SHORT TERM OPTIMIZATION OF COGENERATION SYSTEMS CONSIDERING HEAT AND ELECTRICITY DEMANDS

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Abstract – In this paper, a new approach for the optimal scheduling of cogeneration systems is presented. Cogeneration plants deal with the simultaneous production of heat and electricity, leading to a better global efficiency of energy systems and a decrease in polluting emissions. Nevertheless, complexity of such plants can be a serious disadvantage. Thus, the short term optimization of these systems has become a crucial point for Energy Industries. The initial issue is a huge, non linear mixed integer programming problem. Due to high computation times, a meta-heuristic based method has to be used to compute a very often suitable solution. A new approach is presented here. The main idea is to reformulate the global problem, to obtain a well suited model for which the exact solution remains tractable. Considering heat and electricity demands, the optimization problem is divided into smaller ones, allowing the use of exact mixed integer methods. In order to have reasonable computation times, the problem is reformulated in a linear frame with the help of extra variables. The approach is tested on a benchmark cogeneration system designed by 'Electricité de France'. Results show that a very convenient solution can be computed, with low computation times.

Keywords: Cogeneration, Short term scheduling, Reformulation, Mixed integer programming

1 INTRODUCTION

Short term optimal scheduling of power systems has appeared to be of great interest for Energy Industries. It can be explained by the following motives. First, energy markets are more and more open and competitive: each producer has to find a convenient and coordinated management of its production units, so as to produce with the lowest costs and/or to earn the maximum income. Secondly, new technologies have been developed (cogeneration, load predictors, energy storage tanks…), which potentially allow a better management of power systems. Finally, environmental laws compel producers to decrease polluting emissions. It can also be achieved with a suitable management of production sites.

In such a technical, economical and environmental context, the short term scheduling of cogeneration plants has emerged as a crucial point for Energy Industries. Due to high global efficiencies of such production sites, it is possible to produce electric and thermal power with lower costs. High efficiencies lead also to a decrease in fuel consumption and so in polluting emissions.

The short term optimization of cogeneration systems is a mixed integer programming problem, similar to the classical "Unit Commitment" problem. Integer variables refer to the on/off status of production units during the temporal horizon. Real variables are the amounts of energy they produce. The main difficulty is the number of binary optimization variables in the global problem, which leads to the highly intractability of the exact full problem.

Many methods have already been applied to get a suitable solution to the "Unit Commitment" problem. For example, they are listed in [1]. The exact methods such as extensive enumeration, priority list, exact mixed integer programming ([2]) or dynamic programming ([3]) suffer from the curse of combinatory complexity. It is also very difficult to take into account temporal constraints such as minimum time up and minimum time down constraints. A classical method to deal with this question is Lagrangian relaxation. This method is well depicted and successfully applied in [4] and [5]. Nevertheless, the minimization of production costs is obtained via the maximization of the dual optimization problem. Because of integer variables, the objective function of the optimization problem is non convex, and no guarantee can be given for the duality gap, and so for the actual quality of the solution.

For tractability purposes, stochastic methods have also been applied. For example, genetic algorithms ([6]) or simulated annealing ([7]) are used to solve the "Unit Commitment" issue. With such methods, computation times can become relatively low, but there is no guarantee on the optimum quality. Furthermore, as tentative solutions are randomly tried, tests have to be made to provide the feasibility of the solution.

In this paper, a new approach is presented. The main idea is to use a priori knowledge of the system to get an efficient reformulation of the initial optimization problem. Thus, instead of looking for an approximate solution of the initial optimization problem, an approximate problem is established for which the computation of the exact solution remains tractable. Thus, to validate optimization results, it is no use checking the algorithm behavior anymore.

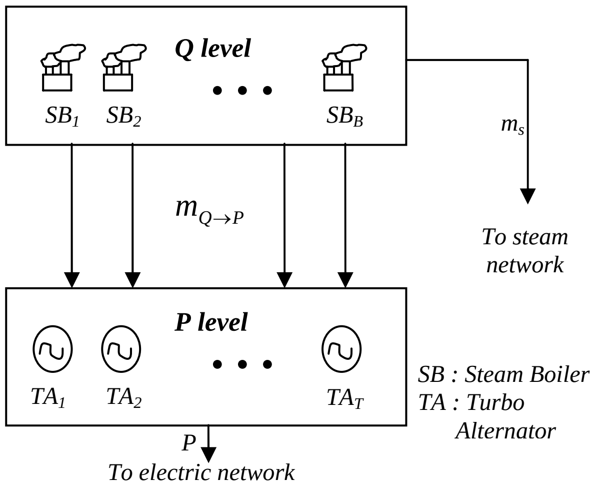

Figure 1: Schematic picture of a cogeneration system

The validation can be made through the validation of the approximate model, which can be done from a priori knowledge and physical comments about the system.

