ \sum_k \lambda_{ik} = 1,\ \lambda_{ik} \geq 0 \tag{7}$$

second-order network model

$$P_i - P_{Di} + \sum_{j \in k(i)} a_{ij} (\delta_i - \delta_j) \geq \sum_{j \in k(i)} \tfrac{1}{2} \hat{g}_{ij} (\delta_i - \delta_j)^2 \tag{8}$$

line flow limits

$$-\overline{T_{ij}} \leq T_{ij} = -a_{ij} (\delta_i - \delta_j) \leq \overline{T_{ij}} \tag{9}$$

power generation / purchase limits

$$\underline{P_i} \leq P_i \leq \overline{P_i} \tag{10}$$

where

$k(i)$ = set of nodes connected to node i
N_G = number of generators in the utility
N_P = number of power purchase transactions
P_i = power generation / purchase at node i
$\overline{P_i}$ = maximum limit of P_i
$\underline{P_i}$ = minimum limit of P_i
P_{Di} = power demand at node i
T_{ij} = average power flow on line ij
$\overline{T_{ij}}$ = capacity of line ij
V_i = voltage magnitude at node i
y_{ij} = line admittance= $g_{ij} + \sqrt{-1} b_{ij}$,
$a_{ij} = V_i V_j b_{ij}$, $\hat{g}_{ij} = V_i V_j g_{ij}$
δ_i = voltage angle at node i, $\delta_1 = 0$

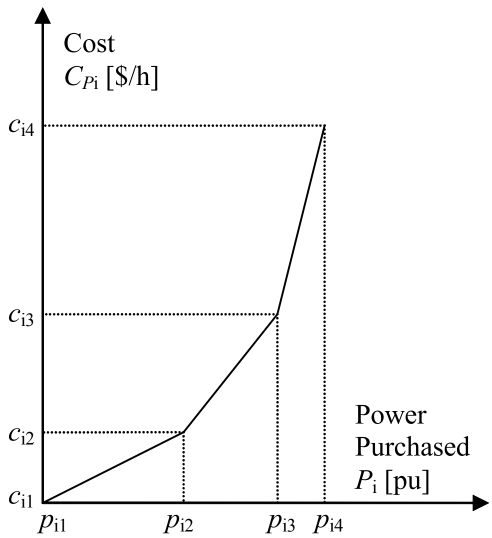

Figure 2: Cost Curve of Purchased Power

The problem formulated in equations (5) through (10) corresponds to an OPF dispatch in which the utility schedules the amount of generation produced by its own units and the amount of power purchased from neighboring utilities. In the total cost function (6), the cost coefficients of the purchased power are assumed to be known a-priori. In practice, these coefficients may be forecasted by the utility for performing market analysis [1]. For instance, when a utility is considering taking a certain transaction opportunity, it should consider other possible future transaction opportunities which may be more profitable [3]. In such a case, a forecast of the cost coefficients of potential future transactions is required for making an economical decision. The cost coefficient forecast has a degree of uncertainty specified by a covariance matrix. Denton et al. indicate that a deterministic solution taking the expected price as given will not in general produce the correct expectation since it ignores volatility [14]. Consequently, a method of incorporating price volatility in the OPF solution is described next.

3 ELLIPSOIDAL UNCERTAINTY SET

Equation (6) corresponding to the total cost function can be rearranged as

$$t \geq \sum_{i=1}^{N_G} C_{Gi} + \sum_{i=1}^{N_P} \sum_{k} c_{ik}\lambda_{ik} = \sum_{i=1}^{N_G} C_{Gi} + c^T \lambda \tag{11}$$

where c and λ are vectors with appropriate dimension. To model the volatility in the price of the purchased power, assume that the vector c of the coefficients is affected by random perturbations:

$$c = c^n + \zeta \tag{12}$$

where c^n is the vector of forecasted coefficients and ζ is a random perturbation vector with zero mean and covariance matrix V_c. In this case, $c^T\lambda$ becomes a random variable with expected value $(c^n)^T\lambda$ and standard deviation $\sqrt{\lambda^T V_c \lambda}$ [15]. To reduce risk, a safe version of (6) would be

$$t \geq \sum_{i=1}^{N_G} C_{Gi} + (c^n)^T \lambda + \Omega\sqrt{\lambda^T V_c \lambda} \tag{13}$$

where Ω is a safety parameter. Using the Tchebycheff inequality [16], it follows that with a probability at least $1 - 1/\Omega^2$, t is an upper bound on the uncertain total cost function (right hand side (rhs) of equation (11) subject to random perturbations (12)) evaluated at λ:

$$\text{Prob}\left[\sum_{i=1}^{N_G} C_{Gi} + c^T\lambda \leq t\right] \geq 1 - \frac{1}{\Omega^2} \tag{14}$$

