TWO-STATION EQUIVALENT OF HYDRO POWER SYSTEMS

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Abstract - This paper studies the possible gains of representing hydropower systems with a new model consisting of two stations instead of the usual one station representation. The equivalent is obtained by solving an optimization problem where the aim is to find parameters which minimizes the difference in the production plans obtained from the two optimization problems corresponding to the original system and the equivalent respectively. This is in it self an uncommon optimization problem type. The paper also provides numerical examples.

Keywords - Hydro power, equivalents, reduced model, optimization

1 Introduction

When planning the energy production of a hydropower system or simulating such a system, good models are essential. However, using too exact models can result in large mathematical problems which are simply too time consuming to solve. In, e.g. Sweden there are about 200 hydropower plants with an installed capacity greater than 10 MW. When considering systems of this magnitude or greater, a common technique is to reduce the size of the problem by representing plants in the same river system or in several river systems with a single aggregated plant. Although this is effective in reducing the size of the problem, the approach has its drawbacks. The model only deals with the total energy in the system and the total production of the system and therefore fails to accurately represent the bottlenecks within the system. For example, if all water inflow to the reservoirs in the river is stored, sooner or later one of them will be spilled and energy will be forced to be produced in the hydro plant with the full reservoir. Unless all reservoirs are filled simultaneously, the aggregated plant will not be forced to produce energy since the total system is not full. This illustrates a case when the result from using a simpler model might diverge from the result obtained with a more detailed model.

The aim of this paper is to examine the possible gains of using a model with not one but two stations linked together to represent a larger system. For example, the aggregated representation could consist of two plants with one large and one small reservoir compared to the inflow of water. This would result in a model, which more often would be forced to produce energy because of the small reservoir.

The aim of the two-station equivalent is to make a model that gives a production plan as close as possible to that of the original system, when supplied with the same input data. This data being the spot price of energy each time-step during the planning period and a value for stored energy at the end of the planning period. The idea used in the paper is to try to find values for the design flows and the storage capacities of two virtual plants, which achieves this. This will be formulated as an optimization problem where the aim is to find parameters which minimizes the difference between the result of two optimization problems.

There is not so much published concerning equivalents for hydro system, but there are some papers. The most common method is to make a one-station representation of a given system (see [1] or [2]). The station representing the whole system receives, stores and releases potential energy instead of water. At each hydroplant in the original system, the water stored there is converted to its total energy generating capability. Total generating capability takes into account that discharged water at each plant of the actual system also can be used to generate energy at plants located downstream. The sum of all the generating capabilities is the total energy in the one-station equivalent. In the same way, the inflows and outflows from the system is converted to potential energy. The model could then be completed by a statistical description of energy inflow and a generation function, which based on statistical data, converts the one-stations released potential energy into actual production [1]. In section 3, a version of this composite model will be constructed to be compared with the two-station equivalent.

In the last two decades, many commercial models for production planning in complex power systems have been developed. Two of the most widely used in the nordic electricity market, are the `EMPS` and the `EOPS` models from `SINTEF` [3]. They can be used for generation scheduling (both long- and mid-term) and are designed mainly for electrical systems with a significant part being hydropower. The minimum time step is one week and the maximum planning period is three years. The `EOPS` model is especially designed for planning by the individual producer, while the `EMPS` model is specialized for analyzing larger systems.

Since these programs are commercial, obviously no exact information is available about how the simplified models of the detailed systems are constructed. But both the `EMPS` and the `EOPS` models use, what they call, standard plant/reservoir modules for modelling hydropower. Several hydropower plants are put together to form modules. Each of these modules can then have the following characteristics (plus a few not in the list):

- a reservoir
- inflow, both storable and non-storable

- a plant, with discharge capacity and generation correction for water head

- separate destinations for plant discharge (spillage and plant discharge)

- constraints on reservoir levels and water flow

Modules of this kind can then be linked together. Although the specifics are unknown, the approach seems similar to that of the composite model, where several power plants are put together to form one single unit.

In section 2 the original hydro system model is presented and in section 3 the common one-station equivalent is examined. In section 4 the theory of the new two-station equivalent is developed and in sections 5-7 some numerical examples are shown.