This strategy can be summarized as follows. Considering the spatial partition (electric/thermal) of cogeneration systems, the optimization problem is first divided into smaller ones. Using suited reformulation, estimations of parts of the objective function, and extra variables, costs and temporal constraints (such as minimum time down/time up) can be expressed in a linear frame. Thus, each sub problem is a linear mixed integer programming problem, with few integer variables (compared with the initial problem), which can be quickly and exactly solved with an exact mixed integer programming method ("Branch and Bound" for example).

The proposed optimization strategy is fully presented in section 2. This strategy has been tested on a benchmark example designed by 'Electricité de France'. This case study is depicted in section 3. Numerical results are then given in section 4. They show that a convenient solution can be computed by the proposed optimization strategy with very low computation times compared with the size of the optimization problem. A discussion about the advantages and drawbacks of the method is the core of section 5. Finally, conclusions are drawn in section 6.

2 OPTIMIZATION STRATEGY

2.1 Nomenclature

The following notations will be used in this paper:

- N: length of the temporal horizon (hours).
- n (superscript) : time interval $[n\text{-}1,n]$.
- Q: thermal power.
- P: electric power.
- B: number of steam boilers.
- T: number of turbo alternators.
- c_{el}^{n} : price of sold electricity.
- u_k^n : on/off status of production unit k.
- $m_{SB_b}^{n}$: steam mass flow produced by steam boiler b.
- $m_{TA_t}^{n}$: steam mass flow through turbo alternator t.

2.2 Initial optimization problem

A schematic picture of a cogeneration system is drawn on figure 1. Steam boilers are brought together in a primary network, labeled "*Q level*". This level is made of heat-only units. A part of the produced thermal power is given to the "*P level*" where the electric power is produced and then sold to the electric network.

The initial optimization problem can be stated as the minimization of production costs:

$$\min_{\substack{\left\{m_{SB_b}^n, u_{SB_b}^n\right\} \\ \left\{m_{TA_t}^n, u_{TA_t}^n\right\}}} \sum_{n=1}^{N} \left(\begin{array}{l} \displaystyle\sum_{b=1}^{B} \left(\begin{array}{l} c_{SB_b}^{prod}\left(m_{SB_b}^n, u_{SB_b}^n\right) \\ + c_{SB_b}^{on/off}\left(u_{SB_b}^n, u_{SB_b}^{n-1}\right) \end{array} \right) \\ + \displaystyle\sum_{t=1}^{T} \left(\begin{array}{l} c_{TA_t}^{prod}\left(m_{TA_t}^n, u_{TA_t}^n\right) \\ + c_{TA_t}^{on/off}\left(u_{TA_t}^n, u_{TA_t}^{n-1}\right) \\ - P_{TA_t}^{prod}\left(m_{TA_t}^n, u_{TA_t}^n\right) c_{el}^n \end{array} \right) \end{array} \right) \tag{1}$$

In this objective function,

- $c_{SB_b}^{prod}\left(m_{SB_b}^n, u_{SB_b}^n\right)$ is the thermal production cost of steam boiler b.
- $c_{SB_b}^{on/off}\left(u_{SB_b}^n, u_{SB_b}^{n-1}\right)$ is the start up and shut down cost of steam boiler b.
- $c_{TA_t}^{prod}\left(m_{TA_t}^n, u_{TA_t}^n\right)$ is the electrical production cost of turbo alternator t.
- $c_{TA_t}^{on/off}\left(u_{TA_t}^n, u_{TA_t}^{n-1}\right)$ is the start up and shut down cost of turbo alternator t.
- $P_{TA_t}^{prod}\left(m_{TA_t}^n, u_{TA_t}^n\right)$ is the electric power produced by turbo alternator t.

Optimization problem constraints are:

- Capacity constraints of production units, which can be expressed for unit k as:

$$u_k^n m_k^{\min} \le m_k^n \le u_k^n m_k^{\max} \tag{2}$$

- Minimum time up and time down constraints of production units.
- Network constraints (steam balance), as steam boilers and turbo alternators are brought together in primary networks.
- Satisfaction of heat demand (and in some cases of electricity demand):

$$m_s^n \ge m_{dem}^n \tag{3}$$

The initial optimization problem is a mixed integer programming one. The objective function is non linear (start up costs for example), and so are some of the constraints (minimum time up/time down constraints for instance). Furthermore, there are numerous binary variables: $N(B+T)$. For tractability purposes, this optimization problem has to be reformulated and divided up.