On the other hand, it can be seen that using (13) instead of (6) in the OPF formulation results in the robust counterpart of the uncertain OPF problem. In the corresponding uncertainty set [9], the cost coefficients lie in a given ellipsoid E_c:

$$\mathrm{E}_c = \left\{c^n + \Omega D_c^T u : u^T u \leq 1\right\} \text{ and } D_c^T D_c = V_c \tag{15}$$

The risk-averted OPF problem is therefore defined by the optimization program in equations (5), (13) and (7) through (10). The solution of this program for successive values of Ω increasing towards infinity results in the rhs of (14) approaching 1. The implication is that for sufficiently large values of Ω, t will always be an upper bound on the uncertain total cost. This is possible if the standard deviation of the uncertain total cost approaches zero. In fact

$$\Omega \to \infty \Rightarrow \sqrt{\lambda^T V_c \lambda} \to 0 \text{ and } t \to \sum_{i=1}^{N_G} C_{Gi} \tag{16}$$

Letting Ω vary continuously generates a curve of optimal solutions. This curve is called the efficient frontier. In section 4, it is shown that the risk-averted OPF problem is equivalent to a conic quadratic program.

3.1 Covariance Matrix

The covariance matrix V_c can be estimated from historical and predicted values by making use of the exponentially weighted moving average model [17]. In this research, it is assumed that the forecasted cost coefficients of curves pertaining to different power suppliers are independent of each other. The cost coefficients of the same curve are however highly correlated. If the correlation coefficients for each cost curve are equal to one, then [16]

$$\lambda^T V_c \lambda = \sum_{i=1}^{N_P}\left(\sum_{k=1}^{4} \sigma_{ik}\lambda_{ik}\right)^2 \tag{17}$$

where σ_{ik} is the standard deviation of the forecasted cost coefficient c_{ik}. It follows from (15) that

$$D_c = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} & & & & \\ & & & & \cdots & & & \\ & & & & & \sigma_{P1} & \sigma_{P2} & \sigma_{P3} & \sigma_{P4} \end{bmatrix} \tag{18}$$

4 SECOND-ORDER CONE PROGRAM

One way of generalizing a linear optimization problem is to include a constraint of the form

$$x \in C \tag{19}$$

in the problem definition where C is required to be a convex cone [18]. Let each element x_i of the vector x be a member of exactly one of the vectors x^t, $t = 1,\ldots,k$. Condition (19) is satisfied if each one of the vectors x^t belongs to one of the following cones:

(i) the R set (set of real numbers)

(ii) the quadratic cone:

$$C^q = \left\{x \in R^m : x_1 \geq \sqrt{\sum_{j=2}^{m} x_j^2}\right\} \tag{20}$$

(ii) the rotated quadratic cone:

$$C^r = \{x \in R^m : 2x_1x_2 \geq \sum_{j=3}^{m} x_j^2, x_1, x_2 \geq 0\} \tag{21}$$

where m is the length of vector x. Considering the above definitions, the risk-averted OPF problem is equivalent to the following conic quadratic program:

minimize t subject to (22)

safe version of total cost function

$$t \geq \sum_{i=1}^{N_G}(a_i + b_i P_i) + p^c + (c^n)^T\lambda + \Omega u \tag{23}$$

$$v = D_c\lambda,\ u \geq \sqrt{v^T v} \tag{24}$$

$$q^c = 1/2,\ w_i^c = \sqrt{c_i}P_i \tag{25}$$

$$2p^c q^c \geq \sum_{i=1}^{N_G}(w_i^c)^2,\ p^c \geq 0 \tag{26}$$

interpolatory variable constraints

$$P_i = \sum_k p_{ik}\lambda_{ik},\ \sum_k \lambda_{ik} = 1,\ \lambda_{ik} \geq 0 \tag{27}$$