2 Original system

The "original system" consists of a detailed description of the system that is to be optimized. The aim is to find a plan that maximizes profits of the system during a period of T hours. Energy can either be sold during each of the hours to the spot price corresponding to that hour or stored in the reservoirs at the end of the planning period to a specified value. Some important simplifications of reality assumed in the description here are:

- A linear model for power generation
- No delay time for water flowing from one plant to a plant directly downstream
- Deterministic inflow, spot price and water value

The linear model implies that the production planning becomes a standard linear optimization problem. Since a linear program is always convex, an optimal point for the problem is sure to be found, not necessarily unique however. The linear model for power generation is here used instead of the common used piecewise-linear to simplify the description.

The variables for this problem are u_{it}, x_{it} and s_{it}. u_{it} represents the discharge from plant i during hour t. s_{it} is the spillage from plant i during hour t. x_{it} represents the stored water in plant i at the beginning of hour t. The variables are all measured in hour equivalents, (HE or $m^3/s \cdot h$). The objective function is the total profit. $spot_t$ is the spot price during hour t (SEK/MWh). val is the value of the energy in reservoirs at the end of the planning period (SEK/MWh). γ_i is the production equivalent of plant i (MWh/HE).

With these simplifications, the production planning problem for a system of 3 stations located in a river after each other, can be expressed mathematically as the optimization problem in equation 1.

$$
\begin{aligned}
&\max \textstyle\sum_{t=1}^{T} \sum_{i=1}^{3} spot_t \gamma_i u_{it} + \\
&+val((\gamma_1+\gamma_2+\gamma_3)x_{1T+1} + (\gamma_2+\gamma_3)x_{2T+1} + \\
&+\gamma_3 x_{3T+1}) \\
\\
&x_{1t+1} = x_{1t} - u_{1t} - s_{1t} + flow_1, \\
&t \in \{1, ..., T\} \\
&x_{2t+1} = x_{2t} - u_{2t} - s_{2t} + u_{1t} + s_{1t} + flow_2 - \\
&flow_1, t \in \{1, ..., T\} \\
&x_{3t+1} = x_{3t} - u_{3t} - s_{3t} + u_{2t} + s_{2t} + flow_3 - \\
&flow_2, t \in \{1, ..., T\} \\
&x_{i1} = \delta_i store_i, \quad \forall i \\
&0 \le u_{it} \le desflow_i, \quad \forall i, \quad t \in \{1, ..., T\} \\
&0 \le x_{it} \le store_i, \quad \forall i, t \quad t \in \{1, ..., T+1\} \\
&0 \le s_{it} \le spill_i, \quad \forall i, \quad t \in \{1, ..., T\} \\
&i \in \{1, 2, 3\}
\end{aligned}
\tag{1}
$$

The first three sets of constraints set the amount of water in the reservoirs the following hour. $flow_i$ is the total inflow upstreams plant i (HE). The next set of constraints see to that the reservoirs all are filled to $\delta_i/100\%$ at the beginning of the planning period. $store_i$ is the storage capacity in reservoir i (HE). The following three sets of constraints ensures that the variables stay in their allowed domains. $desflow_i$ and $spill_i$ is the maximum level of discharge and spillage possible at plant i (HE).

There are available methods (e.g. [4, 5, 6]) that provides better plans than the one obtained with the formulation in equation 1. The aim here is though to evaluate the performance of the equivalent and because of that it is enough to use the simplified model.

3 Composite system

For a two-station equivalent to be of any use, it must give more accurate results than a simpler one-station model, such as the composite model. Because of this, a composite representation is created to serve as a reference when evaluating the accuracy of the two-station model.

The system can be transformed to a single plant containing potential energy. The level of energy is determined by how much energy the water in the system at the start of the period could produce. In the same manner, the flow of water into the original system is converted into an amount of potential energy entering the system in each period of time. This plant can then release potential energy to satisfy an objective such as maximizing profits or minimizing cost while satisfying a load. In the case of trying to maximize profits while selling energy on the spot market the result is the maximization problem shown below.

