# ESTIMATING VOLTAGE QUALITY IN DISTRIBUTION SYSTEMS USING CFA-MATRIX DESCRIPTION FOR NON-LINEAR LOADS

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Abstract - In this paper a new method for voltage quality calculation in power networks with non-linear loads is presented. Using the measured data the equivalent models of the non-linear loads are constructed in the form of <u>Crossed-Frequency-A</u>dmittance-Matrix (CFAM). These models together with the network equivalent representation are integrated into the computing algorithm that iteratively calculates voltages and currents in the analyzed network using nodal equations. An illustrative example with resistive welding machines as non-linear loads is presented and discussed. As a result of the investigations a tool for the analysis of nonlinear loads in distributions systems with reference to voltage quality is obtained. The developed method allows the voltage quality to be estimated taking into account the interaction effects in non-linear loads.

Keywords: Power System, Power Quality, Nonlinear Load, Modeling, CFA-Matrix

# **1 INTRODUCTION**

In most voltage quality analyses power systems are modeled as linear systems with passive elements separately excited by a sinusoidal voltage with constant magnitude and frequency and/or current sources representing the non-linear loads [1], [2]. However, in distribution networks a certain participation of the harmonics appears which comes from interaction effects between different harmonic orders that take place in nonlinear loads. This phenomenon cannot be analyzed by separate calculations at each harmonic order. Therefore, to improve calculation accuracy and to better understand this interaction phenomena a model of the nonlinear load has to be considered that properly reflects the interdependence between currents and voltages at the load clamps and additionally incorporates such features that allow for a flexible application in power circuit calculations. These requirements can be met by using a modeling approach that treats nonlinear loads as linear stationary transformations in a matrix form. The matrix reflects the harmonic interactions between currents and voltages and is called Crossed-Frequency-Admittance-Matrix (CFAM) [3]. This non-linear load model was developed and successfully applied for small-power devices such as energy-saving lamps [3], [4] and was then extended to devices with pulsed power, whereby a measured-based approach has been used to parameterize the non-linear load [5], [6]. Thereby,

studies in [3], [4], [5] and [6] have concentrated on the non-linear behavior representation at the load clamps.

This article introduces a novel method that is based on an enhanced CFAM-algorithm that allows for the inclusion of more than one non-linear load. The nonlinear load description is then integrated into an iterative computation algorithm that calculates voltages and currents within the analyzed network with the help of nodal equations.

As a matter of form within the next section the basic CFAM-algorithm together with the parameterization procedure are shown. The sections thereafter present the basics of the proposed computation algorithm as well as its discussion on an illustrative example with resistive welders as non-linear loads.

## 2 BASIC CFAM-ALGORITHM

The basic algorithm for computating a CFA-Matrix for non-linear loads representation was proposed in [3]. This method involved physically-based load modeling using direct measurement by a network simulation amplifier and was therefore limited to small consumer loads like electronic ballast, small inductions motors or fluorescent lamps [4]. For bigger loads like resistive welders an improvement of the method is necessary. The resistive welder is characterized by big pulsed power with pulses rising about 0.5A/us and peak values around 1000A. The maximal apparent power of  $S_{max}$ =350kVA classifies it to the significant loads among loads at the distribution level. Since the network simulation amplifiers do not have enough output power to supply such a big device like a resistive welder and there was no possibility to change the parameters of the supply voltage in the distribution network, the measurementbased simulation model of the non linear load (welder) has to be created to calculate the coefficients of the CFA-Matrix for such load. Matlab<sup>@</sup>-Simulink<sup>@</sup> environment was chosen for this [5], [6].

Since the exact electrical data of a quasi-linear part of the welder is unknown, the welding process with transformer should be identified and incorporated into the simulation model [5]. The identification approach presented in [5] assumed that the object to parameterize consists of a serial connection of the resistance R and the inductance L. This approach can be used only for small systems, and therefore in this contribution an improved method is proposed.

#### 2.1 Improved parameter identification method

Fig.1 presents the measuring arrangement for the identification of the welding process parameters. The welder is connected between phase L1 and L2 as it is by major industry applications to reach greater welding current. In Figure 2 the currents injected by welder at different power level are shown. The dependency of the power of the machine on current distortion can be observed from the shape of the current. The device decreases its non linear character according to the growth of the power and becomes nearly linear for the maximum power.

