HIGHER-ORDER NORMAL FORMS ANALYSIS OF STRESSED POWER SYSTEMS: A NON-RECURSIVE APPROACH

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Abstract— In this paper, a new procedure based on a modified normal form (NF) approach is proposed for determining the effects of the higher-order nonlinear terms of the power system representation on system dynamic performance.

A general nonlinear analysis technique which avoids the use of center manifold reduction is first developed for calculating the NF representation and the associated nonlinear transformations for resonant systems. Using this representation, analytical expressions are obtained that provide approximate solutions to system performances and indices for interpreting nonlinearity in terms of modal interaction are given. The derived formulations are suitable for the analysis of practical systems and result in more accurate solutions than existing procedures.

The application of these procedures is illustrated on a 68-bus, 16-machine model of the NPCC system. The efficiency and accuracy of this approach is demonstrated by comparisons to fully nonlinear computations.

Keywords: Normal form analysis, nonlinear modal interaction

1 INTRODUCTION

Nonlinear power system analysis by means of perturbation theory has been the subject of considerable interest over the years. The mathematical analysis of nonlinear system behavior begins with the derivation of a nonlinear system model obtained by approximating the center manifold of the power system model at an equilibrium point by a truncated power series [1].

Conventional analysis techniques which are based on a second-order approximation of the system model may fall short of providing adequate information for the detection and quantification of nonlinear behavior. In systems where nonlinearities are strong, or highly stressed, the higher degree terms can not be neglected, and the low-order approximation may yield inaccurate results.

Recent studies suggest that higher dimensional representations may be needed to fully extract system nonlinear power system behavior especially under heavy stress operating conditions [2,3]. The approximation of nonlinear systems to higher degrees by linear systems has been treated in [4] and more recently in [5] using NF analysis.

In the literature several approaches to this problem have been developed. The techniques used for the determination of the NF system representation can be broadly classified into recursive [6] and non-recursive depending on whether center manifold reduction is used or not [7]. The latter approach is explored here.

The present work builds upon the NF procedure developed by Martínez *et al.* [5] and the non-recursive approached developed in [7]. This approach has been successfully used for the analysis of second-order degree systems exhibiting the inter-area mode phenomenon. The extension of this approach to deal with higher-dimensional systems, however, is very challenging.

A general nonlinear analysis technique which avoids the use of center manifold reduction is first developed for calculating the NF representation and the associated nonlinear transformations for resonant systems. Using this representation, analytical expressions are obtained that provide approximate solutions to system performances and indices for interpreting nonlinearity in terms of modal interaction are given. The derived formulations are suitable for the analysis of practical systems and result in more accurate solutions than existing procedures.

The application of these procedures is illustrated on 68-bus, 16-machine model of the NPCC system. Attention is restricted to the study of the influence of third-order effects in the series expansion of the power system representation on system behavior, but the theory and analysis methods can be easily generalized to account for more general system models.

This technique is shown to be effective and useful for nonlinear modeling even though some practical limitations arise when the order of the system is too large

2 HIGHER ORDER NF ANALYSIS

The method of NF enables to study the behavior of a vector field near a singularity by reducing it via a suitable change of coordinates to a simpler form.

Consider an n-dimensional nonlinear dynamical system described by the differential equation

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \tag{1}$$

Without loss of generality, we assume that $\mathbf{f}$ can be expressed as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) = \mathbf{f}_1(\mathbf{x}) + \sum_{k=2}^{q} \mathbf{f}_k(\mathbf{x}) + O(q+1) \tag{2}$$

where $\mathbf{x}$ is an n-dimensional vector of system states; $\mathbf{f}_1(\mathbf{x})$ contains the linear part of the original vector field and $\mathbf{f}_k(x)$, $k = 2,...,q$ contains the nonlinear part; q represents the desired order of approximation and $O(q+1)$ are the terms in $\mathbf{x}$ of order $q+1$ and higher. It is assumed that the function $\mathbf{f}$ is continually differentiable up to order q, and that the system in (1) has an equilibrium point at $\mathbf{x} = \mathbf{x}^o$, such that $\mathbf{f}(\mathbf{x}^o) = \mathbf{0}$. Higher order NF methods may be obtained from this basic representation, but the complexity of the resulting model depends directly on the structure of the fundamental model.

2.1 Reduction to the NF: Conventional Approach

Existing approaches to higher normal form analysis are based on recursive computation of the NF coefficients and the associated nonlinear transformations. Conventional NF analysis (CNF) uses the k-th order near identity transformation to remove the k-th order nonlinear terms. The derivation of higher-order terms is described in [5,6] and is summarized here for completeness.