2.3 Physical partition

Cogeneration systems management is highly influenced by the price of sold electricity: "*Q level*" has first to satisfy the heat demand of the steam network m_s, but if the price of electricity is sufficiently high, it is economically interesting to produce more steam so as to produce electricity. This is the extra mass flow, feeding "*P level*" $m_{Q\to P}$.

Considering this physical partition, the initial optimization problem (1) is divided into 2 optimization problems: electricity production and steam production.

To solve the electricity production problem, it is necessary to have an estimated function $\hat{c}_Q^{prod}$ of thermal production costs ("*Q level*"). Thus, the first problem, electricity production, can be stated as the following optimization problem:

$$\min_{\substack{\left\{ m_{TA_t}^n, u_{TA_t}^n \right. \\ \left. m_{Q\to P}^n \right\}}} \sum_{n=1}^{N} \left(\begin{array}{l} \hat{c}_Q^{prod}\left(m_s^n + m_{Q\to P}^n\right) \\ + \sum_{t=1}^{T} \left(\begin{array}{l} c_{TA_t}^{prod}\left(m_{TA_t}^n, u_{TA_t}^n\right) \\ + c_{TA_t}^{on/off}\left(u_{TA_t}^n, u_{TA_t}^{n-1}\right) \\ - P_{TA_t}^{prod}\left(m_{TA_t}^n, u_{TA_t}^n\right) c_{el}^n \end{array} \right) \end{array} \right) \tag{4}$$

The constraints of this first problem are:

- Minimum time up and time down constraints of turbo alternators.
- Network constraints in "*P level*".
- Satisfaction of heat demand (and in some cases of electricity demand).

$$m_Q^{\max} \geq m_{dem}^n + m_{Q\to P}^n \tag{5}$$

Where $m_Q^{\max}$ is the maximum steam mass flow which can be produced by "*Q level*".

The second optimization problem, steam production, is a classical "Unit Commitment" problem:

$$\min_{\left\{ m_{SB_b}^n, u_{SB_b}^n \right\}} \sum_{n=1}^{N} \left(\sum_{b=1}^{B} \left(\begin{array}{l} c_{SB_b}^{prod}\left(m_{SB_b}^n, u_{SB_b}^n\right) \\ + c_{SB_b}^{on/off}\left(u_{SB_b}^n, u_{SB_b}^{n-1}\right) \end{array} \right) \right) \tag{6}$$

The following constraints have to be satisfied:

- Time up/time down constraints of steam boilers.
- Network constraints in "*Q level*".
- Satisfaction of heat demand: $m_{dem}^n + m_{Q\to P}^n$.

After solution of both optimization problems, the "Unit Commitment" is stored, and the "Economic Dispatch" of the whole problem is solved again. This can be done quickly as there is no binary variable anymore.

2.4 Optimization method

To solve electricity and steam production an exact mixed integer programming method is used: "Branch and Bound" (see [2] for instance). Nevertheless, for tractability purposes, it is necessary to have a linear frame for optimization problems. The linear construction of the model is depicted in section 2.5.

2.5 Model for optimization

Electricity and steam production problems are quite similar. As mentioned above, it is necessary to have a linear model of the optimization problem so as to have low computation times. That is why production costs are assumed to be linearly expressed:

$$\begin{cases} \hat{c}_{prod}^{Q}\left(m_s^n + m_{Q\to P}^n\right) = \lambda^1\left(m_s^n + m_{Q\to P}^n\right) + \lambda_0 \\ c_{SB_b}^{prod}\left(m_{SB_b}^n, u_{SB_b}^n\right) = \alpha_{SB_b}^1 m_{SB_b}^n + \alpha_{SB_b}^0 u_{SB_b}^n \\ c_{TA_t}^{prod}\left(m_{TA_t}^n, u_{TA_t}^n\right) = \beta_{TA_t}^1 m_{TA_t}^n + \beta_{TA_t}^0 u_{TA_t}^n \\ P_{TA_t}^{prod}\left(m_{TA_t}^n, u_{TA_t}^n\right) = \gamma_{TA_t}^1 m_{TA_t}^n + \gamma_{TA_t}^0 u_{TA_t}^n \end{cases} \tag{7}$$

Start up costs can be expressed for unit k by the quadratic expression:

$$c_k^{start}\left(u_k^n, u_k^{n-1}\right) = c_k^{start}\, u_k^n \left(1 - u_k^{n-1}\right) \tag{8}$$