Here the variables are $Pout_t$ and E_t which represents the energy released during hour t and the energy stored in the system at the beginning of hour t. The first variable is measured in MWh/h and the latter in MWh. The parameters have already been defined in the previous section.

$$\max \textstyle\sum_{t=1}^{T} spot_t Pout_t + val E_{T+1} \tag{2}$$

$$
\begin{aligned}
&E_{t+1} = E_t - Pout_t + \textstyle\sum_i^3 flow_i\gamma_i, \quad t \in \{1,...,T\}\\
&E_1 = \textstyle\sum_i^3 \delta_i store_i \sum_{j\geq i}\gamma_j\\
&E_{max} = \textstyle\sum_i^3 store_i \sum_{j\geq i}\gamma_j\\
&P_{min} \leq Pout_t \leq \textstyle\sum_i^3 desflow_i\gamma_i, \quad t \in \{1,...,T\}\\
&0 \leq E_t \leq E_{max}, \quad \forall t \in \{1,...,T+1\}\\
&i \in \{1,2,3\}
\end{aligned}
\tag{3}
$$

Once again the objective represents the total profit. The first set of constraints give the energy in the system the following hour. The next two equations calculate the total starting energy in the system and the total storable energy in the system. The next two sets of constraints see to that no more energy can leave the system than when all plants are working at full capacity and finally that no more energy can be contained in the system than when the reservoirs are full. As stated in the equation there may be a lower limit, P_{min}, corresponding to non-storable energy in run-of-the-river plants. In the examples below this parameter will though be set to zero. Although the solution of this problem results in a production plan, it does not give a plan on how to operate each single hydro plant just the total amount of energy that all the plants combined should produce each hour.

4 Two-station equivalent

The aim of the two-station equivalent is to make a model that gives a production plan as close as possible to that of the original system, when supplied with the same input data. This data are the spot price of energy each time-step during the planning period and a value for stored energy at the end of the period. The idea used here is to try to find values for the design flows and the storage capacities of the two virtual plants which achieves this.

Given a specific spot price series and a value of stored water, the production planning problem for a setup of three adjacent hydro plants on the same river is shown in equation 1.

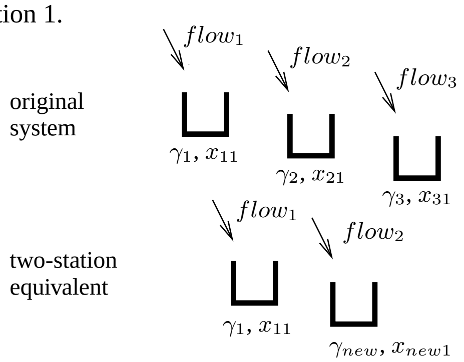

Figure 1: Formation of two-station equivalent

One way of constructing a two-station model of the system is by putting station 2 and 3 together to form a new imaginary station and removing $flow_3$. This new station receives a γ given by the formula $\gamma_{new} = \frac{flow_2\gamma_2 + flow_3\gamma_3}{flow_2}$. That way, when all the plants work as run of the river plants (i.e. plants with no reservoirs), the same amount of energy is produced in the 2-station equivalent as in the original problem ($flow_1\gamma_1 + flow_2\gamma_2 + flow_3\gamma_3 = flow_1\gamma_1 + flow_2\gamma_{new}$). Also, to preserve initial energy the starting energy in the new station x_{new1} is set according to $x_{new1} = ((\gamma_2 + \gamma_3 - \gamma_{new})x_{11} + (\gamma_2+\gamma_3)x_{21} + \gamma_3 x_{31})/\gamma_{new}$. This results in the following production planning problem for the two-station equivalent:

$$
\begin{aligned}
&\max \textstyle\sum_{t=1}^T \sum_{i=1}^2 spot_t\gamma_i u_{it} +\\
&+val((\gamma_1+\gamma_{new})x_{1T+1} + \gamma_{new}x_{newT+1})
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
&x_{1t+1} = x_{1t} - u_{1t} - s_{1t} + flow_1, \quad t \in \{1,...,T\}\\
&x_{newt+1} = x_{newt} - u_{2t} - s_{2t} + u_{1t} + s_{1t} +\\
&+flow_2 - flow_1, \quad t \in \{1,...,T\}\\
&x_{11} = \delta_1 store_1\\
&x_{new1} = ((\gamma_2+\gamma_3-\gamma_{new})\delta_1 store_1 +\\
&+(\gamma_2+\gamma_3)\delta_2 store_2 + \gamma_3\delta_3 store_3)/\gamma_{new}\\
&0 \leq u_{it} \leq \underline{?}, \quad \forall i, \quad t \in \{1,...,T\}\\
&0 \leq x_{it} \leq \underline{?}, \quad \forall i, \quad t \in \{1,...,T+1\}\\
&0 \leq s_{it} \leq spill_i, \quad \forall i, \quad t \in \{1,...,T\}\\
&i \in \{1,2\}
\end{aligned}
\tag{5}
$$

Now, the challenge is to set the design flows and the storage capacity of the two new plants, so that the model produces a production plan as similar as possible to that of the original system. The idea for finding good values in this paper is to use probable spot price curves to try to calibrate the design flows and the storage capacities of the two-station model. By finding a two-station equivalent with design flows and storage capacities that gives a production schedule as close to that of the original system as possible for the given spot curve curves, the idea is that this also will give an accurate two-station equivalent for all other reasonable spot price curves. The procedure in more detail is as follows:

1. Consider a set of probable spot price curves.
2. Solve the production planning problems for the original system and obtain the total energy production each hour corresponding to each of these spot price curves.
3. Guess values for the design flows and the storage capacities of the two-station equivalent
4. Solve the production planning problems for the two-station equivalent with these values and obtain the total energy production each hour corresponding to each of the spot price curves.
5. Calculate the difference between the energy production of the original system and that of the equivalent for each hour and spot price curve. Square these differences and calculate the sum of them.
6. Try to minimize this sum as a function of the design flows and the storage capacities of the two-station equivalent.

The sum of the squares of the differences is not the only objective function that could be chosen to represent the deviation. For example, the absolute value could have been chosen just as well, but the square rewards an "even" result, since larger deviations are penalized harder than smaller.

In the problem formulated below $\overline{u}_i$, $\overline{x}_i$ represent the maximum allowed discharge and the storage capacity of plant i for the two-station equivalent. $\overline{u}$ and $\overline{x}$ are the vectors $(\overline{u}_1 \quad \overline{u}_2)$ and $(\overline{x}_1 \quad \overline{x}_2)$. The extra index s on all variables and $spot_t$ represents the different spot price scenarios. This means that the discharge and reservoir contents can differ for different spot price curves but their maximum values can not. For each spot price curve and given values for $\overline{u}_i$, $\overline{x}_i$ a production plan (values for u_{sit} and x_{sit}) that maximizes profits is found. $Paim_{st}$ is the total energy production of the original three-station problem, during hour t with spot curve s.

The problem can now be formulated as:

$$\min_{\overline{u},\overline{x}} \sum_{st}(Paim_{st} - \sum_i \gamma_i u_{sit})^2 \tag{6}$$

$$\begin{cases} \max_{u_{sit},x_{sit},s_{sit}} \sum_{t=1}^{T}\sum_{i=1}^{2} spot_{st}\gamma_i u_{sit}+ \\ +val((\gamma_1+\gamma_{new})x_{s1T+1}+\gamma_{new}x_{snewT+1}) \\ \\ \quad x_{s1t+1} = x_{s1t} - u_{s1t} - s_{s1t} + flow_1, \\ \quad t \in \{1,...,T\} \\ \quad x_{snewt+1} = x_{snewt} - u_{s2t} - s_{s2t} + u_{s1t}+ \\ +s_{s1t} + flow_2 - flow_1, \quad t \in \{1,...,T\} \\ x_{s11} = \delta_1 store_1 \\ x_{snew1} = ((\gamma_2+\gamma_3-\gamma_{new})\delta_1 store_1 + \\ +(\gamma_2+\gamma_3)\delta_2 store_2 + \gamma_3\delta_3 store_3)/\gamma_{new} \\ \quad 0 \le u_{sit} \le \overline{u}_i, \quad \forall i, \quad t \in \{1,...,T\} \\ \quad 0 \le x_{sit} \le \overline{x}_i, \quad \forall i, \quad t \in \{1,...,T+1\} \\ \quad 0 \le s_{sit} \le spill_i, \quad \forall i, \quad t \in \{1,...,T\} \\ \quad i \in \{1,2\} \end{cases} \tag{7}$$