To obtain the NF system the following steps are taken:

1). Substitute the linear transformation $\mathbf{x} = \mathbf{U}\mathbf{y}$ into (2). Upon this transformation the system in (2) becomes

$$\dot{\mathbf{y}} = \mathbf{f}(\mathbf{y}) = \mathbf{\Lambda}\mathbf{y} + \sum_{k=2}^{q} \mathbf{F}_k(\mathbf{y}) + O(q+1) \qquad (3)$$

where $\mathbf{y} \in C^n$ is the vector of Jordan form coordinates; matrix $\mathbf{\Lambda} = \mathbf{U}^{-1}\mathbf{f}_1\mathbf{U}$ is the diagonal matrix of system eigenvalues; $O(q+1)$ are the terms in $\mathbf{y}$ of order higher to $q+1$ and the vectors $\mathbf{F}_k(\mathbf{y}) = \mathbf{U}^{-1}\mathbf{f}_k(\mathbf{U}\mathbf{y})$ are complex-valued polynomial vectors of order k in $\mathbf{y}$.

2). Use the kth-order nonlinear transformation to simplify the kth-order nonlinear terms of the system. Starting with $k = 2$ consider a formal nonlinear near-identity coordinate change of the form

$$\mathbf{y} = \mathbf{z}_2 + \mathbf{h}_2(\mathbf{z}_2)$$

where $\mathbf{z}$ is the vector of normal form coordinates and $\mathbf{h}_2(\mathbf{z}_2)$ is a complex vector-valued function whose components are homogeneous polynomials of order 2 with coefficients to be determined so that the system in (3) becomes as simple.

The vector field (3) now becomes

$$\dot{\mathbf{z}}_2 = \mathbf{\Lambda}\mathbf{z}_2 + \hat{\mathbf{F}}_2(\mathbf{z}_2) + \sum_{k=3}^{q} \hat{\mathbf{F}}_k(\mathbf{z}_2) \qquad (4)$$

where the first few terms are given by

$$\begin{aligned}
&\hat{\mathbf{F}}_2(\mathbf{z}_2) = \mathbf{F}_2(\mathbf{z}_2) + \mathbf{\Lambda}\mathbf{h}_2(\mathbf{z}_2) - D\mathbf{h}_2(\mathbf{z}_2)\mathbf{\Lambda}\mathbf{h}_2(\mathbf{z}_2) \\
&\hat{\mathbf{F}}_3(\mathbf{z}_2) = \mathbf{F}_3(\mathbf{z}_2) - D\mathbf{h}_2(\mathbf{z}_2)\mathbf{\Lambda}\mathbf{h}_2(\mathbf{z}_2) - D\mathbf{h}_2(\mathbf{z}_2)\mathbf{F}_2(\mathbf{z}_2) \\
&\qquad + \left(D\mathbf{h}_2(\mathbf{z}_2)\right)^2 \mathbf{\Lambda}\mathbf{z}_2 \\
&\hat{\mathbf{F}}_4(\mathbf{z}_2) = \mathbf{F}_4(\mathbf{z}_2) + (D\mathbf{h}_2(\mathbf{z}_2))^2 \mathbf{\Lambda}\mathbf{h}_2(\mathbf{z}_2) - D\mathbf{h}_2(\mathbf{z}_2)\mathbf{F}_3(\mathbf{z}_2) \\
&\qquad + \left(D\mathbf{h}_2(\mathbf{z}_2)\right)^2 \mathbf{F}_2(\mathbf{z}_2) \\
&\cdots
\end{aligned}$$

and use has been made of the approximation

$$[\mathbf{I} + D\mathbf{h}_2(\mathbf{z}_2)]^{-1} \approx \mathbf{I} - D\mathbf{h}_2(\mathbf{z}_2) + [\,D\mathbf{h}_2(\mathbf{z}_2)\,]^2 \qquad (5)$$

3). Determine the second order normal form coefficients, $\mathbf{h}_2(\mathbf{z}_2)$, by solving the homological equation $\mathbf{F}_2(\mathbf{z}) = D\mathbf{h}_2(\mathbf{z}_2)\mathbf{\Lambda}\mathbf{z}_2 - \mathbf{\Lambda}\mathbf{h}_2(\mathbf{z}_2)$. Under this approximation the second order NF system becomes

$$\dot{\mathbf{z}}_2 = \mathbf{\Lambda}\mathbf{z}_2 + \mathbf{F}_2^r(\mathbf{z}_2) + \hat{\mathbf{F}}_3(\mathbf{z}_2) + O(4) \qquad (6)$$

where the term $\hat{\mathbf{F}}_3(\mathbf{z}_2)$ indicates third-order terms that have been affected by the transformation in Eq. (4), $\mathbf{F}_2^r(\mathbf{z}_2)$ represents the second order resonant terms and the terms $O(4)$ represent fourth-order and higher terms introduced by the nonlinear transformation.

