DECOMPOSITION ALGORITHM FOR OPTIMAL SECURITY-CONSTRAINED POWER SCHEDULING

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Abstract - This paper presents an approach to determine the optimal daily generation scheduling in a competitive environment taking into account dispatch constraints (power reserve, ramp rate limits, real and reactive power output limits), network and power system security constraints (real and reactive power flow equations, bus voltages and transmission flow limits in pre and post-contingency states).

The proposed approach considers Benders decomposition for solving the security-constrained optimal generation scheduling problem. It has been developed using market algorithm applied to a 'Single System Operator' Model which considers the daily market and the technical constraints resolution process as a single one. The objective function minimizes the energy purchase cost subject to previously defined constraints, in a market where all available generating units must make energy bids arranged in different blocks with their own price. As a result of the optimization process, the on/off status of the generating units and voltage control devices, the active and reactive power output of each generating unit and the tap transformer value are provided.

Mathematically the power scheduling is defined as a non-linear mixed-integer optimization problem with linear objective function, binary decision variables (on/off status of the generating units, reactors and capacitors), continuous variables for the operation processes, time couplings (ramp rate limits and minimum up/down time) and non-linear constraints as complete flow load equations.

The model has been tested in different cases of the IEEE 24-bus Reliability Test System and an adapted IEEE 118-bus Test System. It has been programmed in GAMS mathematical modelling language, using CONOPT and CPLEX solvers for non-linear and linear mixed-integer programming problems, respectively.

Keywords - Power system regulation, electricity markets, non-linear mixed-integer optimization problem, Benders decomposition, preventive security analysis

1 INTRODUCTION

THIS paper presents a Benders decomposition approach [1] to define the optimal security-constrained daily unit commitment in a competitive environment.

Electricity markets can be organized in different ways. The analyzed model is based on the structure of certain electricity markets [2, 3] that solve jointly the generation dispatch problem and the network constraints. A single entity, System Operator (SO), determines the daily generation scheduling minimizing the energy purchase cost and considering the secure operation of the power system.

The market participants submit hourly energy multi-block price bids. The dispatcher (SO) problem is to select the cheapest offers from the set of the supply-side offers to match the real power demand (demand-side bids are not considered in this study). The solution sets the accepted offers, the committed energy and the hourly price paid to every committed generating unit. The committed units can be paid at their offer prices or at the spot prices, which are directly provided by the solution.

The secure power scheduling involves two classical problems in the electric systems operation: the unit commitment and the security-constrained optimal power flow. In order to define a preventive secure power scheduling, dispatch constraints (generation limits, power reserve, ramp rate limits, minimum up and down time) as well as network (complete load flow equations) and security constraints (bus voltages and transmission flow limits in pre and post contingency states) have been included in the model. This preventive security criterion also incorporates the commitment of voltage control devices, such as reactors or capacitors.

Therefore, in this work, the daily power scheduling is a mixed-integer non-linear optimization problem that includes:

- linear objective function based on hourly energy multi-block price bids,
- binary decision variables (off-line or on-line generating units, reactors and capacitors),
- continuous variables for the operation processes (real and reactive power, transformer taps, voltage magnitude and phase angle),
- time couplings (ramp rate limits, minimum up and down time),
- non-linear network constraints (complete power flow equations, transmission flow limits, reactive power injected by voltage control devices).
- non-linear security constraints (post-contingency real and reactive power flow equations, transmission flow limits, . . .).

This optimization problem has been solved using Generalized Benders decomposition [4, 5]. This partition algorithm is a decomposition technique in two-levels, master and slave. The master problem solves the unit commitment problem whereas the slave problem deals with the security-constrained optimal power flow (SCOPF). This Benders method allows to broach the presence of binary variables in a non-linear model. Besides, the time couplings are arranged so that it can be treated in a optimal way by the Benders algorithm.

This method improves presently available approaches as it supplies an optimal 24-hour scheduling considering a precise model of the transmission network and a complete security analysis, providing an optimal and preventive real power dispatch with voltages control devices. The model has been tested in the IEEE 24-bus Reliability Test System [6] and an adapted IEEE 118-bus System [7]. The execution times and iterations number are provided and some results for the IEEE 24-bus System are reported. The model has been programmed in GAMS [8], using CONOPT and CPLEX solvers [9] for non-linear and linear mixed-integer programming problems, respectively.

This paper is organized in the following way. The notation used throughout the article is provided in Section 2. Section 3 expounds the model formulation. The structure of the Benders decomposition algorithm and its application to the optimal secure power scheduling are explained in Section 4. Section 5 presents the test systems and the results and finally, Section 6 states the conclusions.