To determine the parameters it can be generally assumed that the welding process is characterized by the discrete transmittance (admittance) determined by current  $I_s$  and voltage  $U_s$ .



Figure 1: Mesuring arrangement for the identification of the welding process parameters.



Figure 2: Waveforms of the current demanded by the welder by distinct welding power.

The discrete character comes directly from the available measurement system that also has a discrete character. Therefore, the relationship (1) can be written:

$$\frac{I_{s}(z^{-1})}{U_{s}(z^{-1})} = Y(z^{-1}) = \frac{\sum_{n=0}^{N} b_{n} z^{-n}}{1 + \sum_{n=0}^{M} a_{m} z^{-m}},$$
 (1)

where:  $N \leq M$ . For simplicity at this stage N=M=1 can be used, since even for these parameters the discrete transmittance approximates the measured current signals well at every power level. In order to incorporate the above equation into the identification algorithm, the discrete transmittance must be reformulated, as in (2):

$$I_{s}(T \cdot k) = b_{a}U_{s}(T \cdot k) + b_{b}U_{s}(T \cdot (k-1)) - a_{1}I_{s}(T \cdot (k-1)), \quad (2)$$

where T signifies the sample time and k is an algorithm step. For linear parameter identification, the Recursive Least Squares method has been used.

Substituting the *Z*- transformation in (1) by the Tustin function in the form:

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$$=\frac{\frac{2}{T}+s}{\frac{2}{T}-s}$$
(3)

the discrete transmittance can be written in the Laplace domain:

$$Y(s) = \frac{s(\frac{T}{2}b_0 - \frac{T}{2}b_1) + b_0 + b_1}{s(\frac{T}{2} - a_1\frac{T}{2}) + a_1 + 1}$$
(4)

From (4), the admittance can be simplified and the load synthesis can be formulated, as in (5):

$$Y(s) = \frac{1}{R_o} + \sum_{i=1}^{1} \frac{\frac{1}{L_i}}{s + L_i R_i}.$$
 (5)

This transmittance (5) reflects the serial connection of a resistance  $R_I$  and an inductance  $L_I$  which both are connected in parallel with a resistance  $R_0$ . The obtained structure of the parameterized object (Figure 1) can be directly applied into the Matlab<sup>®</sup>-Simulink model, as described in [5], [6]. In general, an analogous procedure can be used for parameter identification of all unknown linear elements appearing in a distribution system.

As can be seen in Figure 3 the identified parameters  $L_I$ ,  $R_I$  have different values at different power levels of the machine and this dependency changes non-linearly. The resistance  $R_0$  is not presented because it has a very high value with reference to the other parameters and can be treated therefore as an open circuit. The different values of the  $L_I$ ,  $R_I$  parameters at different power levels result from the behavior of the welding transformer.



**Figure 3:** Parameter  $L_1$  and  $R_1$  identified as a function of the welding power.

#### 2.2 Derivation of the CFA-Matrix. Basic algorithm

The computation algorithm is based on the assumption that the output current of the welder is a non linear function of the supply voltage and the load behavior can be simulated around its working point by the CFA-Matrix  $\underline{Y}_{CFA}$ . The matrix reflects the cross-frequency influence of the supply voltage on the current that is injected by the machine. It can be written as in (6):

$$i_{s} = F(u_{L12}, S_{w}) \Longrightarrow \underline{I}_{s} = \underline{\mathbf{Y}}_{CFA(U_{L12}, S_{w})} \underline{U}_{L12}, \tag{6}$$

where:  $S_w$  is an apparent power of the welder, which is depended on a firing angle of the thyristors in the welder;  $i_S$  and  $u_{L12}$  being current and voltage in time domain, respectively. This can be interpreted as a transformation of the non linear function F into the linear dependence of current and voltage in the form of the matrix  $\underline{Y}_{CFA}$  around each operation point of the machine. The CFA-Matrix transformation takes place in the frequency domain. Writing again the right side of (6)  $\underline{Y}_{CFA}$  can be represented by (7):