Although this might look like an ordinary optimization problem at first glance, it is not. It is actually several optimization problems (one for each spot curve) within an optimization problem. Unfortunately, no literature dealing with this subject has been found. Unlike multi-objective optimization where different, and often conflicting, objective functions are considered simultaneously, here we want to minimize a function that depends on several optimization problems.

To find values for design flow and maximum storage that results in a two-station equivalent that gives good approximation to the desired production plan, the following method has been applied:

1. Given starting values for $\overline{u}$ and $\overline{x}$ and a set of spot price curves, the program calculates the production plans for the equivalent and then obtains the value for the objective function.

2. The program takes a step in each direction in $\overline{u}_1$ and $\overline{u}_2$, solves these new optimization problems and checks if the new value of objective function is lower than the original one (the step length is set before the program starts and is the same for all $\overline{u}$- and $\overline{x}$-directions).

3. The direction in $\overline{u}_1$ and $\overline{u}_2$ which gives the lowest value of the objective is then saved (including the direction 0). If a step in both $\overline{u}_1$ and $\overline{u}_2$ results in a decrease in the objective, the value when moving in those two directions at the same time is checked (moving diagonally).

4. The movement that results in the lowest value of the objective is chosen to be the starting point for an iteration in $\overline{x}$-space. The same procedure, as for $\overline{u}$, is used for this iteration.

5. If the objective has not been lowered after a search in both $\overline{u}$- and $\overline{x}$-space the program terminates, otherwise a new iteration starts. The most recent moving directions are kept track of to avoid unnecessary checking of points that have already been checked in the previous iteration.

It is important to realize that, although the objective function is derived from linear programs, which are both concave and convex and therefore have global maximums, the problem to minimize the objective, which depend on these convex linear programs, is not a convex function. Therefore many different starting points has to be tried to find a local minimum with a good value for the objective. Once a local minimum i found there is no way to confirm if it is indeed a global minimum.

For the practical solution of the problem, an in-house used MATLAB-routine based on the SIMPLEX-method has been used for the linear optimization problems in step 2 (i.e. the constraint eq. 7), while the MATLAB routine `fminsearch` [7] has been used for the solution of the whole optimization problem with the objective eq. 6.

5 Example of basic model

All of the parameters except the spot price and value of stored energy are listed below.

$$\begin{aligned} &flow = (147 \quad 148 \quad 148) \\ &desflow = (300 \quad 300 \quad 300) \\ &store = (1 \quad 1 \quad 2)\cdot 10^6/3600 \\ &spill = (760 \quad 760 \quad 600) \\ &\gamma = (.2 \quad .2 \quad .2) \\ &\delta = (.5 \quad .5 \quad .5) \end{aligned}$$

In the example described in this section three spot price curves were used to calibrate the two-plant model. In figure 2, the three spot curves can be seen together with

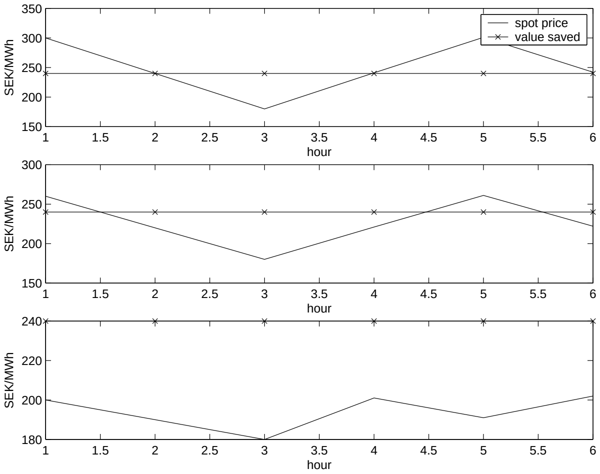

Figure 2: The three different spot price curves and the value of saved energy

the value of the energy stored at the end of the planning period (the horizontal line). A producer trying to maximize profits will try to discharge water when price is over the horizontal line and store the water in reservoirs when the price is under the horizontal line. Remember that the aim is to minimize the function