2 NOMENCLATURE

The notation used throughout the paper is:

Variables

$P_{t,i}$	real power output of unit *i* at period *t*.
$P^{g}_{t,j}$	real power injected by all the generating units connected at the bus *j* at period *t*.
$P_{t,i,b}$	real power of block *b* offered by unit *i* at period *t*.
$rt_{t,nj}$	transformer tap for transformer (*nj*) at period *t*.
$Q_{t,k}$	reactive power output of generating unit, reactor or capacitor *k* at period *t*.
$Q^{g}_{t,j}$	reactive power injected by the generating units, reactors and capacitors connected at the bus *j* at period *t*.
$u_{t,k}$	decision variable (0/1) that represents the commitment state of unit, reactance or capacitor *k* at period *t*.
$x_{t,i}$	number of hours that generating unit *i* has been on (+) or off (-) at the end of hour *t*.
$\mathbf{V}_{t,n}$	bus voltage at bus *n* during period *t*.
$V_{t,n}$	bus voltage magnitude at bus *n* during period *t*.
$\delta_{t,n}$	phase angle at bus *n* during period *t*.
$G_{t,jn}$	real term of the element *j,n* in bus admittance matrix.
$B_{t,jn}$	imaginary term of the element *j,n* in bus admittance matrix.
$G^{c}_{t,jn}$	real term of the element *j,n* in bus admittance matrix for the 'c' post-contingency state.
$B^{c}_{t,jn}$	imaginary term of the element *j,n* in bus admittance matrix for the 'c'post-contingency state.
$\lambda^{m}_{t,k}$	dual variable supplied by the slave subproblem in each iteration *m*, which is associated to the decision of connecting of unit *k* at period *t*.
α^{*}_{t}	underestimation of the operation costs computed in the slave subproblem at period *t*.

Parameters

$p^{\bullet}_{t,i,b}$	price offered by unit *i* at hour *t* for block *b*.
$P^{max}_{t,i,b}$	maximum real power offered by the generating unit *i* for the block *b* at period *t*.
P^{max}_{i}	maximum real power output of generating unit *i*.
P^{min}_{i}	minimum real power output of generating unit *i*.
$P^{max}_{t,i}$	maximum real power output of generating unit *i* at period *t*.
$P^{min}_{t,i}$	minimum real power output of generating unit *i* at period *t*.
Q^{max}_{i}	maximum reactive power output of unit *i*.
Q^{min}_{i}	minimum reactive power output of unit *i*.
$P^{d}_{t,n}$	real load demand at bus *n* during period *t*.
$Q^{d}_{t,j}$	reactive load demand at bus *n* during period *t*.
UR_{i}	ramp-up and start-up rate limit of unit *i*.
DR_{i}	ramp-down and shut-down rate limit of unit *i*.
UT_{i}	minimum up time of unit *i*.
UT_{i}	minimum down time of unit *i*.
$U^{m}_{t,k}$	state (on/off) of the unit *k* at period *t* for iteration *m*.
$P^{m}_{t,i}$	real power of the unit *i* at period *t* for iteration *m*.
R_{t}	reserve requirement during period *t*.
V^{min}	minimum voltage at any node *n* and any period *t* for *n* state.
V^{max}	maximum voltage at any node *n* and any period *t* for *n* state.
$\overline{V}^{min}$	minimum voltage at any node *n* and any period *t* for any (*n*-1) state.
$\overline{V}^{max}$	maximum voltage at any node *n* and any period *t* for any (*n*-1) state.
Rt^{min}_{jn}	minimum transformer tap at any period *t*.
Rt^{max}_{jn}	maximum transformer tap at any period *t*.
B^{sh}_{j}	susceptance of the capacitor or reactor *k* connected at bus *n*.
S^{max}_{jn}	maximum transmission capacity (MVA) at line (*jn*) for *n* state.
$\overline{S}^{max}_{jn}$	maximum transmission capacity (MVA) at line (*jn*) for any (*n*-1) state.
y_{jn}	line series admittance.
y'_{jn}	charging admittance.
ε	cost tolerance.

Sets

G	set of indexes of all generating units.
GR	set of indexes of all generating units, reactors and capacitors.
RC	set of indexes of all reactors and capacitors.
B	set of indexes of energy sale blocks.
N	set of indexes of all buses.
Nc	set of all load buses.
C	set of selected contingencies.
T	set of indexes of all periods in hours.
Φ	set of all system branches and transformers.
Φ_{n}	subset of all system branches connected at bus *n*.
Φ^{RT}_{n}	subset of all transformers connected at bus *n*.
Ψ_{n}	subset of all generating units at bus *n*.
M	set of iteration indexes.