second-order network model

$$P_i - P_{Di} + \sum_{j\in k(i)} a_{ij}(\delta_i - \delta_j) = p_i \quad (28)$$

$$q_i = 1/2\,,\; w_{ij} = \sqrt{\hat{g}_{ij}/2}(\delta_i - \delta_j) \quad (29)$$

$$2p_i q_i \geq \sum_{j\in k(i)} w_{ij}^2,\; p_i \geq 0 \quad (30)$$

line flow limits

$$-\overline{T_{ij}} \leq T_{ij} = -a_{ij}(\delta_i - \delta_j) \leq \overline{T_{ij}} \quad (31)$$

power generation / purchase limits

$$\underline{P_i} \leq P_i \leq \overline{P_i} \quad (32)$$

Note that the R set is not specified explicitly, because any variable which does not belong to one of the quadratic or rotated quadratic cones is a member of this cone. An advantage of this formulation is that one can benefit from the time and effort that has been put into high-powered software tools like MOSEK to solve the conic quadratic optimization problem [13]. It has been shown that primal-dual interior-point methods developed for linear optimization can be generalized to the conic quadratic case while maintaining their efficiency [18]. Indeed, the MOSEK implementation [13] is based on the best known theoretical algorithm. Moreover, it can reliably detect problem infeasibility by virtue of using the homogeneous model.

5 NUMERICAL RESULTS

Numerical simulations were performed on two test systems: a 5-bus system and a 30-bus system.

5.1 5-bus system

The network capacity, line and load data for the 5-bus test system shown in Figure 3 were obtained from [1]. For this system, line resistances were not specified and consequently the network losses were neglected. The characteristics of the local generators connected at bus-bars 3 and 5 are given in Table 1 [1]. There are two offers from neighboring utilities, each having a predicted cost curve as shown in Figure 2. The breakpoints of the cost curves of purchased power are given in Table 2. To model the uncertainty in the prices, the costs in Table 2 are assumed to be the forecasted values. The standard deviation of the cost coefficients are given as 15 and 11% of the forecasted costs of power purchased from utilities connected to bus-bars 1 and 2, respectively. These values are consistent with the typical price forecast error [19]. The corresponding D_c is

$$D_c = \begin{bmatrix} 0 & 29.69 & 66.28 & 103.12 & & & & \\ & & & & 0 & 8.37 & 18.09 & 27.50 \end{bmatrix}.$$

To obtain the optimal amount of locally generated and purchased power, the conic optimizer [13] was used to solve the risk-averted OPF program defined by equations (22) through (32).

bus	a ($/h)	b ($/MWh)	c ($/MW^{2}h)	P_{max} (MW)
3	0	1	0.008	130
5	0	3	0.015	110

Table 1: Characteristics of local generators (5-bus system)

bus	p_{i1} (MW)	p_{i2} (MW)	p_{i3} (MW)	p_{i4} (MW)
1	0	87.5	175	250
2	0	35	70	100
bus	**c_{i1} ($/h)**	**c_{i2} ($/h)**	**c_{i3} ($/h)**	**c_{i4} ($/h)**
1	0.00	197.97	441.88	687.50
2	0.00	76.13	164.50	250.00

Table 2: Cost curves of purchased power (5-bus system)