$$\min_{\overline{u}_i,\overline{x}_i} \sum_{st} (Paim_{st} - \sum_i \gamma_i u_{sit})^2 \qquad (8)$$

subjected to constraints. $Paim_{st}$ is the total production of the original three station system during hour t and price scenario s and $\sum_i \gamma_i u_{sit}$ is the total production of the two-station equivalent. To find a local optimum for this function, iterations were started from different values of $\overline{u}$ and $\overline{x}$ using fairly large step lengths (5-10 HE). Then, when an area was found which seemed to contain a globally optimal point, smaller steps (1 HE) were taken in that area to find a local optimum.

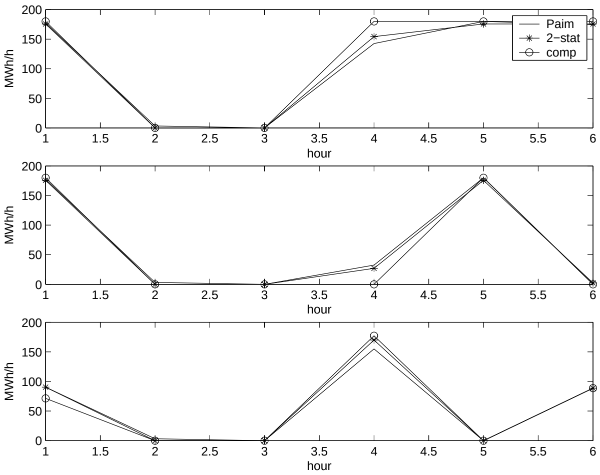

Figure 3: Production plans for the different models corresponding to the three spot price curves. Setup 1.

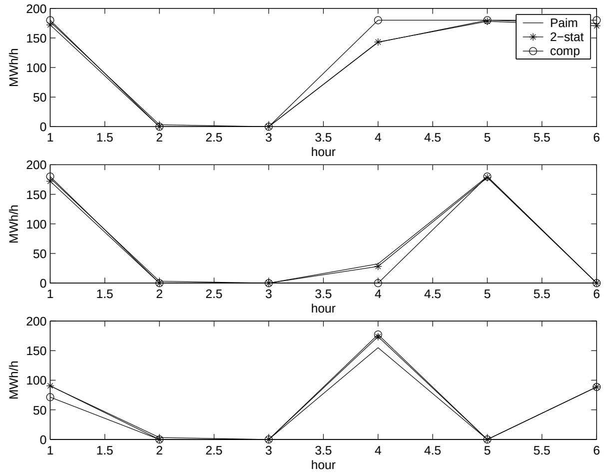

Figure 4: Production plans for the different models corresponding to the three spot price curves. Setup 2.

In figures 3 and 4 the production plan of the two station equivalent with the two best setups of $\overline{u}$ and $\overline{x}$ found can be seen, for each of the three used spot price curves, together with the production plan of the original problem and the production plan obtained with the Composite model. The objective of equation 8 is equivalent of trying to position the production curve of the two-station equivalent, *2-stat*, exactly on that of the original system, *Paim*, for all spot price curves. The fit is clearly much better for the two-station model than the Composite model in both cases.

If we concentrate on figure 3 some of the problems with the Composite model can be seen. Due to the fact that the model only keeps track of total energy in the system without regard to inner limitations both overproduction and underproduction (compared to the original system) is made possible. In the upper picture of figure 3 the Composite model is able to maximize production during hour 4 even though the original system clearly can not do this. What has happened is that at least one of the reservoirs in the original system does not have enough water to maximize production the last three hours, but obviously there is still enough energy left in the total system for the Composite model to produce at maximum capacity. In the middle picture of figure 3, we see that the Composite model do not produce during hour 4 while the original system does. Somewhere a reservoir is full, or will be full if the plant does not produce, but the Composite model does not recognize this fact because the total system is not yet full.