It should be noticed that the superscript 'c' in a state variable ($V^c_{t,n}$, $Q^c_{t,j}$, $\delta^c_{t,n}$) represents the value of that variable in the 'c'post-contingency state. The variable $V^c_{t,n}$ is only defined for *Nc* buses.

3 PROBLEM FORMULATION

After the liberalization of the electricity industry, the unit commitment is solved as a market problem based on offer prices, instead of the cost-based minimization of the classical model. In the analyzed model, the OS operates the power system in a centralized way. Its aim is to minimize the energy purchase cost taking into account the bids submitted by the generating agents into the market, the hourly demand and the different constraints of the power system. In this market, all available generating units must submit energy bids arranged in different power-prize blocks for each hour.

Mathematically the model can be stated as follows:

$$\text{Minimize}_{P_{t,i,b}} \qquad \sum_{t\in T}\sum_{i\in G}\sum_{b\in B} p^{\bullet}_{t,i,b}\cdot P_{t,i,b} \tag{1}$$

The objective function includes the energy bids $P_{t,i,b}$ divided in blocks as well as their bid prices, $p^{\bullet}_{t,i,b}$. It should be noticed that the model assumes, by simplicity, that the minimum power of thermal plant, P^{min}_i, is always offered as the first block, which is considered as indivisible energy block through the decision binary variable $u_{t,i}$. The acceptance of this first block involves the generating unit start-up. With this consideration, the objective function (1) could be formulated as follows:

$$\text{Minimize}_{u_{t,i};P_{t,i,b}}$$
$$\sum_{t\in T}\sum_{i\in G} p^{\bullet}_{t,i,1}\cdot u_{t,i}\cdot P^{min}_i + \sum_{t\in T}\sum_{i\in G}\sum_{\substack{b\in B\\ b>1}} p^{\bullet}_{t,i,b}\cdot P_{t,i,b} \tag{2}$$

The injected power to or drawn from the system is limited by a set of constraints, which can be classified in three different groups:

- Dispatch constraints: (3)-(10),
- Network constraints: (11)-(17),
- Security constraints: (18)-(25).

The network constraints include a complete AC model. The *n*-1 security constraints are additional equality and inequality constraints associated with those preselected prevailing outages. These outages produce a violation of the security limits and they are characterized by a new set of nodal power flow equations and transmission system operating limits (n-1 state equations), in which the control variables (generator real power, generator voltage magnitude and transformer taps) are kept in equal value that in the base-case (n state). The exception would be the generating unit outage, in which the lost generation will be supplied by the rest of the committed generating units (all of them or a subset, simulating the action of the P-f regulation) according to the equation (22). In the case of line or transformer outage, it is verified that $G^c_{t,jn}=G_{t,jn}$ and $B^c_{t,jn}=B_{t,jn}$ except for the terms of the bus admittance matrix related to the lost element.

Voltage magnitudes are incorporated as they are a critical factor in some real power systems. By simplicity, for (n-1)-state constraints, it has been considered in the formulation all voltage magnitudes as $V^c_{t,j}$. This is only true in PQ buses.

The previously mentioned constraints are the following ones:

a) Energy blocks limits:

$$0\leqslant P_{t,i,b}\leqslant P^{max}_{t,i,b} \qquad \forall t\in T, \forall i\in G, \forall b\in B/\{1\} \tag{3}$$

b) Relation between energy blocks and real power output:

$$P_{t,i}=u_{t,i}\cdot P^{min}_i+\sum_{\substack{b\in B\\ b>1}} P_{t,i,b} \qquad \forall t\in T, \forall i\in G \tag{4}$$

c) Real power output limits:

$$u_{t,i}\cdot P^{min}_i\leqslant P_{t,i}\leqslant u_{t,i}\cdot P^{max}_i \qquad \forall t\in T, \forall i\in G \tag{5}$$

d) Reactive power output limits:

$$u_{t,i}\cdot Q^{min}_i\leqslant Q_{t,i}\leqslant u_{t,i}\cdot Q^{max}_i \qquad \forall t\in T, \forall i\in G \tag{6}$$

e) Ramp rate limits:

$$-DR_i\leqslant P_{t,i}-P_{t-1,i}\leqslant UR_i \qquad \forall t\in T, \forall i\in G \tag{7}$$

f) Minimum starting up times:

$$[x_{t-1,i}-UT_i]\cdot[u_{t-1,i}-u_{t,i}]\geqslant 0 \qquad \forall t\in T, \forall i\in UG \tag{8}$$

g) Minimum starting down times:

$$[x_{t-1,i}+DT_i]\cdot[u_{t,i}-u_{t-1,i}]\leqslant 0 \qquad \forall t\in T, \forall i\in UG \tag{9}$$

h) System operating reserve requirements:

$$\begin{aligned}\sum_{i\in G} u_{t,i}\cdot P^{max}_i &\geqslant \sum_{n\in N} P^d_{t,n}+R_t && \forall t\in T\\ \sum_{k\in GR} u_{t,k}\cdot Q^{max}_k &\geqslant \sum_{n\in N} Q^d_{t,n} && \forall t\in T\end{aligned} \tag{10}$$

i) *n*-state real power flow equations:

$$\sum_{n\in N} V_{t,j}V_{t,n}(G_{t,jn}\cos(\delta_{t,j}-\delta_{t,n})+B_{t,jn}\sin(\delta_{t,j}-\delta_{t,n}))=$$
$$=P^g_{t,j}-P^d_{t,j} \qquad \forall t\in T,\ \forall j\in N: P^g_{t,j}=\sum_{i\in\Psi_j} P_{t,i} \tag{11}$$

j) *n*-state reactive power flow equations:

$$\sum_{n\in N} V_{t,j}V_{t,n}(G_{t,jn}\sin(\delta_{t,j}-\delta_{t,n})-B_{t,jn}\cos(\delta_{t,j}-\delta_{t,n}))=$$
$$=Q^g_{t,j}-Q^d_{t,j} \qquad \forall t\in T,\ \forall j\in N: Q^g_{t,j}=\sum_{k\in\Psi_j} Q_{t,k} \tag{12}$$

k) *n*-state transmission capacity limits of lines:

$$\left|\mathbf{V}_{t,j}\cdot[(\mathbf{V}_{t,j}-\mathbf{V}_{t,n})\cdot y_{jn}]^{*}+\mathbf{V}_{t,j}\cdot\left(\mathbf{V}_{t,j}\cdot\left(\frac{1}{2}\cdot y'_{jn}\right)\right)^{*}\right|\leqslant$$
$$\leqslant S^{max}_{jn} \qquad \forall t\in T,\ \forall j,n\in N:(jn)\in\Phi_j \tag{13}$$

l) Transformer tap limits:

$$Rt_{jn}^{min} \leqslant rt_{t,jn} \leqslant Rt_{jn}^{max} \quad \forall t \in T, \forall j, n \in N : (jn) \in \Phi_j^{RT} \quad (14)$$

m) Pre-contingency reactive power injected by shunt reactors or capacitors:

$$Q_{t,k} = u_{t,k} \cdot B_{k,j}^{sh} \cdot V_{t,j}^2 \quad \forall t \in T, \forall k \in RC, j \in N : B_{k,j}^{sh} \neq 0 \quad (15)$$

n) *n*-state bus voltage magnitude limits:

$$V^{min} \leqslant V_{t,n} \leqslant V^{max} \qquad \forall t \in T, \forall n \in Nc \quad (16)$$

o) *n*-state bus angle limits:

$$\begin{aligned} -\pi \leqslant \delta_{t,n} \leqslant \pi & \qquad \forall t \in T, \forall n \in N/\{ns\} \\ \delta_{t,ns} = 0 & \qquad ns\text{: swing bus} \quad (17) \end{aligned}$$

p) (*n*-1)-states real power flow equations:

$$\sum_{n \in N} V_{t,j}^c V_{t,n}^c \left(G_{t,jn}^c \cos\left(\delta_{t,j}^c - \delta_{t,n}^c\right) + B_{t,jn}^c \sin\left(\delta_{t,j}^c - \delta_{t,n}^c\right)\right) =$$
$$= P_{t,j}^g - P_{t,j}^d \quad \forall c \in C, \forall t \in T, \forall j \in N : P_{t,j}^g = \sum_{i \in \Psi_j} P_{t,i} \quad (18)$$

q) (*n*-1)-states reactive power flow equations:

$$\sum_{n \in N} V_{t,j}^c V_{t,n}^c \left(G_{t,jn}^c \sin\left(\delta_{t,j}^c - \delta_{t,n}^c\right) - B_{t,jn}^c \cos\left(\delta_{t,j}^c - \delta_{t,n}^c\right)\right) =$$
$$= Q_{t,j}^g - Q_{t,j}^d \quad \forall c \in C, \forall t \in T, \forall j \in N : Q_{t,j}^g = \sum_{i \in \Psi_j} Q_{t,i} \quad (19)$$

r) (*n*-1)-states transmission capacity limits of lines:

$$\left| \mathbf{V}_{t,j}^c \cdot \left[\left(\mathbf{V}_{t,j}^c - \mathbf{V}_{t,n}^c \right) \cdot y_{jn} \right]^* + \mathbf{V}_{t,j}^c \cdot \left(\mathbf{V}_{t,j}^c \cdot \left(\frac{1}{2} \cdot y'_{jn} \right) \right)^* \right| \leqslant$$
$$\leqslant S_{jn}^{max} \quad \forall c \in C, \forall t \in T, \forall j, n \in N : (jn) \in \Phi_j \quad (20)$$