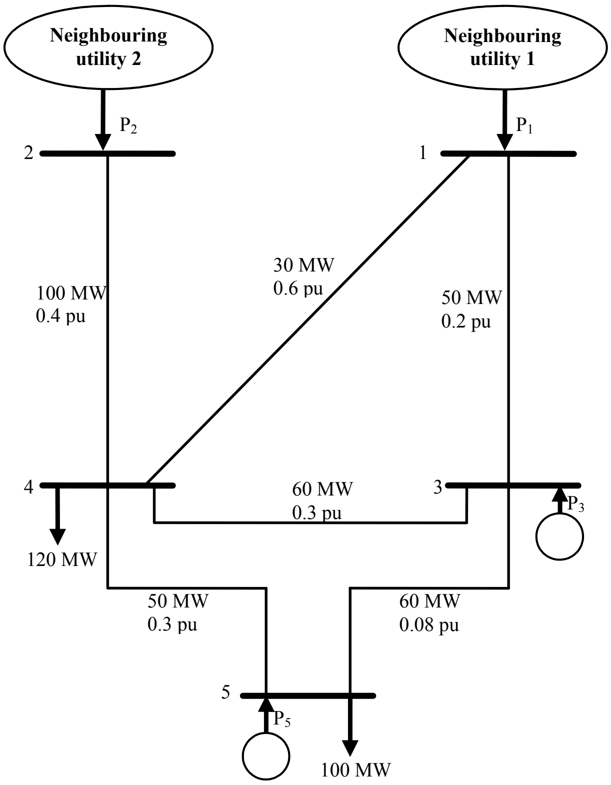

Figure 3: 5-bus test system

Table 3 shows the optimal generation dispatch for several choices of the safety parameter Ω. In this table, the values of mean cost and standard deviation are in $/h. It is seen that for a sufficiently large value of Ω (Ω = 18), the standard deviation becomes zero and the mean cost approaches the cost of dispatch from local generators only. The behavior of the optimal solution as a function of Ω is presented in Figure 4. Three characteristics are shown in Figure 4: the mean cost, the standard deviation and the robust optimal cost (rhs of (13)). Recall that for Ω = 6, the Tchebycheff bound insures that there is a maximum of 2.8% chance that the actual cost exceeds the robust cost of 565.35 $/h. For this values of Ω, the average cost increases by 8.4% compared to the schedule for Ω = 0.

The mean profit due to power purchase for a certain value of Ω is defined by:

mean profit(Ω) = mean cost(Ω→∞) – mean cost(Ω)

The corresponding values of mean profit and standard deviation are plotted in Figure 5. This curve is the efficient frontier. Any generation dispatch that produces a mean-variance combination that does not lie on the efficient frontier can be improved either by increasing its mean without changing the variance or by decreasing the variance without changing the mean [20]. Therefore, the system operator should only consider generation schedules that lie on the efficient frontier.

Ω	P_1 MW	P_2 MW	P_3 MW	P_5 MW	mean cost	std. dev.
0	17.4	100.0	65.3	37.3	521.60	28.13
1	26.9	71.5	73.3	48.3	525.87	20.69
2	20.7	70.0	79.7	49.6	527.58	19.41
3	17.6	63.4	86.3	52.7	533.32	17.33
4	14.9	44.5	99.6	60.9	551.46	12.13
5	11.3	35.0	108.4	65.3	564.05	9.21
6	9.5	35.0	110.0	65.5	565.36	8.97
7	7.7	30.6	114.2	67.6	573.55	7.77
8	4.1	14.8	126.5	74.6	603.27	3.80
9	3.2	10.1	130.0	76.7	612.61	2.66
10	3.4	9.8	130.0	76.8	613.04	2.61
11	3.6	9.5	130.0	76.9	613.38	2.58
12	3.8	9.2	130.0	77.0	613.65	2.56
13	3.9	9.1	130.0	77.0	613.88	2.54
14	4.0	8.9	130.0	77.1	614.07	2.53
15	4.1	8.8	130.0	77.1	614.23	2.51
16	4.0	8.3	130.0	77.8	616.13	2.39
17	1.9	3.9	130.0	84.3	637.18	1.12
18	0.0	0.0	130.0	90.0	656.70	0.00

Table 3: Optimal dispatch for several choices of Ω

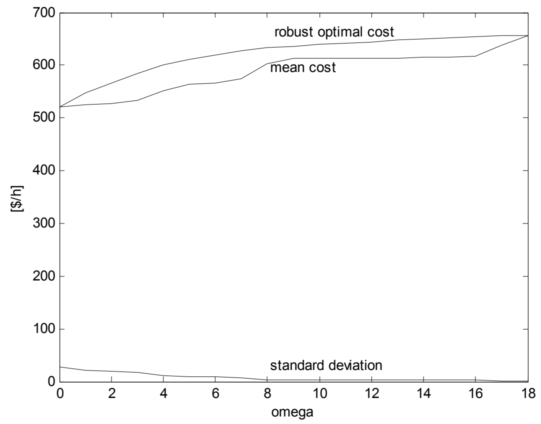

Figure 4: Characteristics of the Optimal Policy (5-bus)

