ELEN0037
Microelectronics
Tutorials

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Tutorial 4: Switched-Capacitor circuits, Filters

Exercise 1 (1st, P10.5/2nd, P14.11)

Find the capacitance values needed for a first order switched-capacitor circuit such that its 3-dB point is at $10\,kHz$ when a clock frequency of $100\,kHz$ is used. Use the bilinear transform. It is also required that the discrete-time zero is at $z = 0$ and that the DC gain be unity. Assume $C_A = 10\,pF$.[1][2] What is the gain at $f = 50\,kHz$?[3] The transfer function of a first order switched-capacitor circuit is given by:

$$H(z) = \frac{-\left(\frac{C_1+C_2}{C_A}\right)z + \frac{C_1}{C_A}}{\left(\frac{C_A+C_3}{C_A}\right)z - 1},$$

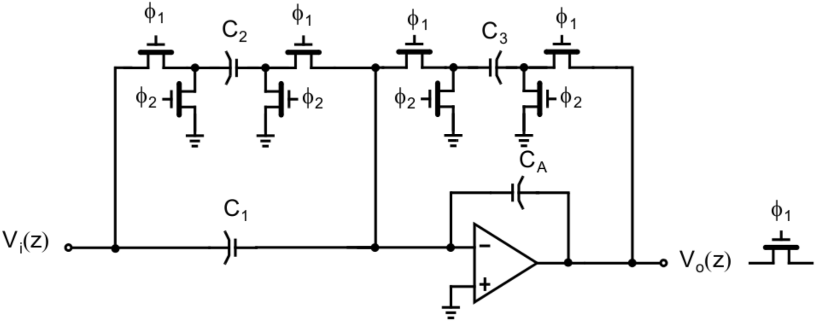
with a zero at $\frac{C_1}{C_1+C_2}$, and a pole at $\frac{C_A}{C_A+C_3}$.

[1] $H(z) = \frac{0.46673}{z-0.53327}$

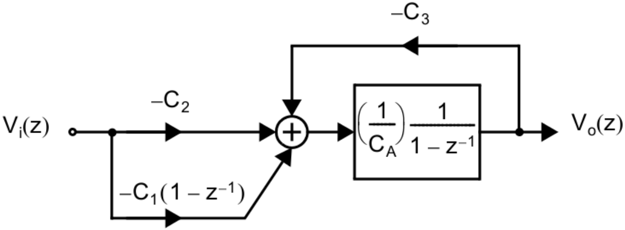
[2] $C_1 = 0,\ C_A = 10\,pF,\ C_3 = 8.752\,pF,\ C_2 = -C_3$ ($\Rightarrow$ differential input)

[3] $G = |H(-1)| = 0.304 = -10.33\,dB$

Exercise 1 (first order switched-capacitor circuit)



(*a*)



Exercise 2 (first order switched-capacitor circuit)

Based on the previous exercise:

1. What is the gain at $f = 10\,kHz$?[4]
2. What would you expect, knowing that $f = f_{3-dB}$?[5]
3. Re-compute the transfer function, by using the exact transform $z = e^{j\omega T}$ in order to find the 3-dB point exactly at $10\,kHz$.[6]
4. What is the real 3-dB frequency in the first approach?[7]
5. Derive the discrete-time relationship of this first order switched-capacitor circuit and propose an implementation in a digital system (DSP, µC, FPGA).[8]

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[4] $G = \left|H(e^{j2\pi\frac{10}{100}})\right| = 0.719$

[5] We expect $G = \frac{1}{\sqrt{2}} = 0.707\ldots$

[6] $H(z) = \frac{0.45589}{z-0.54411}$, $\left(z_p = \alpha = 2 - \cos(\omega_0 T) - \sqrt{(\cos(\omega_0 T) - 2)^2 - 1}\right)$

[7] $f_0 = 10.354\,kHz$, $\left(f_0 = \frac{1}{2\pi} f_s \arccos\left(\frac{4\alpha - \alpha^2 - 1}{2\alpha}\right)\right)$

[8] $y[k] = \alpha y[k-1] + (1-\alpha)x[k] = y[k-1] + (1-\alpha)(x[k] - y[k-1])$

Exercise 3 (1st, P10.6/2nd, P14.12)

Find the capacitance values needed for the same first order switched-capacitor circuit such that its 3-dB frequency is at $1\,kHz$ when a clock frequency of $50\,kHz$ is used. Use the exact transform. It is also required that the discrete-time zero is at $z = 0$ and that the DC gain be unity. Assume $C_A = 50\,pF$.[9] What is the gain (in dB) at $f = 25\,kHz$?[10]