s) Post-contingency reactive power injected by shunt reactors or capacitors:

$$Q_{t,k}^c = u_{t,k} \cdot B_{k,j}^{sh} \cdot (V_{t,j}^c)^2$$
$$\forall c \in C, \forall t \in T, \forall k \in RC, j \in N : B_{k,j}^{sh} \neq 0 \quad (21)$$

t) Real power output of units after generating unit outage:

$$P_{t,k}^c = P_{t,k} + \frac{P_k^{max} - P_{t,k}}{\sum_{k \neq i} (P_k^{max} - P_{t,k})} \cdot P_{t,i} \quad \forall k \in G : P_{t,k} > 0$$
$$P_{t,i}^c = 0 \qquad i \in G : P_{t,i} > 0 \quad (22)$$

u) (*n*-1)-states bus voltage magnitude limits:

$$\overline{V}^{min} \leqslant V_{t,n}^c \leqslant \overline{V}^{max} \quad \forall c \in C, \forall t \in T, \forall n \in Nc \quad (23)$$

v) (*n*-1)-states reactive power output limits:

$$u_{t,i} \cdot Q_i^{min} \leqslant Q_{t,i}^c \leqslant u_{t,i} \cdot Q_i^{max} \quad \forall c \in C, \forall t \in T, \forall i \in G \quad (24)$$

w) (*n*-1)-states bus angle limits:

$$\begin{aligned} -\pi \leqslant \delta_{t,n}^c \leqslant \pi & \qquad \forall c \in C, \forall t \in T, \forall n \in N/\{ns\} \\ \delta_{t,ns}^c = 0 & \qquad ns\text{: swing bus} \quad (25) \end{aligned}$$

4 BENDERS DECOMPOSITION

The short-term power scheduling addressed in this paper is a mixed-integer non-linear optimization problem which includes linear objective function, binary decision variables, continuous variables for operation processes, time couplings and linear and non-linear constraints.

The difficulties associated to the resolution of non-linear optimization problems with binary and/or integer variables force to make use of partitioning techniques. The Benders partition algorithm is a decomposition technique in two-levels, master and slave. The master level represents the decision problem, unit commitment, whereas the slave level deals with the operation problem: SCOPF.

The Benders algorithm defines an iterative procedure between both levels. The master problem is formulated as a linear mixed-integer problem which determines the committed generating units, reactors and capacitors. This schedule is transferred to the slave subproblem –non-linear optimization problem– which calculates the operating cost and the dual values associated to the scheduling decision previously taken by the master problem. This new information is supplied to the master problem through the Benders cuts in order to improve the new decision of the master problem.

The slave level solves the operation problem by means of a security constrained AC optimal power flow. In daily scheduling, the slave problem can be decomposed in 24 subproblems (one per hour), which are sequentially solved. This is possible because of the ramp rate limits, time coupling constraints, are formulated as power limits [10] that are updated after solving each hourly slave subproblem so that production limits for each generating unit are complied within the following hourly slave subproblem.

Therefore, this method allows to broach the non-convexity associated with the binary variables and to divide the global problem into two smaller problems easier to solve. Besides, the time couplings can be treated in an optimal way. The algorithm optimizes jointly the 24-hour problem (unit commitment with SCOPF) providing better results than those obtained in case of solving the constraints separately hour by hour, as it has usually been solved. The procedure followed in this paper includes the steps illustrated in the flowchart of the Figure 1.

4.1 Master Problem

The master level solves the decision problem, a mixed-integer linear problem, whose solution provides the on/off state of the generating units and voltage control devices given by the binary variables $u_{t,k}$. The master problem contains any constraint with binary variables in its formulation. At the same manner, the objective function is made of those terms of the function (2) that include binary variables.

The objective function is defined as follows:

$$\text{Minimize}_{u_{t,i},\, \alpha_t^*} \quad \sum_{t \in T} \sum_{i \in G} p_{t,i,1}^{\bullet} \cdot u_{t,i} \cdot P_i^{min} + \sum_{t \in T} \alpha_t^* \quad (26)$$