In an analogous fashion, third- and higher-order terms are removed by using the nonlinear transformation $\mathbf{z}_{k-1} = \mathbf{z}_k + \mathbf{h}_k(\mathbf{z}_k)$, $k = 3,...,q$.

4). Repeat the process order by order and choose the components of the q-*th* transformation successively to eliminate or simplify as many terms as possible. Details of this recursive procedure are given in Ref. [4].

At q-*th* order the approximate NF representation for the system in (3) may be written as

$$\dot{\mathbf{z}}_q = \mathbf{\Lambda}\mathbf{z}_q + \sum_{k=2}^{q} \mathbf{F}_k^r(\mathbf{z}_k) + O(q+1) \qquad (7)$$

where $\mathbf{\Lambda} = diag[\lambda_1, \lambda_2, ..., \lambda_n]$, $\mathbf{F}_k^r$ are resonance terms that can not be eliminated by the nonlinear transformations and $O(q+1)$ denotes an expression containing residual terms in $\mathbf{z}_q$ of order $q+1$ and higher.

This approach has several disadvantages:

- At each step of the NF computation, higher order terms are generated which are not fully accounted for in successive computations.
- It is difficult to determine the exact order at which to truncate (5).

Further, finding the explicit formulas for the normal form coefficients in terms of the original nonlinear system is difficult and time consuming. To overcome these limitations we next explore the use of a non-recursive approach which avoids the center manifold reduction.

3 A NON-RECURSIVE APPROACH TO NORMAL FORM ANALYSIS

Let the system be represented by (3). Assume further that the system is truncated at order q. Consider now the nonlinear transformation

$$\mathbf{y} = \mathbf{z} + \sum_{k=2}^{q} \mathbf{h}_k(\mathbf{z}) \tag{8}$$

where, by definition,

$$\mathbf{h}_2(\mathbf{z}) = \begin{bmatrix} \sum_{l=1}^{n}\sum_{m=l}^{n} h_{2lm}^{1} z_l z_m \\ \sum_{l=1}^{n}\sum_{m=l}^{n} h_{2lm}^{2} z_l z_m \\ \vdots \\ \sum_{l=1}^{n}\sum_{m=l}^{n} h_{2lm}^{n} z_l z_m \end{bmatrix}, \mathbf{h}_3(\mathbf{z}) = \begin{bmatrix} \sum_{l=1}^{n}\sum_{m=l}^{n}\sum_{p=m}^{n} h_{3lmp}^{1} z_l z_m z_p \\ \sum_{l=1}^{n}\sum_{m=l}^{n}\sum_{p=m}^{n} h_{3lmp}^{2} z_l z_m z_p \\ \vdots \\ \sum_{l=1}^{n}\sum_{m=l}^{n}\sum_{p=m}^{n} h_{3lmp}^{n} z_l z_m z_p \end{bmatrix}$$

$$,\dots$$

Introducing (8) in (3) gives the normal form system

$$[\mathbf{I} + D\sum_{k=2}^{q} \mathbf{h}_k(\mathbf{z})]\dot{\mathbf{z}} = \mathbf{\Lambda z} + \mathbf{\Lambda}\sum_{k=2}^{q} \mathbf{h}_k(\mathbf{z}) + \sum_{k=2}^{q} \hat{\mathbf{F}}_k(\mathbf{z}) + O(q+1) \tag{9}$$

In conventional normal form theory, the nonlinear transformation coefficients, $\mathbf{h}_k$, in (8), are obtained by approximating the term $[\mathbf{I} + D\sum_{k=2}^{q} \mathbf{h}_k(\mathbf{z})]^{-1}$ by a truncated power series [4]. A more accurate approach is obtained herein by using Eq. (7) instead. Substituting Eq. (7) in (9) and rearranging, one obtains the modified homological equation

$$\begin{aligned} &\sum_{k=2}^{q} \mathbf{F}_k^r(\mathbf{z}) + \mathbf{\Lambda}\sum_{k=2}^{q} \mathbf{h}_k(\mathbf{z}) = \\ &\sum_{k=2}^{q} \hat{\mathbf{F}}_k(\mathbf{z}) + D\sum_{k=2}^{q} \mathbf{h}_k(\mathbf{z})\mathbf{\Lambda z} + D_z\sum_{k=2}^{q} \mathbf{h}_k(\mathbf{z})\sum_{k=2}^{q} \mathbf{F}_k^r(\mathbf{z}) \end{aligned} \tag{10}$$