Extra binary optimization variables are added:

$$\delta_k^n = u_k^n \left(1 - u_k^{n-1}\right) \tag{9}$$

Note that these binary variables can also be expressed with the following linear inequalities:

$$\begin{cases} \delta_k^n \leq u_k^n \\ \delta_k^n \leq \left(1 - u_k^{n-1}\right) \\ \delta_k^n \geq u_k^n - u_k^{n-1} \end{cases} \tag{10}$$

Similarly, shut down costs can be reformulated with extra binary variables defined by:

$$\varepsilon_k^n = u_k^{n-1}\left(1 - u_k^n\right) \Leftrightarrow \begin{cases} \varepsilon_k^n \leq u_k^{n-1} \\ \varepsilon_k^n \leq \left(1 - u_k^n\right) \\ \varepsilon_k^n \geq u_k^{n-1} - u_k^n \end{cases} \tag{11}$$

Time up and time down constraints can also be linearly reformulated, using variables δ_n^k and ε_n^k:

$$\begin{aligned} \delta_k^n = 1 &\Rightarrow \left(u_k^{n+1} = 1, u_k^{n+2} = 1, \ldots, u_k^{n+T_k^{up}-1} = 1 \right) \\ &\Leftrightarrow \sum_{j=0}^{T_k^{up}-1} u_k^{n+j} \geq T_k^{up} \delta_k^n \end{aligned} \tag{12}$$

$$\begin{aligned} \varepsilon_k^n = 1 &\Rightarrow \left(u_k^{n+1} = 0, u_k^{n+2} = 0, \ldots, u_k^{n+T_k^{down}-1} = 0 \right) \\ &\Leftrightarrow \sum_{j=0}^{T_k^{down}-1} \left(1 - u_k^{n+j}\right) \geq T_k^{down} \varepsilon_k^n \end{aligned} \tag{13}$$

Note that the number of binary variables has been multiplied by 3, which could be quite a disadvantage for the linear model. However, equations (10) and (11) show that δ_n^k and ε_n^k variables are automatically assigned to binary values when u_n^k variables have binary values. Thus, this property can be used to fight against combinatory complexity, using a suited programming of "Branch and Bound" method.

So, the optimization problem (4) can equivalently be stated as:

$$\min_{\left\{\begin{array}{c} m^n_{TA_t},u^n_{TA_t} \\ \delta^n_{TA_t},\varepsilon^n_{TA_t} \\ m^n_{Q\to P}\end{array}\right\}} \sum_{n=1}^{N} \left(\begin{array}{l} \lambda^1\left(m^n_s+m^n_{Q\to P}\right)+\lambda^0 \\ +\sum_{t=1}^{T} \left(\begin{array}{l} \left(\beta^1_{TA_t} m^n_{TA_t}+\beta^0 u^n_{TA_t}\right) \\ +c^{start}_{TA_t}\delta^n_{TA_t}+c^{down}_{TA_t}\varepsilon^n_{TA_t} \\ -\left(\gamma^1_{TA_t}m^n_{TA_t}+\gamma^0_{TA_t}u^n_{TA_t}\right)c^n_{el}\end{array}\right)\end{array}\right) \qquad (14)$$

And the optimization problem (6) is:

$$\min_{\left\{\begin{array}{c} m^n_{SB_b},u^n_{SB_b} \\ \delta^n_{SB_b},\varepsilon^n_{SB_b}\end{array}\right\}} \sum_{n=1}^{N} \left(\sum_{b=1}^{B} \left(\begin{array}{l} \alpha^1_{SB_b} m^n_{SB_b}+\alpha^0_{SB_b} u^n_{SB_b} \\ +c^{start}_{SB_b}\delta^n_{SB_b}+c^{down}_{SB_b}\varepsilon^n_{SB_b}\end{array}\right)\right) \qquad (15)$$

Finally, the developed optimization strategy can be summed up by figure 2. Reformulations have been done for step ② and ③ for tractability purposes.

3 CASE STUDY

The optimization strategy is tested on a benchmark cogeneration system, depicted on figure 3. This is a part of a district heating network which has been completely defined, modeled and simulated in [8].

To illustrate the versatility of the developed method, this cogeneration system is a little bit more complicated than the generic system depicted on figure 1:

- A third kind of unit is present: "cogen unit". This unit produces both steam and electricity. It can be viewed as an alternator and a steam boiler in series.
- There are two thermal demands: one for an industrial consumer, $m^n_{c,dem}$, and one for a district heating network $m^n_{dh,dem}$.