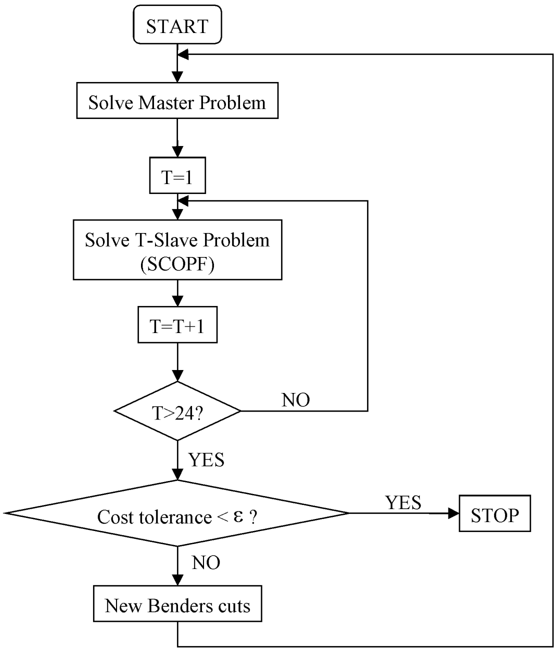

Figure 1: Flowchart of the decomposition procedure

The objective function consists of two terms, the first one represents the "start-up"cost of the unit *i* at its minimum power and the second one is a lower estimation of the operation costs computed in the slave problem. Therefore, the optimization variables are α_t^* and $u_{t,k}$.

This function is subject to the constraints (8), (9), (10) and the Benders linear cuts which are formulated as:

$$\alpha_t^* \geqslant \alpha_t\left(U_{t,k}^{m-1}\right) + \sum_{k \in GR} \lambda_{t,k}^{m-1} \cdot \left(u_{t,k} - U_{t,k}^{m-1}\right) \;\; \forall t \in T, \forall m \in M \quad (27)$$

The key issue in Benders decomposition is located at these Benders linear cuts. Both levels, master and slave, are coupled by the Benders cuts that are updated at each iteration for all operation problems. These cuts includes the term $\alpha_t(U_{t,k}^{m-1})$, which represents the cost of the slave subproblem corresponding to the period *t* for the generation scheduling established by the master problem in the previous iteration, and a second term related to the sensitivity associated to the unit commitment provided by the master problem in the previous iteration. "24"new cuts are added to the master problem at each iteration.

On the other hand, the minimum and maximum starting up times –constraints (8) and (9)– are linearized to be included in the master problem [10].

4.2 Slave Problem

The slave level solves the operation problem by means of a security constrained optimal power flow. The slave problem is decomposed in hourly subproblems, which are sequentially solved. Each hourly slave subproblem solves the system operation problem, a SCOPF, minimizing the cost of the production bids (except for the first block bid) submitted to the market by each generating unit committed by the master problem at each hour.

Therefore, the objective function for each hourly subproblem is:

$$\text{Minimize}_{P_{t,i,b}} \quad \sum_{i \in G} \sum_{\substack{b \in B \\ b>1}} p^{\bullet}_{t,i,b} \cdot P_{t,i,b} \qquad t \in T \quad (28)$$

subject to the rest of constraints that have not included in the master problem, in addition to:

$$u_{t,k} = U_{t,k}^m : \lambda_{t,k}^{m+1} \qquad t \in T, \forall k \in GR \quad (29)$$

To consider the constraint (29) in the slave problem, it is necessary to define temporally the integer variable $u_{t,k}$ as a continuous variable to obtain its sensitivity or dual value $\lambda_{t,k}^{m+1}$.

Each slave problem determines the values of the operation variables ($P_{t,i}$, $Q_{t,k}$, $V_{t,n}$, $r_{t,nj}$, $\delta_{t,n}$, $\bar{Q}_{t,k}$, $\bar{V}_{t,n}$, $\bar{\delta}_{t,n}$) at each period for *n* and *n*-1 states.

4.3 Benders convergence criterion

The iterative Benders decomposition procedure stops when the value of the objective function computed in the master problem reaches the same value than the 'start-up'costs (first bid block) plus operating costs computed through the slave problem, except for a small tolerance cost ε. Actually, as the 'start-up'costs are the same in both problems, slave and master, they can be omitted in the convergence criterion (CC), considering only the operating costs of both problems.

The final convergence criterion is established as the equation (30) shows.

$$\text{CC} = \frac{\sum_{t \in T} \left(\alpha_t\left(U_{t,k}^m\right) - \alpha_t^*\right)}{\sum_{t \in T} \alpha_t\left(U_{t,k}^m\right)} \leqslant \varepsilon \qquad \forall m \in M \quad (30)$$

4.4 Slave problem feasibility

Feasibility cuts have been added to the master problem to enforce the feasibility of the hourly slave subproblems. However, since voltage control is a local problem, there could be some cases where the reactive power constraint (32) does not guarantee the problem feasibility. It would force to add fictitious sources at some PV buses or at buses with voltage control devices and to include them with a high penalty factor in the objective function of the slave problem so that it is optimized the global cost minimizing the cost of the infeasibilities as well.