The comparison between the two-station model and the Composite model can be further illustrated by the following data:

- Setup 1
  - $\overline{u} = (316 \quad 287), \overline{x} = (300 \quad 484)$
  - average power production for original system 78.2 MWh/h
  - average error of power production for two-station equivalent 2.88 MWh/h

- average error of power production for Composite model 7.36 MWh/h

- Setup 2
  - $\overline{u} = (317 \quad 294)$, $\overline{x} = (312 \quad 417)$
  - average power production for original system 78.2 MWh/h
  - average error of power production for two-station equivalent 3.19 MWh/h
  - average error of power production for Composite model 7.36 MWh/h

The results seem promising, using the Composite model results in errors more than twice as large as using the two-station equivalent for these spot prices.

6 Example with variable γ and starting water levels in reservoirs

So far, the second station in the equivalent has been made to represent the second and third station in the original system (in terms of water inflow, amount of water at startup and production equivalent). The amount of water in reservoir two and three have been transferred to the second reservoir in the equivalent. This means that the second reservoir tend to be rather large. Also, only the production equivalent of the second station, γ_{new}, has been modified. This means that station two has a large amount of water in its reservoir at the start of the planning period it is also able to produce more energy because of the higher production equivalent.

This inflexibility of the two-station equivalent may reduce the achievable accuracy of it. For example, maybe better results could be obtained if some of the starting water in the second reservoir could be in the first or the other way round. Or, maybe the upper plant which tends to have a smaller reservoir should have most of the energy production capabilities. Because of these questions, in this example γ_i for both virtual plants as well as the starting water levels in the reservoirs are considered as variables. γ, or the production equivalents of the plants, combined with the $\overline{u}$ decides how much the individual plants in the two-station equivalent can produce. The starting water levels, besides redistributing energy within the system, sets the minimum level that $\overline{x}$ can take since they must be able to hold the starting water level.

The demand that the energy at start is preserved remains. So does the demand that the energy production when both virtual plants function as run of the river plants is the same as for the original system. This gives rise to the following equations:

$$\begin{aligned}
&TE = (\gamma_1 + \gamma_2 + \gamma_3)x_{11} + (\gamma_2 + \gamma_3)x_{21} + \gamma_3 x_{31} \\
&EP = \gamma_1 flow_1 + \gamma_2 flow_2 + \gamma_3 flow_3 \\
&\widetilde{\gamma} = \beta, \quad 0 \leq \beta \leq 1, \\
&\widehat{\gamma} = (EP - \widetilde{\gamma} flow_2)/flow_1 \\
&\widehat{x}_1 = TE\alpha/(\widehat{\gamma} + \widetilde{\gamma}) \\
&\widetilde{x}_1 = (TE(1-\alpha))/\widetilde{\gamma}, \quad 0 \leq \alpha \leq 1
\end{aligned} \tag{9}$$

TE is the total energy in the original system at the start. If α percent of this energy is placed in the upper reservoir at the beginning, $\widehat{x}_1$, then $(1-\alpha)TE$ must be placed in the lower reservoir to preserve energy. In terms of amounts of water this becomes $TE\alpha/(\widehat{\gamma} + \widetilde{\gamma})$ and $(TE(1-\alpha))/\widetilde{\gamma}$. EP is the energy production of the original system when used as a run of the river system. If $\widetilde{\gamma} = \beta$ is the production equivalent at the lower site, then the production equivalent at the upper site, $\widehat{\gamma}$, must be $\widehat{\gamma} = (EP - \widetilde{\gamma} flow_2)/flow_1$. Else, the condition that the run of the river production should be the same as for the original system would not hold.

To see if what gains could be made from altering α and β the setups found in the section 5 were used. Although, ideally, one would like to make iterations in α- and β-space part of the program this proved too complicated because of the increased degrees of freedom. Therefore β and α were here altered manually.