Carrying out the operations indicated in (10), and collecting terms up to third order degree, we obtain

$$\mathbf{F}_2^r(\mathbf{z}) = \hat{\mathbf{F}}_2(\mathbf{z}) - D\mathbf{h}_2(\mathbf{z})\mathbf{\Lambda z} + \mathbf{\Lambda h}_2(\mathbf{z})$$

$$\mathbf{F}_3^r(\mathbf{z}) = \hat{\mathbf{F}}_3(\mathbf{z}) - D\mathbf{h}_3(\mathbf{z})\mathbf{\Lambda z} + \mathbf{\Lambda h}_3(\mathbf{z}) - D\mathbf{h}_2(\mathbf{z})\mathbf{F}_2^r(\mathbf{z})$$

or, equivalently,

$$\begin{aligned} \mathbf{F}_j^r(\mathbf{z}) = &\hat{\mathbf{F}}_j(\mathbf{z}) - D\mathbf{h}_j(\mathbf{z})\mathbf{\Lambda z} + \mathbf{\Lambda h}_j(\mathbf{z}) \\ &- \sum_{k=2}^{j-1} D\mathbf{h}_k(\mathbf{z})\mathbf{F}_{j-(k-1)}^r(\mathbf{z}) \end{aligned} \tag{11}$$

for *j=2,3*, where $\sum_{k=2}^{j-1} D\mathbf{h}_k(\mathbf{z})\mathbf{F}_{j-(k-1)}^r(\mathbf{z})$, is a residual term which contains information regarding the resonant terms that can not be annihilated by the previous nonlinear coordinate change, $\mathbf{h}_{j-1}$. We remark that these terms arise only for third (and higher) NF representations.

From (11) the terms of the transformation higher to three are dependent upon the resonance conditions of previous order.

Equating terms of like order, the corresponding second and third order resonant terms become

$$\begin{aligned} r_{2lm}^{j} &= [\lambda_j - (\lambda_l + \lambda_m)]h_{2lm}^{j} + \hat{c}_{2lm}^{j} \\ r_{3lmp}^{j} &= [\lambda_j - (\lambda_l + \lambda_m + \lambda_p)]h_{3lmp}^{j} + \hat{c}_{3lmp}^{j} + \hat{r}_{3lmp}^{j} \end{aligned} \tag{12}$$

In the equations above, r^j_{2lm} and r^j_{3lmp} are the coefficients of the resonant polynomials $\mathbf{F}_2^r(\mathbf{z})$, and $\mathbf{F}_3^r(\mathbf{z})$ respectively; the terms $\hat{c}_{2lm}^{j}$ and $\hat{c}_{3lmp}^{j}$ are the coefficients of the polynomials $\hat{\mathbf{F}}_2(\mathbf{z})$ and $\hat{\mathbf{F}}_3(\mathbf{z})$ respectively, and $\hat{r}_{3lmp}^{j}$ is a coefficient of the residual resonant polynomial $D\mathbf{h}_2(\mathbf{z})\mathbf{F}_2^r(\mathbf{z})$.

Note that, in this approach, nonlinear terms in the equations are assumed to be small but not negligible. Essentially, the technique is able to give an enhanced, more accurate, third-order estimate of the system solution.

In the particular case of non-resonance conditions, i.e., $\lambda_j \neq (\lambda_l + \lambda_m)$ and $\lambda_j \neq (\lambda_l + \lambda_m + \lambda_p)$, the terms of the transformation for the j-th state are determined in conventional form, and are given by

$$h_2^j(\mathbf{z}) = \sum_{l=1}^{n}\sum_{m=l}^{n} \frac{\hat{c}_{2lm}^{j}}{\lambda_l + \lambda_m - \lambda_j} z_l z_m \tag{13}$$

and

$$h_3^j(\mathbf{z}) = \sum_{l=1}^{n}\sum_{m=l}^{n}\sum_{p=m}^{n} \frac{\hat{c}_{3lmp}^{j}}{\lambda_l + \lambda_m + \lambda_p - \lambda_j} z_l z_m z_p \tag{14}$$

This agrees with the results obtained in [4,9] for the particular case in which the nonlinear terms are neglected.

3.1 Determination of the third order NF transformation under resonance conditions

The analysis above shows that it is possible to determine the NF transformation coefficients in terms of the resonance conditions and the previously determined second-order relationships.

Three cases are of particular interest in this study: second order resonances, third order resonances and the simultaneous occurrence of second and third-order resonance conditions.