Nevertheless, these cuts improve the problem convergence, reducing the number of iterations. The cuts are formulated as:

$$\begin{aligned} &\sum_{i \in G} u_{t,i} \cdot P_{t,i}^{max} \geqslant \sum_{n \in N} P_{t,n}^d + R_t && \forall t \in T \\ &\sum_{i \in G} u_{t,i} \cdot P_{t,i}^{min} \leqslant \sum_{n \in N} P_{t,n}^d && \forall t \in T \end{aligned} \quad (31)$$

$$\begin{aligned} &\sum_{k \in GR} u_{t,k} \cdot Q_k^{max} \geqslant \sum_{n \in N} Q_{t,n}^d && \forall t \in T \\ &\sum_{k \in GR} u_{t,k} \cdot Q_k^{min} \leqslant \sum_{n \in N} Q_{t,n}^d && \forall t \in T \end{aligned} \quad (32)$$

5 TEST SYSTEMS AND RESULTS

The test systems are the IEEE 24-bus Reliability Test System and an adapted IEEE 118-bus Test System with standard costs for the generating units. Only results for the IEEE 24-bus system are shown. The test for the IEEE 118-bus system seeks to compare the behaviour of the proposed approach in a larger power system with an important number of voltage control devices (12 capacitors and 2 reactors) and to compare the execution times.

The IEEE 24-bus Test System includes 32 units: nuclear, coal, oil and hydro plants, distributed all over the generating buses and ranging from 12 to 400 MW. The total generation capacity amounts to 3405 MW. The peak load is 2850 MW and occurs in hours T18 and T19. The minimum load is 1682 MW and takes place in hours T4 and T5. A 98% power factor is applied to all load levels.

The transmission network contains 24 load/generating buses connected by 38 lines or transformers at two voltages, 138 and 230 kV. The swing bus is N13. The power system has voltage control devices at buses N14 (synchronous condenser) and N6 (reactor). The transformers taps are modelled as continuous variables. The voltage limits in n-state are 0.95 and 1.05. In a post-fault state the limits are set to 0.93 and 1.11. The flow limits are provided in the reference [6]. The selected prevailing contingencies are shown in Table 1.

Hours (t)	Lines/transformers (*jn*)	Units (*i*)
t= T1,T2,T4, T5,T22,T24	(N7-N8), (N8-N9), (N8-N10)	G9, G10 or G11 at bus N7
t = T3, T6	(N7-N8), (N8-N9)	G9, G10 or G11
t = T23	(N7-N8), (N8-N9), (N8-N10), (N3-N24), (N9-N11)	G9, G10 or G11
t = T7	(N7-N8), (N8-N9), (N8-N10), (N3-N24), (N9-N11), (N11-N14), (N12-N23), (N15-N24)	G9, G10 or G11
t= [T8,T21]	(N7-N8)	

Table 1: Selected contingencies at each period (IEEE 24-Bus System)

The generators bid prices have been taken according to the marginal costs of each energy block. The number of energy blocks is 5 and the bid price rises with the block number. The model manages in the master problem 816 binary variables $u_{t,k}$, 624 integer variables $x_{t,k}$ converted in 1248 new binary variables as a result of the linearization of equations (8) and (9), and 24 continuous variables α_t. On the other hand, each hourly slave problem includes from 357 variables and 644 constraints on, depending on the number of contingencies. Each contingency adds 227 new constraints and 77 new variables. The master problem is solved using CPLEX under GAMS, whereas the slave subproblem is solved using CONOPT. The per unit cost tolerance ε is fixed to 1e-3.

Available solution data include:

- system total cost.
- on/off status of every generating unit, reactor and capacitor per hour.
- active power output of every plant per hour.
- voltage magnitude of every bus per hour.
- reactive power output of every plant and voltage control devices per hour.
- power flow of every line per period.
- spot price per bus and period.

Table 2 shows the evolution of the Benders iterative process. The difference between operating costs of the slave and master problems is progressively decreasing with the number of iterations. The operating costs are very high in the first iterations due to the commitment of fictitious generation sources in some buses. The master total cost reports on the total cost of unit commitment with constraints resolution.