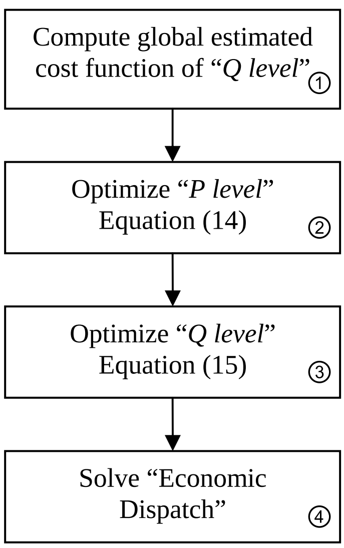

Figure 2: Optimization strategy

3.1 *Estimated objective functions*

In order to separate electric and thermal parts of the system, it is necessary to compute 2 estimated cost functions: one for the block (SB_1, SB_2), and one for the block (SB_3, SB_4).

In this study, it has been decided to use a static characteristic for these estimated functions. For instance, for (SB_1, SB_2), the estimated cost function can be computed as the real optimization problem:

$$\hat{c}^{prod}_{SB_1,SB_2}(m)=\min_{\left(m_{SB_1},m_{SB_2}\right)}\left(c^{prod}_{SB_1}\left(m_{SB_1}\right)+c^{prod}_{SB_2}\left(m_{SB_2}\right)\right) \qquad (16)$$

Constraints are capacity constraints, and the following equation:

$$m_{SB_1}+m_{SB_2}=m \qquad (17)$$

Note that dynamic aspects such as start up and shut down costs and time up and time down constraints are not taken into account in this estimation.

In practice, these functions can only be computed for few values. They are then approximated by a linear expression (for tractability of electricity optimization), using a least square method. Thus,

$$\begin{cases}\hat{c}^{prod}_{SB_1,SB_2}(m)=\lambda^1_{1,2}m+\lambda^0_{1,2} \\ \hat{c}^{prod}_{SB_3,SB_4}(m)=\lambda^1_{3,4}m+\lambda^0_{3,4}\end{cases} \qquad (18)$$

3.2 *Optimization of electricity production*

With the help of these estimated functions, the optimization of electricity production can be expressed by the following equation:

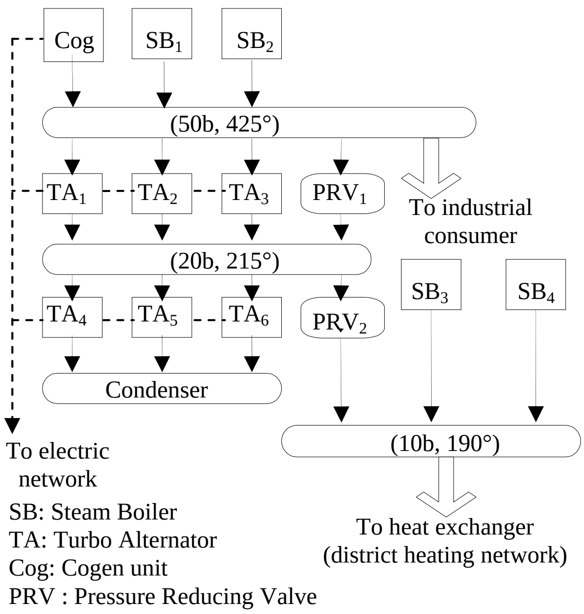

Figure 3: Benchmark cogeneration system.

$$\min_{\left\{\begin{array}{c} m_{TA_i}^n, u_{TA_i}^n \\ m_{Cog}^n, u_{Cog}^n \\ m_{PRV_1}^n, m_{PRV_2}^n \end{array}\right\}} \sum_{n=1}^{N} \left(\begin{array}{l} \lambda_{1,2}^1 \left(\begin{array}{l} \sum_{i=1}^{3} m_{TA_i}^n + m_{PRV_1}^n \\ + m_{c,dem}^n - m_{Cog}^n \end{array} \right) + \lambda_{1,2}^0 \\ + \lambda_{3,4}^1 \left(m_{dh,dem}^n - m_{PRV_2}^n \right) + \lambda_{3,4}^0 \\ + \sum_{i=1}^{6} \left(\begin{array}{l} c_{TA_i}^{prod} \left(m_{TA_i}^n, u_{TA_i}^n \right) \\ + c_{TA_i}^{on/off} \left(u_{TA_i}^n, u_{TA_i}^{n-1} \right) \\ - P_{TA_i} \left(m_{TA_i}^n, u_{TA_i}^n \right) \end{array} \right) \\ + \left(\begin{array}{l} c_{Cog}^{prod} \left(m_{Cog}^n, u_{Cog}^n \right) \\ + c_{Cog}^{on/off} \left(u_{Cog}^n, u_{Cog}^{n-1} \right) \\ - P_{Cog} \left(m_{Cog}^n, u_{Cog}^n \right) \end{array} \right) \end{array} \right) \tag{19}$$