For the first case, $\lambda_j = (\lambda_l + \lambda_m)$ and $\lambda_j \neq (\lambda_l + \lambda_m + \lambda_p)$. Assume that the above conditions hold the corresponding terms of the second and third-order transformation are then

$$\begin{aligned} h_{2lm}^{j} &= 0 \\ r_{2lm}^{j} &= c_{2lm}^{j} \\ \hat{r}_{3lmp}^{j} &= h_{2lp}^{j} c_{2lm}^{j} \\ h_{3lmp}^{j} &= \frac{\hat{c}_{3lmp}^{j} + \hat{r}_{3lmp}^{j}}{[(\lambda_l + \lambda_m + \lambda_p) - \lambda_j]} \\ r_{3lmp}^{j} &= 0 \end{aligned} \tag{15}$$

Note that, in this representation, second-order resonant terms give rise to third order-terms, $\hat{r}^{j}_{3lmp}$, which are accounted for by the respective transformation. The other two cases of resonance are treated in a similar manner.

3.2 Analysis of residual terms

The residual terms in (11) arise from the use of the second-order nonlinear transformation (8) in the NF procedure. In the derived formulation, the third-order terms in (11) are of the form

$$\hat{\mathbf{F}}_3(\mathbf{z}) = \mathbf{F}_3(\mathbf{z}) + 2\mathbf{z}^T \mathbf{F}_2 \mathbf{h}_2(\mathbf{z}) \tag{16}$$

To clarify the nature of these terms, Table 1 compares the expressions for third-order residual terms using CNF theory as a function of the approximate expression (5).

Observe that, by neglecting higher-order terms, $[D\mathbf{h}_2(\mathbf{z})]^2$, important information may be lost. Whilst increasing the number of terms in the series expansion (5) increases accuracy, this becomes highly unpractical especially as the dimension of the system increases. This limits the accuracy of the conventional approach to assess dynamic behavior under stressed conditions.

CNF theory	Third order terms
$\mathbf{I} - D\mathbf{h}_2(\mathbf{z})$	$\hat{\mathbf{F}}_3(\mathbf{z}) = \mathbf{F}_3(\mathbf{z}) + 2\mathbf{z}^T \mathbf{F}_2 \mathbf{h}_2(\mathbf{z}) - D\mathbf{h}_2(\mathbf{z})\mathbf{F}_2(\mathbf{z})$
$\mathbf{I} - D\mathbf{h}_2(\mathbf{z}) + [D\mathbf{h}_2(\mathbf{z})]^2$	$\hat{\mathbf{F}}_3(\mathbf{z}) = \mathbf{F}_3(\mathbf{z}) + 2\mathbf{z}^T \mathbf{F}_2 \mathbf{h}_2(\mathbf{z}) - D\mathbf{h}_2(\mathbf{z})\mathbf{F}_2(\mathbf{z}) + [D\mathbf{h}_2(\mathbf{z})]^2\ \mathbf{\Lambda z}$

Table 1: Nature of NF coefficients as a function of (5)

The discussion of these effects postponed until section 4.

3.3 Third order closed-form time-domain solutions

Closed-form solutions are determined by following the approach in [8]. Assume that via a suitable change of coordinates the system in (3) is taken to the normal form

$$\dot{\mathbf{z}}(t) = \mathbf{\Lambda z}(t) + \mathbf{O}(4) \tag{17}$$

Neglecting higher order terms, one obtains that $z_j(t) = z_j^0 e^{\lambda_j t}$, $j = 1,\dots,n$. Substituting these solutions in (8) yields

$$\begin{aligned} y_j(t) &= z_j^0 e^{\lambda_j t} + \sum_{l=1}^{n}\sum_{m=l}^{n} h^{j}_{2lm} z_l^0 z_m^0 e^{(\lambda_l+\lambda_m)t} \\ &+ \sum_{l=1}^{n}\sum_{m=l}^{n}\sum_{p=m}^{n} h^{j}_{3lmp} z_l^0 z_m^0 z_p^0 e^{(\lambda_l+\lambda_m+\lambda_p)t} \end{aligned} \tag{18}$$

$j = 1,\dots,n$

where the terms z_j^o are the initial conditions in the normal form space.

Note that Eq. (18) reduces to the conventional theory when, in particular, $h^{i}_{3klm} = 0$ for low stress conditions.

Normal form solutions in (18) are then transformed back into the original physical domain by using the linear transformation.

$$\mathbf{x}(t) = \mathbf{U}\mathbf{y}(t) \tag{19}$$

Figure 1 illustrates the nature of the analytical solutions using this procedure.