Iter.	Master Total Cost	Slave Operating Cost	Master Operating Cost	CC
1	200275.38	0.00	12754747.16	1.000
2	212400.71	4826.19	5623158.21	0.999
3	505353.91	253161.62	815471.30	0.690
4	537442.34	292004.22	399780.87	0.270
5	564275.97	318100.91	359046.42	0.114
6	588114.92	336825.69	614480.42	0.452
7	590622.53	341848.27	348124.03	0.018
8	595639.04	348235.17	348513.89	0.001

Table 2: Evolution of the convergence of costs ($) with the iterations

Figure 2 shows the hourly maximum price obtained without constraints (Case1) and the hourly maximum price with constraints solution (Case2). It highlights the incidence of constraints solution in the daily market. Obviously, this last maximum price is determined by some generating unit committed to solve constraints. The generating units located at buses N7 and N13, which are more expensive, determine the final price at each period. It may be deduced that these units have a privileged location in the network to solve constraints. It stands to reason that the higher difference in prices between both cases occurs in the valley load periods, as it is necessary to connect new generators to fulfill the network and security constraints.

Figure 2: System maximum price with or without constraints

Spot prices per bus at periods T18 (peak load) and T2 (valley load) are shown in the Figure 3. Figure 4 details the time evolution of the spot price in the buses N2 (PV bus) and N19 (PQ bus). Spot prices per bus and hour show, respectively, their space distribution (maximum spot prices at critical buses N7 and N8) and the correlation between the time evolution of total generation and the time evolution of spot prices –spot prices per hour follow approximately the load curve–.

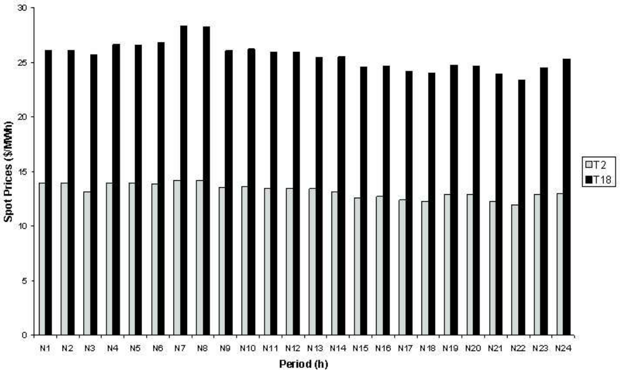

Figure 3: Time evolution of the spot prices (periods T2 y T18)

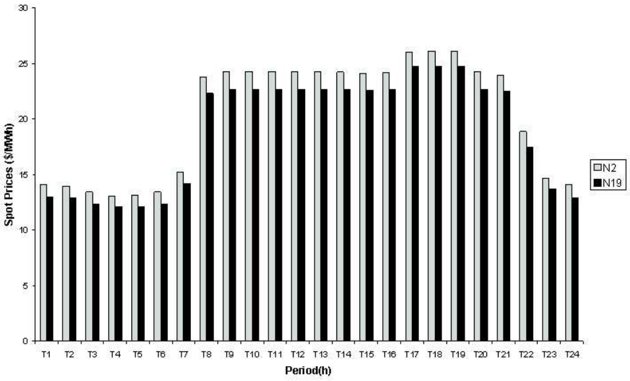

Figure 4: Spot prices per bus (buses N2 and N19)

The number of iterations to reach the convergence and the total CPU time required carrying out the study cases on an INTEL P-IV (3.06 GHz.) are shown in Table 3. The method has been applied to real systems but it is necessary to improve its performance in the future to reduce the computation time. In large-scale power system, the execution time is highly dependent on the precision required to the linear mixed-integer optimizer, the number of contingencies analyzed and the computation efficiency of non-linear optimizer.

	Computation time	Iterations
IEEE 24-Bus System	322”	8
IEEE 118-Bus System	2h. 01’ 21”	18

Table 3: Computation and number of iterations in test systems

6 CONCLUSIONS

The Generalized Benders decomposition method is used to solve a multi-period dispatch problem with security constraints in a wholesale market model. This model solves the daily market and constraints solution process in a single stage. The method shows good convergence properties for the developed application.

The proposed algorithm improves presently available approaches in the following respects: a daily unit commitment is considered simultaneously with a AC precise model of the transmission network including load flow equations, line capacity limits and voltage limits in both pre and post-contingency states; branches overload and bus voltages problems are jointly analyzed; and the model provides an optimal and preventive real power schedule with voltages control devices commitment. Finally, the algorithm optimizes jointly the 24-hour problem (unit commitment with SCOPF) providing better results than those obtained solving each hourly problem separately.

A small-scale case study based on the IEEE 24-bus is analyzed and the computation time and number of iterations are compared with the IEEE 118-bus System. The hourly maximum price allows to know the last generating unit committed to solve constraints in each period. Spot prices per bus and hour show, respectively, their space distribution and the correlation between the time evolution of total generation and the time evolution of spot prices.

Time computation problems can be detected in large-scale power system. This execution time is highly dependent on the precision required to the linear mixed-integer optimizer, the number of contingencies analyzed and the computation efficiency of non-linear optimizer. Parallel computation and/or reactive reserve by areas could be applied to improve the time computation solution.

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