$$\begin{bmatrix} \underline{I}_{S(1)} \\ \underline{I}_{S(2)} \\ \vdots \\ \underline{I}_{S(K)} \end{bmatrix} = \begin{bmatrix} \underline{y}_{11} & \underline{y}_{12} & \cdots & \underline{y}_{1N} \\ \underline{y}_{21} & \underline{y}_{22} & \cdots & \underline{y}_{2N} \\ \vdots \\ \underline{y}_{K1} & \underline{y}_{K2} & \cdots & \underline{y}_{KN} \end{bmatrix}_{(\alpha)} \cdot \begin{bmatrix} \underline{U}_{L12(1)} \\ \underline{U}_{L12(2)} \\ \vdots \\ \underline{U}_{L12(N)} \end{bmatrix}, \quad (7)$$

where K and N denote the maximal order of the analyzed current and voltage harmonic order, respectively.

The element  $\underline{y}_{ii}$  describes the effect of changes of the  $i^{th}$  harmonic of the voltage on the  $i^{th}$  harmonic of the current. The element  $\underline{y}_{ij}$  describes the effect of changes of the  $i^{th}$  harmonic of the voltage on the  $j^{th}$  harmonic of the current. The diagonal components describe a linear behavior of the load, and non-diagonal ones reflect non linear phenomena in load.

The computation algorithm for the derivation of the CFA-Matrix from the developed model is described in

[3] and applied in [4], [5] and [6] for the single non-linear load.

## **3** COMPUTATION OF VOLTAGE QUALITY INDICES USING AN ITERATIVE ALGORITHM

The previous approach allows for the study of the disturbance in a single non linear load environment [3], [4], [5], [6]. In order to investigate the voltage quality in the distribution system with multiple non-linear loads an enhancement algorithm has to be developed. It should take into account any number of linear and non linear loads appearing in some wider area in the chosen distribution network [7].

Therefore, the iterative algorithm is proposed in this paper. The algorithm is based on the matrix-nodalequation. Dependent on the construction and contents of the CFA-Matrix that reflects behavior of the machine in the interesting frequencies range, the nodal equation must be created for this frequency range, as well. This means that the solution is sought for the selected group of harmonic disturbances only. Moreover, all loads must be represented by equally dimensioned square matrices. In order to better understand a construction process of the nodal-equation, two elementary types of the electrical branches are presented in Figure 4.



Figure 4: Two types of electrical branch representation: a) non linear; b) linear.

A schematic interpretation of non-linear branch connected between node j and node l of the studied circuit for  $i^{th}$  harmonic order is presented in Figure 4a. It consists of a passive part, which is described as a linear admittance for  $i^{th}$  harmonic (diagonal elements of the CFA-matrix) and an active part, which is interpreted as voltage controlled current sources. All of the generated signals for frequencies given by the voltage controlled currents are linear combinations of the harmonic voltage at the load clamps with weights established by a certain coefficient of the CFA-Matrix. The Kirchhof law for the circuit in Figure 4a for the  $i^{th}$  frequency can be written as in (8):

$$I_{(j,l),i} = (\underline{V}_{j,i} - \underline{V}_{l,i}) \underline{Y}_{CFM(j,l)}(i,i) + \sum_{k=l,k\neq i}^{N} (V_{j,k} - V_{l,k}) Y_{CFM(j,l)}(i,k),$$
(8)

where:  $V_{j,i}$  is the value of potential in node *j* for the *i*<sup>th</sup> harmonic,  $\underline{Y}_{CFA(j,l)}(i,k)$  is the coefficient of the CFA-Matrix respectively from the *i*<sup>th</sup> -row and *k*<sup>th</sup>-column.