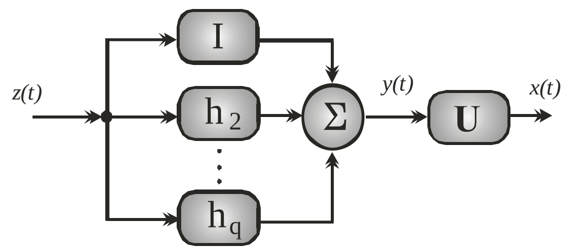


Figure 1: Block diagram illustrating the computation of closed-form solutions

The following approach is used to compute initial conditions in the $\mathbf{z}^o$ coordinates:

1. Given an initial operating condition $\mathbf{x}^o$, determine the initial conditions in the Jordan space from $\mathbf{y}^o = \mathbf{U}^{-1}\mathbf{x}^o$, and
2. Compute $\mathbf{z}^o$ by solving the nonlinear equations
   $$\mathbf{f}(\mathbf{z}^o) = \mathbf{z}^o + \mathbf{h}_2(\mathbf{z}^o) + \mathbf{h}_3(\mathbf{z}^o) - \mathbf{y}^{\mathbf{0}} = \mathbf{0}$$
   using a Newton-based iterative technique.

Once the initial conditions $\mathbf{z}^o$ have been determined, compute approximate time-domain closed-form solutions by using (18).

3.4 Quantification of higher- order nonlinearities

The proposed procedure can be used to generalize the notion of nonlinear interaction indices. Following [8,9] a third-order index of modal interaction can be defined as

$$I_3^j(z_j^o) = \max\left[\max_{l,m}\left|\frac{h_2^j(\mathbf{z}^0)}{z_j^0}\right| \ ,\ \max_{l,m,p}\left|\frac{h_3^j(\mathbf{z}^0)}{z_j^0}\right|\right] \tag{20}$$

Large solutions for the second and third parts in (18) compared to the linear part (z_j^o) are identified to exhibit strong interactions between mode j and modes l, m, and modes l, m, p, respectively in the Jordan form variables. These expressions are used in this work to assess nonlinear behavior and modal interaction.

4 SIMULATIONS OBTAINED: APPLICATION TO A LARGE POWER SYSTEM WITH CONTROLS

The normal form method was tested on the 16-machine 68-bus NPCC system [10]. A one-line diagram of the study system is given in Fig. 2 showing major coherent area.

The NPCC system consists of five coherent areas designated as Area 1 (A1), Area 2 (A2), Area 3 (A3), Area 4(A4), and Area 5 (A5).

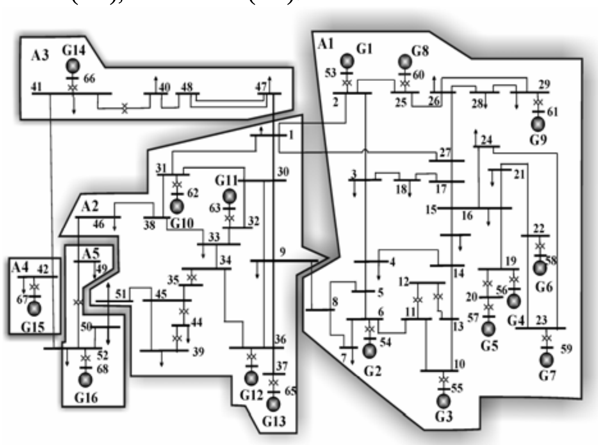


Figure 2: One line diagram of the five-area sixteen generators test system.

For the purpose of this analysis, all machines are represented by a fourth order model and a simple gain dc exciter. The system operating conditions and machine data are taken from [10].

The NPCC system exhibits four lightly damped inter-area modes. Table 2 lists the main characteristics of these modes showing their oscillation patterns and damping ratios. For the case base only exist interchange of generation from Area 5 to Area 2 through the inter-tie 50-51 and 46-49 of around of 1200 MW. In these simulations, a base case condition with modifications on major inter-tie power flows to increase system stress was used. The loads are represented as constant impedances.

4.1 Small Signal Analysis

Eigenvalue analysis of the linear system model identifies five lightly damped inter-area modes with damping ratios below 5%. Table 2 lists the five slowest eigenvalues along with swing pattern and their associated swing frequency.

Eigenvalue (Mode)	Freq. (Hz)	Dominant machines
-0.0686±j2.66 (48)	0.424	9_{A1} 13_{A2} vs 14_{A3} 15_{A4} 16_{A5}
-0.1230±j3.47 (46)	0.553	$14_{\mathbf{A3}}$ vs $16_{\mathbf{A5}}$
-0.0819±j4.62 (44)	0.735	$9_{\mathbf{A1}}$ vs $12_{\mathbf{A2}}$ $13_{\mathbf{A2}}$
-0.2360±j5.04 (42)	0.802	$14_{\mathbf{A3}}$ $15_{\mathbf{A4}}$ vs $16_{\mathbf{A5}}$
-0.2436±j6.75 (40)	1.075	$2_{\mathbf{A1}}$, $3_{\mathbf{A1}}$, $5_{\mathbf{A1}}$

Table 2: Swing pattern for the five slowest modes of the system

4.2 Third order normal form analysis

Normal form analyses were conducted to investigate the potential for nonlinear behavior arising from the interaction of the major inter-area modes as well as to assess the influence of higher order terms on system dynamic performance.