The constraints for the electric part are:

- Capacity constraints (equation (2)).
- Minimum time up/time down constraints.
- Network constraints:

$$\sum_{i=1}^{3} m_{TA_i}^n + m_{PRV_1}^n = \sum_{i=4}^{6} m_{TA_i}^n + m_{PRV_2}^n \tag{20}$$

- Satisfaction of industrial and district heating network demands:

$$\begin{cases} m_{Cog}^n + m_{SB_1}^{\max} + m_{SB_2}^{\max} \geq \sum_{i=1}^{3} m_{TA_i}^n + m_{PRV_1}^n + m_{c,dem}^n \\ m_{PRV_2}^n + m_{SB_3}^{\max} + m_{SB4}^{\max} \geq m_{dh,dem}^n \end{cases} \tag{21}$$

Reformulations depicted in section 2.5 are used to obtain a linear model, for which exact solution remains tractable. The cogeneration system may have to satisfy an exact electricity demand P_{dem}^n instead of maximizing the income earned by electricity selling. If so, the following set of constraints is added:

$$\sum_{i=1}^{6} P_{TA_i}\left(m_{TA_i}^n, u_{TA_i}^n\right) + P_{Cog}\left(m_{Cog}^n, u_{Cog}^n\right) \geq P_{dem}^n \tag{22}$$

For a long time horizon $(N \geq 12)$ it may be necessary to divide it into smaller time intervals so as to keep reasonable computation times.

3.3 *Optimization of heat production*

As there are two heat demands, 2 optimization problems have to be solved. SB_1 and SB_2 have to satisfy demand $m_{dem,1}^n$ and SB_3 and SB_4 demand $m_{dem,2}^n$:

$$\begin{cases} m_{dem,1}^n = \sum_{i=1}^{3} m_{TA_i}^n + m_{PRV_1}^n + m_{c,dem}^n - m_{Cog}^n \\ m_{dem,2}^n = m_{dh,dem}^n - m_{PRV_2}^n \end{cases} \tag{23}$$

Note that these heat demands can be satisfied since constraints (21) have already been taken into account in the optimization of electricity production. For instance, for steam boilers 1 and 2, the problem can be stated as:

$$\min_{\left\{\begin{array}{c} m_{SB_i}^n \\ u_{SB_i}^n \end{array}\right\}} \sum_{n=1}^{N} \left(\begin{array}{l} c_{SB_1}^{prod}\left(m_{SB_1}^n, u_{SB_1}^n\right) + c_{on/off}^{SB_b}\left(u_{SB_1}^n, u_{SB_1}^{n-1}\right) \\ + c_{SB_2}^{prod}\left(m_{SB_2}^n, u_{SB_2}^n\right) + c_{on/off}^{SB_b}\left(u_{SB_2}^n, u_{SB_2}^{n-1}\right) \end{array} \right) \tag{24}$$

Optimization problems are then reformulated as explained in section 2.5. Linear mixed integer programming problems are obtained and exactly solved with a "Branch and Bound" method.

3.4 *Economic Dispatch*

Step ④ of the optimization strategy is the "Economic Dispatch" solution. Note that this last optimization problem has no binary variable anymore. Thus, it is possible to use the initial model (without reformulations and estimations of section 2) to compute real variables. The computation is achieved with a classical gradient optimization method.

3.5 *Overall optimization*

Each stage of optimization generates an optimal solution for that stage only. After 4 steps of optimization there is no guarantee that the solution is optimal for the overall problem. Thus, a fifth step is added: the previous solution is the initial point of a discrete descent method which is performed for the overall problem. This leads to a local minimum of the optimization problem. Finally, the cost of the solution slightly decreases.

4 NUMERICAL RESULTS

To illustrate the quality of optimization results, a complete example of solution is presented in this section. The industrial steam demand and the district heating network one are depicted on figure 4. The price of electricity is constant: $c_{el}^n = 40$ €/MWh. This price corresponds to the current French situation from November the 1st to March the 31st.

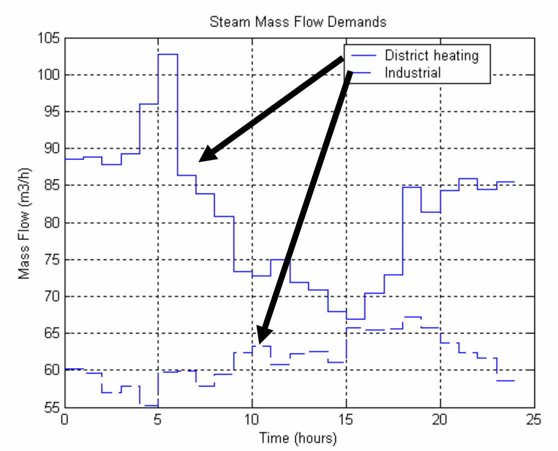

Figure 4: Industrial and district heating network demands.