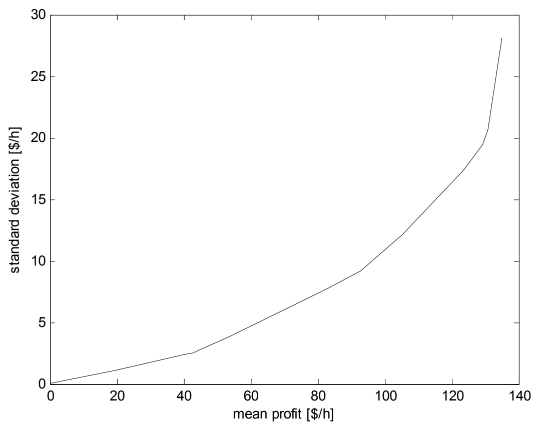

Figure 5: Efficient Frontier (5-bus)

To verify the above results, a Monte-Carlo simulation was used under the assumption that the cost coefficients of purchased power are normally distributed with the previously indicated mean and covariance matrix. Running 100,000 simulations (for each value of Ω) resulted in the statistical values in Table 4. The values in columns 3 and 5 are in agreement with the mean and standard deviation in Table 3, which were computed by the conic optimizer. It is evident from the results in Table 4 that the optimal solution for Ω = 0 results in a generation dispatch which is much more risky, although on average, better than any of the schedules given by a solution for which Ω>0. As an example, Figures 6 and 7 show the distributions of the total cost for Ω = 0 and Ω = 6, respectively.

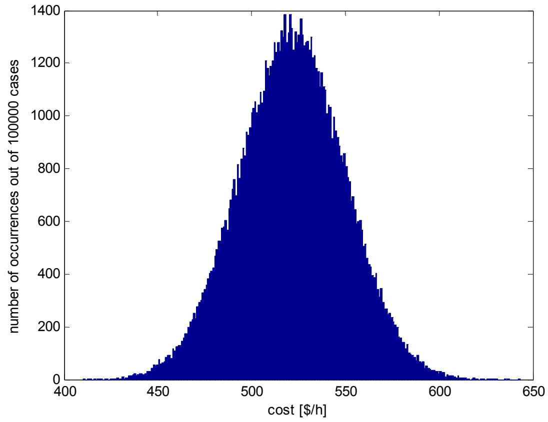

Figure 6: Histogram of total cost for Ω = 0

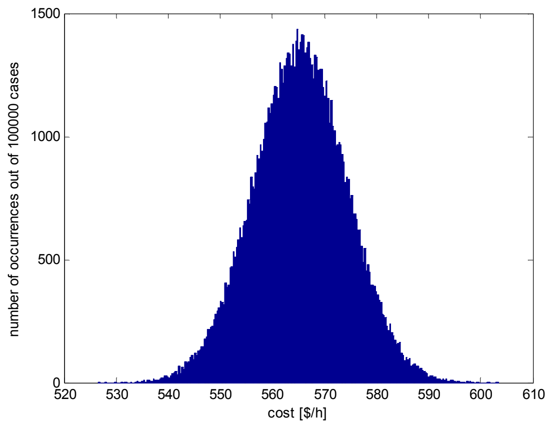

Figure 7: Histogram of total cost for Ω = 6

Ω	min ($/h)	mean ($/h)	max ($/h)	std. dev. ($/h)
0	410.25	521.61	643.26	28.12
3	456.98	533.36	609.72	17.30
6	526.59	565.34	603.40	9.00
9	600.95	612.60	623.90	2.66
12	603.14	613.65	624.61	2.56
15	603.38	614.22	624.82	2.51
18	656.70	656.70	656.70	0.00

Table 4: Total cost statistics for the 5-bus test system

5.2 30-bus system

Similar numerical simulations were carried out for the 30-bus system [21]. In this case, line losses were accounted for using the second-order network model. The utility has three local generators with sufficient capacity to serve the total load of 283.4 MW. As shown in Figure 8, power purchase is also possible from three neighboring utilities connected at bus-bars 5, 11, and 13. The cost curves of local generators and of purchased power are given in Tables 5 and 6, respectively. For the forecasted cost coefficients in Table 6, the standard deviations are 8 % (at bus 5), 10% (at bus 11), and 12% (at bus 13). The remaining data required for simulation can be found in [21]. The corresponding characteristics of the optimal policy and the efficient frontier are given in Figure 9 and 10, respectively. As an example, $\Omega = 16$ results in an average profit of 90.81 \$/h and a standard deviation equal to 4.85 \$/h. Assuming that the actual cost coefficients are normally distributed and running 100,000 simulations results in the following:

Minimum profit = 69.38 \$/h
Mean profit = 90.81 \$/h
Maximum profit = 111.54 \$/h
Standard deviation = 4.85 \$/h

A system operator looking for higher average profit would choose a lower value of Ω.