[9] $C_1 = 0,\ C_3 = 14.202\,pF,\ C_2 = -C_3$ ($\Rightarrow$ differential input)

[10] $G = |H(-1)| = 0.1244 = -9\,dB$

Exercise 4 (1st, P10.7/2nd, P14.13)

Find the transfer function of the first order switched-capacitor circuit, when $C_1 = 0\,pF$, $C_2 = 2\,pF$, $C_3 = 2\,pF$, and $C_A = 20\,pF$.[11] What is the magnitude and phase of the gain at DC, ${}^{f_s}/_4$, and ${}^{f_s}/_2$?[12][13] The transfer function is given by (same as before):

$$H(z) = \frac{-\left(\frac{C_1+C_2}{C_A}\right)z + \frac{C_1}{C_A}}{\left(\frac{C_A+C_3}{C_A}\right)z - 1},$$

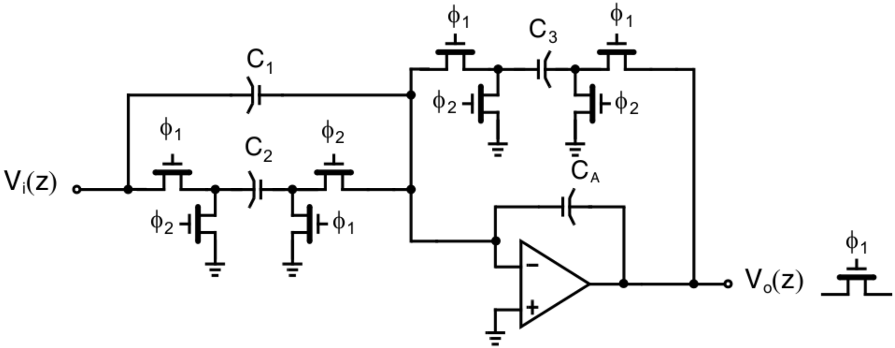
[11] $H(z) = \frac{-0.1z}{1.1z-1}$

[12] $G_{\frac{f_s}{4}} = |H(e^{j\frac{\pi}{2}})| = 0.0673,\ \angle = 137.73^\circ$

[13] $G_{\frac{f_s}{2}} = |H(e^{j\pi})| = 0.0476,\ \angle = 180^\circ$

Exercise 5 (1st, P10.8/2nd, P14.14)

Show that the transfer function of the following SC circuit

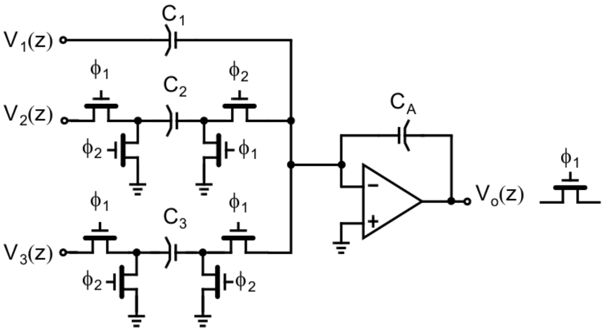


is given by:

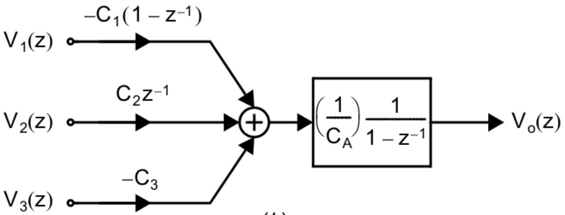
$$H(z) = \frac{-\left(\frac{C_1}{C_A}\right) z + \frac{C_1+C_2}{C_A}}{\left(\frac{C_A+C_3}{C_A}\right) z - 1}.$$

Use the signal-flow-graph analysis. Compare this transfer function to the one used previously.

Exercise 5 (Signal-Flow-Graph Analysis review)



(a)



(b)

Exercise 6 (1st, P10.9/2nd, P14.16)

Show that when $K_6 = 0$ in the low-Q switched-capacitor biquad circuit, if the poles are complex, they lie precisely on the unit circle (i.e. the circuit is a resonator).[14] The transfer function of the low-Q SC biquad is given by:

$$H(z) = -\frac{(K_2 + K_3)\, z^2 + (K_1 K_5 - K_2 - 2K_3)\, z + K_3}{(1 + K_6)\, z^2 + (K_4 K_5 - K_6 - 2)\, z + 1}.$$

[14] $K_6 = 0 \Rightarrow D(z) = z^2 + (K_4 K_5 - 2)\, z + 1 \Rightarrow z_1 z_2 = 1,\ z_2 = z_1^* \Rightarrow |z_1| = 1$