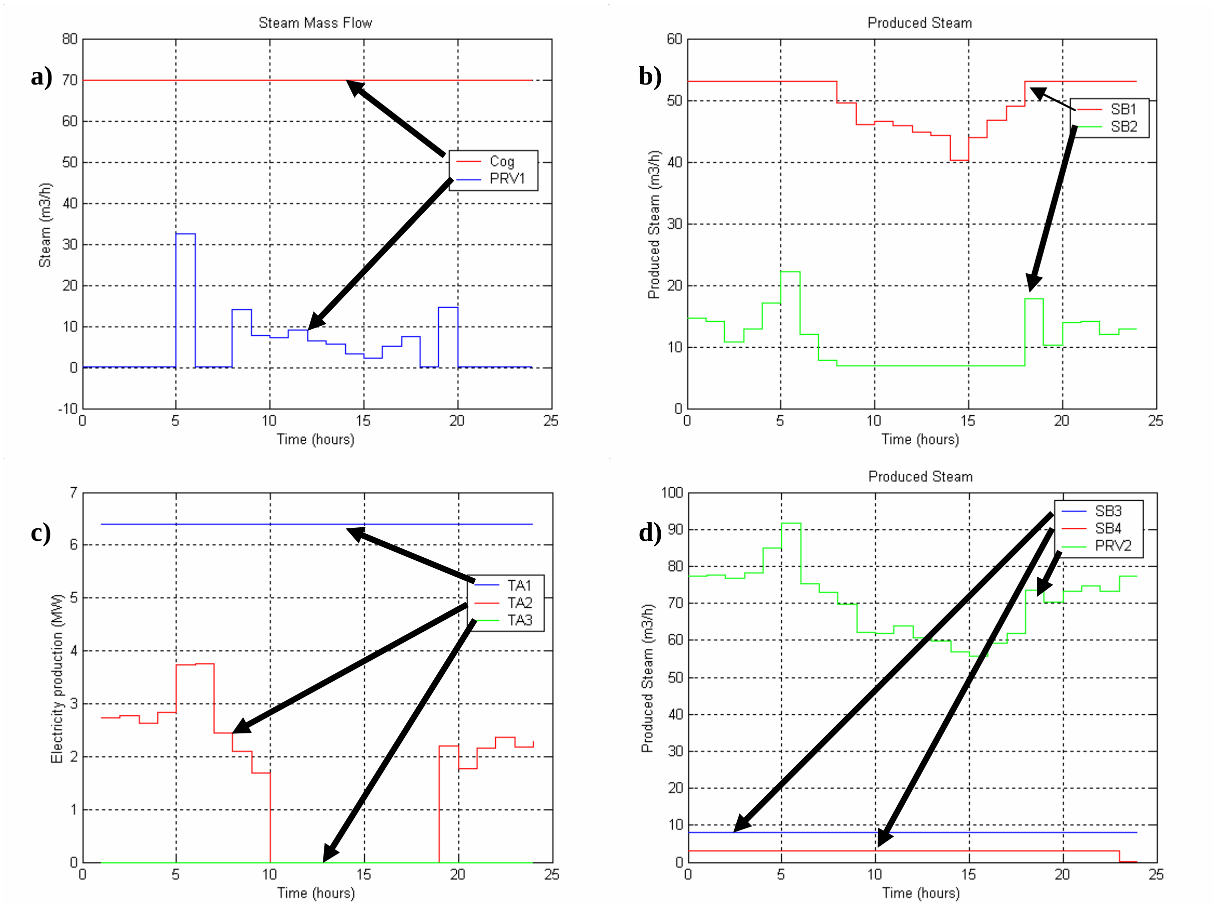

Figure 5: Optimization results. a) Steam mass flow (”Cogen unit” and PRV_1). b) Produced steam (SB_1 and SB_2). c) Produced electricity (TA_1, TA_2, TA_3). d) Produced Steam (SB_3, SB_4 and PRV_2).

The time horizon is N=24 hours. At the initial state, all steam boilers are switched on. Steam boilers 1, 2 and 3 are compelled to be switched on at final time, whereas steam boiler 4 has to be switched off. Computation times are about 190 seconds (Pentium IV, 2.5 GHz). Note that time horizon has been divided for the electric optimization part. Computation times are rather good: the initial optimization problem is a non linear mixed integer programming problem made of 24*11=264 integer variables and 24*13=312 real variables.