The linear part of the system is presented in Figure 4b. It consists of a serial connection of the controlled voltage source and linear impedance with a controlled current source that is connected in parallel to them [7]. Therefore, it can be written:

$$\underline{I}_{(j,l),i} = (\underline{V}_{j,i} - \underline{V}_{l,i} - \underline{E}_{cv(j,l),i}) \frac{1}{\underline{Z}_{(j,l),i}} + \underline{I}_{cc(j,l),i},$$
(9)

where  $\underline{E}_{cv(j,l),i}$  is the controlled voltage source connected between node *j* and node *l* for the *i<sup>th</sup>* frequency which can be regulated by another potential or current of the same harmonic order. Generally,  $\underline{I}_{cc(j,l),i}$ is the controlled current source connected between nodes *j* and *l* for the *i<sup>th</sup>* frequency which can be controlled like the previous voltage source, too.  $\underline{Z}_{(j,l),i}$ represents linear impedance connected between nodes *j* and *l* of the circuit for the *i<sup>th</sup>* harmonic.

For these main types of electrical branches, the following nodal equations at every frequency must be created separately [7]:

$$\underline{I}_{i}^{n-1} = \underline{Y}_{i}^{n} \cdot \underline{V}_{i}^{n} = 
\begin{bmatrix}
\underline{I}_{1,i}^{n-1} \\
\underline{I}_{2,i}^{n-1} \\
\vdots \\
\underline{I}_{2,n-1}^{n-1}
\end{bmatrix} = \begin{bmatrix}
\underline{Y}_{11i} & -\underline{Y}_{12i} & \cdots & -\underline{Y}_{1M,i} \\
-\underline{Y}_{21i} & \underline{Y}_{22i} & \cdots & -\underline{Y}_{2M,i} \\
\cdots & \cdots & \underline{Y}_{mmi} & \cdots \\
-\underline{Y}_{m,i} & -\underline{Y}_{m2,i} & \cdots & -\underline{Y}_{MMi}
\end{bmatrix} \cdot \begin{bmatrix}
\underline{V}_{1,i}^{n} \\
\underline{V}_{2,i}^{n} \\
\vdots \\
\underline{V}_{M,i}^{n}
\end{bmatrix} (10)$$

where the component:

$$\underline{I}_{m,i}^{n-1} = \sum_{l=1;l\neq m}^{M} (\pm \sum_{k=1;k}^{N} (\underline{V}_{m,i}^{n-2} - \underline{V}_{l,i}^{n-2}) \underline{Y}_{CFA(m,l)}(i,k)$$

$$\pm \underline{I}_{cc(m,l),i} \pm \underline{E}_{cv(m,l),i} \frac{1}{\underline{Z}_{(m,l),i}})$$
(11)

determines the coefficients of current vector from the nodal equation (10). The values in this vector result directly from active elements in the considered circuit. These are both linearly controlled sources ( $\underline{L}_{cc(j,l),i}$ ) which come from linear electrical branch representation (9) and current sources, which reflect non linear behaviour of the load (8). The nodal matrix  $\underline{Y}_i^n$  is built by passive elements (diagonal components of the CFA-Matrices), where: *i* is the harmonic order and *n* an iteration step.

The algorithm for computation of the power quality quantities, currents and voltages for studied circuit is presented in Figure 5. At first, the CFA–Matrix model of the load has to be created according to the modelling process that is described in previous sections.



**Figure 5:** Algorithm for calculation of voltage quality quantities using CFA-Matrix models for non-linear loads.

Then the model is incorporated into the studied network, building the respective matrix-nodal-equation, as shown in (10). In the calculation step this equation is computed at every studied frequency, iteratively. As a result of the iteration, voltages and currents in the studied circuit are obtained on the given accuracy level that can be determined (see "*error block*" in Figure 5). These quantities, computed for desired range of frequencies, are then subjected to a procedure that calculates voltage quality indices.

## 4 AN ILLUSTRATIVE EXAMPLE

In this example it was assumed that three resistive welders with the characteristic as described in section 2, are connected to a distribution system as in Figure 6. The three welders work at different power levels of  $S_1=20\%$ ,  $S_2=40\%$  and  $S_3=60\%$ , respectively, representing certain technological process. Moreover, it was assumed, that the voltage supplying the machines contains the harmonic mix of  $3^{\text{rd}}$ ,  $5^{\text{th}}$  and  $7^{\text{th}}$  harmonic with 1%, 5% and 2% of the fundamental component magnitude, respectively [6],[8]:

$$V_{j\in\{1,2,3\}} = \begin{bmatrix} 340\\ 3.4\\ 17.0\\ 6.8 \end{bmatrix}$$
(12)