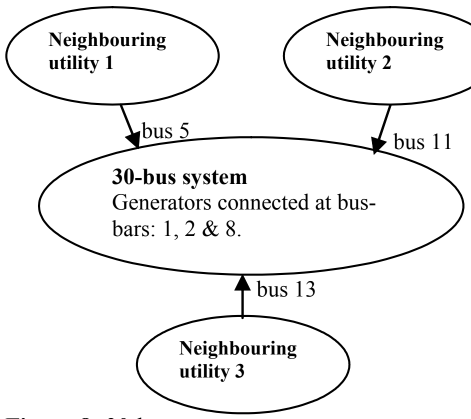

Figure 8: 30-bus test system

bus	a ($/h)	b ($/MWh)	c ($/MW^{2}h)	P_{min} (MW)	P_{max} (MW)
1	0	2.00	0.00375	50	200
2	0	1.75	0.01750	20	80
8	0	3.25	0.00834	10	35

Table 5: Characteristics of local generators (30-bus system)

bus	p_{i1} (MW)	p_{i2} (MW)	p_{i3} (MW)	p_{i4} (MW)
5	0	36.64	111.56	206.25
11	0	16.30	33.71	49.50
13	0	28.78	59.14	86.40
bus	c_{i1} ($/h)	c_{i2} ($/h)	c_{i3} ($/h)	c_{i4} ($/h)
5	0	36.64	111.56	206.25
11	0	34.26	74.03	112.50
13	0	46.90	103.60	160.00

Table 6: Cost curves of purchased power (30-bus system)

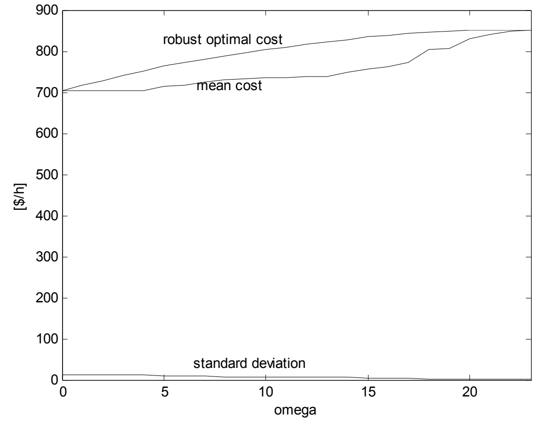

Figure 9: Characteristics of the Optimal Policy (30-bus)

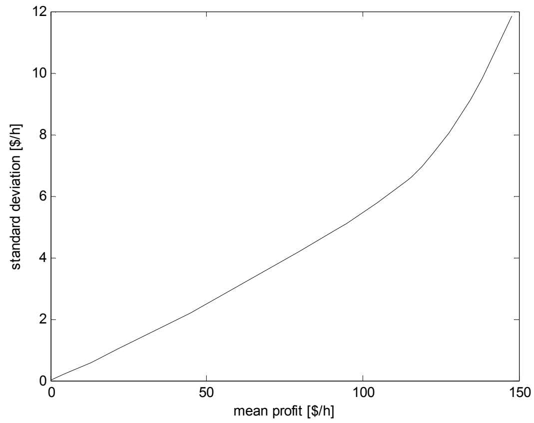

Figure 10: Efficient Frontier (30-bus)

6 CONCLUSION

Decision making under uncertainty has always been a concern in the electric power systems industry. This paper addresses the problem of evaluating power purchases when prices are uncertain. The cost coefficients of power purchase are specified by their mean and variance thus yielding an ellipsoidal uncertainty set. A model for analyzing trade-off between profit and risk faced by a power utility is considered. This model is formulated as second-order cone program and solved using a commercially available interior-point software package. The cone program not only accounts for the ellipsoidal uncertainty, but also includes a quite accurate representation of the power network. Simulation results are reported on a 5-bus and a 30-bus test system. A Monte-Carlo simulation is used to verify the dependence of expected profit on the level of risk.

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