Optimization results are drawn on figure 5. Results appear to be physically coherent. Indeed, a good working order of the cogeneration system is observed. On the first level (SB_1, SB_2 and “Cogen unit”) the “Cogen unit” always produces at its highest rate (figure 5a): this unit has the best efficiency of all steam production units. SB_1 is more profitable than SB_2. Thus, it is used at its maximum rate when needed. It is not the case from hour 8 to hour 17: because of start up and shut down costs, it is more profitable to keep SB_2 switched on, and to decrease SB_1 production when less steam is needed (see figure 4). The produced steam feeds the industrial consumer and the second level (TA_1, TA_2, TA_3 and PRV_1).

On the second level, TA_1 has the best efficiency and TA_3 the worst, which can explain figure 5c. The steam mass flow going from level 1 to level 2 goes either through TA_2 or PRV_1: due to working costs of turbo alternators, the use of turbo alternator (TA_2 in this case) is not profitable if steam mass flow is too low. Electric production of TA_4, TA_5, and TA_6 is zero. Indeed, as the price of electricity is too low, it is more profitable to use the steam going from level 2 to satisfy district heating network demand: less steam will have to be produced by SB_3 and SB_4. This demand is completely fed by steam mass flow in PRV_2: due to electricity power selling, production costs of level 1 are lower than production costs of SB_3 and SB_4. However these 2 steam boilers remain switched on, and work at their minimum rate, because of start up and shut down costs.

Several scenarios have also been tested, leading to coherent behavior of optimization results. For instance, for a nil price of electricity, all turbo alternators are switched off. Industrial demand is satisfied by SB_1 and SB_2, and district demand is satisfied by SB_3 and SB_4. For high prices of electricity, all turbo alternators work at their maximum rate.

5 DISCUSSION

5.1 Polluting emissions

The use of cogeneration systems can be profitable in terms of polluting emissions because of the high global efficiency of production sites. Polluting emissions can of course be explicitly taken into account by additional constraints. For instance, for NO_x emissions the following set of constraints can be added:

$$\sum_{n=1}^{N}\sum_{k=1}^{K} g_k^{NO_x}\left(m_k^n\right) \le g_{\max}^{NO_x} \qquad (25)$$

5.2 Generalization

The proposed method is quite versatile and can be applied to various kinds of cogeneration systems. When the number of production units becomes high, cogeneration systems may have to be agglomerated in smaller systems so as to apply the strategy. In the case study (see section 3), the "Cogen unit" can be viewed as a secondary network of production units. Those production units are agglomerated together, and a global estimated function is computed $\left(c_{Cog}^{prod}\left(m_{Cog}^n, u_{Cog}^n\right)\right)$. Thus, for a given cogeneration system, a relevant modeling has to be defined, based on physical partition and a priori knowledge of the system.

5.3 Two level optimization approach

The proposed method can be applied for generic cogeneration systems. However, it is strongly dependent on the quality of global estimated functions of agglomerated subsystems. The establishment of such functions is a hard task: one may have to estimate the generic solution of partial "Unit Commitment" problems.

In step ③ of the proposed strategy, subsystems are exactly solved. Thus, it is possible to check the quality of estimated functions and to validate the solution. If those exact solutions lead to production costs which are quite different from estimated costs, the optimization problem may be solved again with an updated estimated objective functions. This strategy could be viewed as a two level optimization approach, which is one of the forthcoming works of this study.

6 CONCLUSION

Cogeneration systems can be of great interest for energy Industries: global high efficiencies lead to economical and environmental benefits. However, the optimization of such systems is a tough problem, since it is a huge and non linear mixed integer programming issue, for which exact solution is highly intractable.

In this paper, a new method to optimize cogeneration systems has been presented. Instead of looking for an approximate optimization method, physical partition and reformulations are used to divide the initial problem into smaller ones for which exact solution can be achieved with very low computation times. The advantage of this approach is that the validation of the solution is not made with a fine analysis of optimization algorithms anymore. It is made with the help of producers' expertness: the ways they manage their production sites are guiding principles for the partition procedure.

The partition is mainly achieved with a division into electric and thermal production parts, as the economical management of cogeneration systems is principally based on the price of electricity. However, it may be useful to go on the partition and agglomerate some of production units to keep small optimization problems.

Algebraic reformulations are made to obtain linear optimization problems (objective functions and constraints). They lead to the possibility of exact solution of mixed integer programming problems. The method is also based on estimated objective functions. The establishment of such functions is the main difficulty of the proposed strategy and has been discussed in this paper.

Finally very low computation times are observed. The quality of the solution can be checked with the help of physical and economical reasoning for academic cases (constant values of electricity price).

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