The setups from the previous section were used as starting points for iteration with a slight alteration in β or α. If the program terminated with a lower value of the error than before the alteration, another step in this β- or α-space was taken with the new setup (values for $\overline{u}$ and $\overline{x}$) as a starting point for the program. This approach resulted in new setups with more accurate production. The results became:

- Setup 1
  - before
    - $\overline{u} = (316 \quad 287)$, $\overline{x} = (300 \quad 484)$
    - average power production for original system 78.2 MWh/h
    - average error of power production for two-station equivalent 2.88 MWh/h
    - average error of power production for Composite model 7.36 MWh/h
  - after
    - $\overline{u} = (352 \quad 268)$, $\overline{x} = (396 \quad 353)$, $\alpha = 0.986$, $\beta = 0.400$
    - average power production for original system 78.2 MWh/h
    - average error of power production for two-station equivalent 1.55 MWh/h
    - average error of power production for Composite model 7.36 MWh/h

- Setup 2
  - before
    - $\overline{u} = (317 \quad 294)$, $\overline{x} = (312 \quad 417)$
    - average power production for original system 78.2 MWh/h
    - average error of power production for two-station equivalent 3.19 MWh/h

- average error of power production for Composite model 7.36 MWh/h

– after
  - $\overline{u} = (300 \quad 289)$, $\overline{x} = (422 \quad 609)$, $\alpha = 0.687$, $\beta = 0.228$
  - average power production for original system 78.2 MWh/h
  - average error of power production for two-station equivalent 1.14 MWh/h
  - average error of power production for Composite model 7.36 MWh/h

The average error of the power production of the two-station equivalent have been greatly reduced in both instances. With the first setup as starting point, a setup with 46% less error has been found by altering α and β. The reduction in error is even greater with the second setup, where the error has decreased by 64%.

7 Example with larger systems

The publication `Hydro Power in Sweden` [8] contains all the necessary data about hydro power plants in Swedish river systems, for actual river systems to be used in this paper. Three river systems were chosen for this test. Only parts of the rivers, which starts at outflow into the sea and ends at the point where the river breaks up into two several different flows, are used. This means that no case when a plant is linked to two or more plants directly, is considered. The simulated hydro power stations used, are from the following rivers: *Ångermanälven*, *Skellefte älv* and *Indalsälven*.

These systems all had plants, which reservoirs could be filled during a 24-hour period if no power were produced, which meant that there were bottlenecks in the systems. The size of the systems varies in size from 4 to 11 stations. The number of hours in the planning period was extended to 24.

Three different reduced models for the systems were used, the Composite model, the two-station model of the Composite model and the two-station equivalent. The separation into upper and lower station for the two-station Composite model were at the same point as for the two-station equivalent. The reason for the use of the two-station Composite model was to not overestimate the value of the two-station equivalent. The results became:

River	Case	Composite model		2-station
		1-station	2-stations	equivalent
Indal	1	139	126	80.3
Indal	2	84.7	84.7	37.6
Skell	1	21.7	20.3	9.4
Skell	2	27.2	25.8	12.2
Ånger	1	42.7	17.6	13.3
Ånger	2	61.6	22.5	13.5

Table 1: Average error for 6 cases with 5 test scenarios each (MWh/h)

As shown in the table the best results, i.e. the lowest difference between the schedule obtained with the original model and the reduced model, is obtained with the two-station equivalent.

8 Conclusions

There is always a trade-off between the accuracy of a model ant the simulation time for a routine that uses the model. This paper has presented a new method for how to obtain a reduced model of a hydro power system. The reduced model is in the form of a two-station equivalent. The aim of the equivalent is to mimic a schedule for the original system but with less computational effort.

With an equivalent the computation time of the optimization can be significantly reduced. This conclusion is not based on time comparison since the written code is not optimized. But since a reduced model implies fewer constraints and fewer variables (e.g. in a two station equivalent for an eleven station river), the solution time will be significantly decreased.

The tests performed shows that the two-station equivalent gives better schedules than the commonly used Composite model.

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