In order to study the voltage quality with reference to disturbances introduced by the three working welders, the linear parts of the machines were identified. Matlab<sup>@</sup>-Simulink<sup>@</sup> models were created and CFA-Matrices were computed and integrated into the algorithm as described in the previous section. As a result of calculation the following voltage amplitudes for  $\{1, 3, 5, and 7\}$  harmonics were obtained:

$$|V_{1}| = \begin{bmatrix} 330.1 \\ 6.5 \\ 15.8 \\ 4.9 \end{bmatrix}; |V_{2}| = \begin{bmatrix} 333.9 \\ 4.6 \\ 15.9 \\ 7.1 \end{bmatrix}; |V_{3}| = \begin{bmatrix} 329.2 \\ 5.5 \\ 14.3 \\ 7.8 \end{bmatrix}$$
(13)

Comparing (13) to the supply voltage, which contains a symmetrical certain harmonic content for all phases (12), it can be seen that the machines cause non-symmetrical voltage disturbances at the clamps at every

analyzed frequency. Moreover, voltage for the fundamental frequency falls to about 97% and the magnitude of the 5<sup>th</sup> harmonic is reduced by about 8%. For the 3<sup>rd</sup> harmonic a significant increase of its contents in the voltage by about 50% can be observed. The 7<sup>th</sup> harmonic is more unsymmetrical than the others, but increase of its contents for all phases is small.

In the following, the changes of the THD for the voltage due to the welder working point changes and supply condition are investigated. In Figure 7 the THD for the voltage is presented as a function of the welding power of the two machines connected to node  $V_I$ , the third welder works at 40% of the maximum apparent power. The best conditions for THD are obtained around symmetrical load [9] of about 40% power for all welders. The small shifts from the area of 40% power that can be seen in Figure 7 result from other harmonics existing in the supply voltage. The maximal THD value of about 6.5% appears when the welders work either at 60%, 40% and 80% or at 80%, 40%, 20%. In general, the THD for the first node varies in limits 0.053-0.065.



Figure 6: Simplified circuit with three welders working with different welding power, equated by CFA-Matrices.



Figure 7: Influence of the machines' power connected to the first phase on the voltage THD.



**Figure 8:** Influence of the machines' power connected to the first phase on the  $3^{rd}$  harmonic current injected by the machines.



**Figure 9:** THD for voltage as function of welding power changes and phase changes of the supply voltage 5<sup>th</sup> harmonic.

In Figure 8 the amplitude of the third harmonic current for the first phase is shown. It can be seen, that for the same welding power of both welders connected to the first node, the amplitude of the third harmonic is the smallest. As presumed, the current injected by the welders is tapped in the triangle (Figure 6) and does not flow into the distribution system. This case confirms that the (3n)<sup>th</sup> harmonic currents for this kind of connection are not generated in the network [1], [7], [9].

The THD for the voltage as a function of the welding power changes of the first welder and phase changes of the  $5^{\text{th}}$  harmonic appearing in supply voltage is presented in Figure 9. The local minimum occurs when the welder works at 60% power (symmetrical load according to first node) and the phase shift is about 100 degrees, then THD equals 4.3%. The maximal THD value at the first node appears at 60% of the welder power and at phase shift of about 300 degrees.

# **5** CONCLUSIONS

Distortion of voltage and current waveforms caused by harmonics is one of the major power quality problems. Nowadays, most devices connected to distribution system have a non-linear character and generate higher harmonics in the network [9], [10]. In order to improve the calculation exactness flexible models of the load should be created and used to estimate power quality quantities [7].

In this paper a novel measurement-based non-linear load modeling method was proposed and extended by an iterative load-flow algorithm for the voltage quality estimation in the network. The approach improves the exactness of the calculations and enables a detailed study to be made according to the desired error of simulation. The method is based on the measured data from a non-linear pulsed device (welder) that is used to create an equivalent model in the form of a CFA-Matrix. This representation of the non-linear load is then integrated into a computing algorithm that is based on nodal equations. This extension makes the CFA-Matrix description applicable for voltage quality study in the chosen area of the power network. An illustrative example has shown how an exact solution can be found regarding different modes of operation of the resistive welder.

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