Figures 3 through 5 furnish the second –and third-order interaction indices for the five inter-area modes in Table 2, see (20). For comparison purposes, the interaction indices obtained from conventional normal form theory are presented in Figure 3.

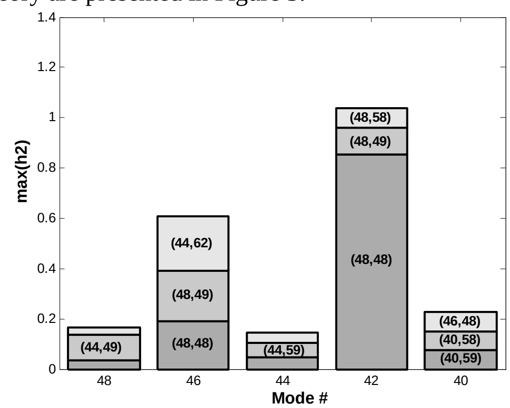


Figure 3: Second order interaccion index. CNF theory

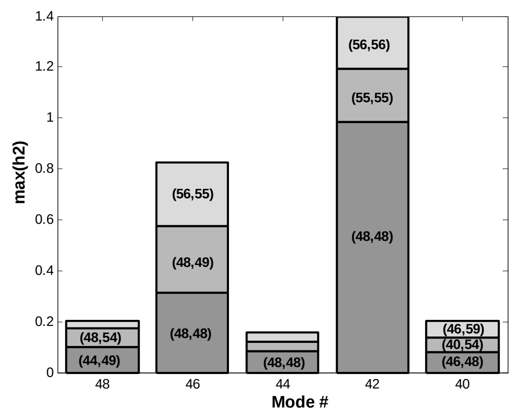


Figure 4: Second order interaccion index. Non-recursive approach.

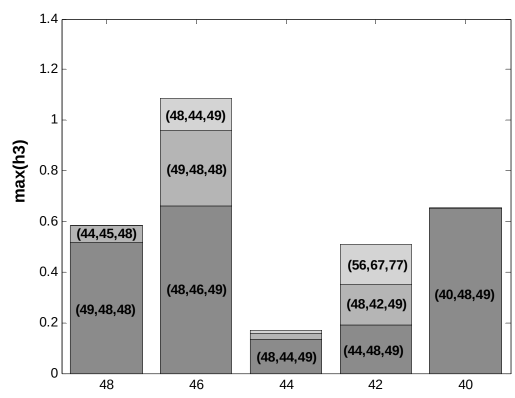


Figure 5: Third order interaccion index. Non-recursive approach

A study of the second-order nonlinear interaction indices in Figures 3 and 4 reveals a strong interaction between mode 48 and 44 and mode 56. Careful inspection of the second-order indices obtained using the proposed procedure, however, discloses the presence of control mode 55 associated with the control of the machine 15, and mode 57 involving the participation of the control states of machines 12 and 14.

The analysis of third-order interaction indices in Figure 5 confirms these findings. It should be observed that, the magnitude of third-order interactions is of the same order that second-order information showing the presence of higher-order nonlinear modal interaction and increased nonlinearity.

4.3 Validation

The method of normal forms is used to investigate system performance following large perturbations in the neighborhood of critical operating conditions obtained from linear system theory.

Based on the analysis of interaction indices, machines # 12, 14, and 15 were selected for study. Three system representations were considered in the study, namely: (1) a linear approximation $\dot{\mathbf{x}} = \mathbf{f}_1\mathbf{x}$ obtained by ignoring higher terms, (2) a second order system representation $(q = 2)$, and (3) a third-order representation $(q = 3)$ obtained from the proposed approach in this paper.

Figure 6 compares the results of the proposed high-order NF solution to the linear and second-order representations for the machines of concern. For completeness, the results are compared with those obtained from detailed step-by-step simulation (full system solution) using a commercial transient stability program.

For machine # 12, examination of the system results in Fig. 6 shows that the response is given mainly by the linear terms; the agreement between the linear solution and the NF solutions is good over the entire study period although some discrepancies are noted. It can also be seen that all solutions remain in phase; the linear solution becomes less accurate as time increases. Similar results are also obtained for other machines.

The analysis of machines #14 and 15, on the other hand, shows that higher-order NF solutions provide a more accurate approximation for the full system solution than the lower order approximations. Of particular interest, simulations results show that third-order NF solutions are in close agreement with the full solutions for the entire study period thus showing the correctness of the proposed procedures. In contrast to this, linear solutions provide a poor approximation to system behavior both in magnitude and phase. Clearly nonlinearity and nonlinear modal interaction are not uniformly distributed and may exhibit complex characteristics.

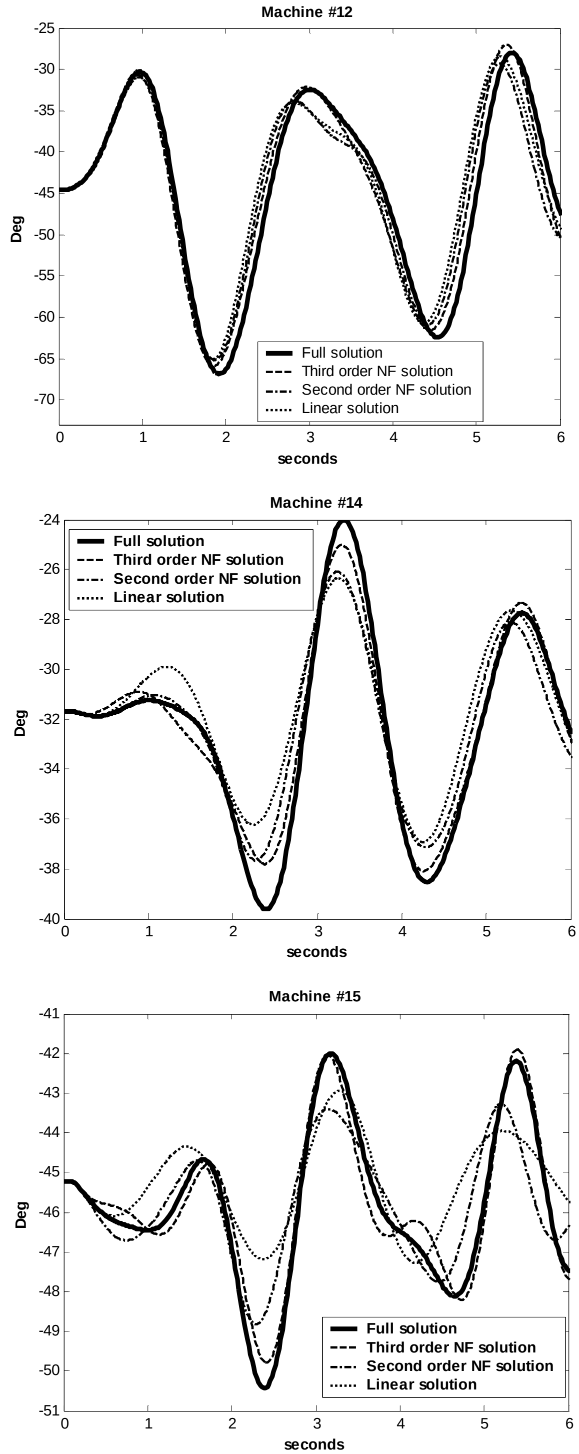
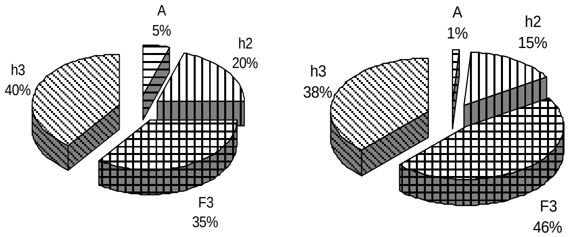


Figure 6: Comparison of relative rotor angle swings.

4.4 Computational aspects

Figure 7 compares the memory storage required for NF analysis using the conventional approach [5,6] and the proposed procedure as a percentage of the total memory usage.



a) Non-recursive b) CNF

Figure 7: Comparison of memory storage requirements.

An attractive feature of the proposed procedure is that reduces the amount of computer memory required to compute the terms $\hat{\mathbf{F}}_3$, since in the conventional approach, it is necessary to determine a larger number of residual terms; memory requirements increase substantially when the order of the transformation increases. This, in turn, causes an increase in CPU time thus limiting its applicability to the study of realistic systems.

5 CONCLUSIONS

In this paper, a systematic analytical tool based on a modified normal form approach to assess the influence of third-order order terms of the power system representation on system dynamic behavior has been proposed. The method avoids use of center manifold reductions and enables the analysis of resonant conditions.

In comparison with conventional normal form approaches, the terms in the k-th order nonlinear transformation are used to simplify not only the k-th order terms in the system, but also used to eliminate higher-order nonlinear terms. This results in an efficient non-recursive formulation that overcomes some of the limitations of existing approaches.

Apart form its simplicity, the method is though to have potentially important applications for dynamic analysis of stressed behavior in nonlinear systems. These include the computation of normal forms and the associated coefficients under resonant conditions and the study of higher-order nonlinear modal interaction.

The generalization of this approach to account for higher dimensional systems deserves further investigation and will be presented in a future paper.

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