OPTIMAL LOCATION OF VOLTAGE REGULATORS IN RADIAL DISTRIBUTION NETWORKS USING GENETIC ALGORITHMS

J. Mendoza, E. López
Department of Electrical Engineering
University of Concepción
Concepción-Chile
jorgemendoza@udec.cl
elopez@die.udec.cl

D. Morales, R. López, M. López, J.C. Vannier
École Supérieure d'Électricité
Paris-France
dario.morales@supelec.fr
rodrigo.lopez@supelec.fr
miguel.lopez@supelec.fr
jean-claude.vannier@supelec.fr

Abstract—. In rural power systems, the automatic voltage regulators help to reduce energy losses and to improve the energy quality of electric utilities, compensating the voltage drops through distribution lines. The use of automatic voltage regulators is constrained, especially in developing countries, due to their elevated investment cost. In order to help electric companies (utilities) in the decision making process, this paper presents a method to define the number of voltage regulators and their optimal position in radial networks. The bi-objective task is based on the minimization of an individual penalty objective function, considering the voltage deviations and energy losses in the network. The optimization program is formulated as Genetic Algorithm problem.

Keywords— Optimization methods, Voltage regulators, Power distribution, Voltage control, Losses, Genetic algorithms.

1 NOMENCLATURE

BE_{NVR}	:	Benefit Index for each NVR case
$\Delta V^{j}_{Regulator}$	:	Voltage variation of each regulator
I_{VRj}	:	Current "j" for VR
$I_{max\ VRj}$	:	Maximal current for j-th VR
i_j	:	Line current
L_T	:	Total power loss index
NL, NB	:	Total lines and node number
NVR	:	Total Voltage regulator number
OF	:	Objective function
$Of_{O.S}$	:	Objective function original system
Rj	:	Series resistance of distribution line
V_D	:	Deviation Voltage index
V_K	:	Voltage for the k-th node
V^{j}_{min}, V^{j}_{max}	:	Minimal and maximal voltage values for each regulator
w_L, w_V	:	Weighting coefficient

2 INTRODUCTION

Keeping voltage profile within certain limits helps to reduce energy losses and improve voltage regulation. Voltage control is a difficult task because voltages are strongly influenced by random load fluctuations. For this reason, Utilities reinforce their power systems in order to have a direct control over voltage variations. Improving system's operation benefits both, Utilities and customers, [1].

Voltage profile can be improved by the use of analytical tools such as optimal power flow, voltage stability, failure indicators analysis etc, and by the installation of devices such as fixed and controlled capacitors banks, automatic voltage regulators and transformers with on-load tap changers, [2]-[3]. The use of new devices is constrained by their elevated investment cost. For these reason, the optimal placement of these devices becomes an important issue.

For many years, researchers have worked to define the optimal number, location and sizing of capacitors banks to achieve voltage control while all operational constraints are satisfied, at different loading levels. Many optimization techniques have been used such as heuristic methods, expert systems, simulated annealing and neural network, [4]. Recently, fuzzy logic, evolutionary algorithms have been used, where the objective function is defined taking into account losses reduction, voltage constraints and total cost, [1] and [5]-[8].

Losses reduction and improvement of voltage profile have been also studied using on-load tap changers. The optimal power flow analysis is used to determine the optimal tap position and the ON/OFF state of capacitor bank, [9]. The same problem is solved in [10] using the loss equation as objective function and voltage inequalities as constraints through the neural network techniques. In [11], the optimal control of tie-switches is also studied. While in [12], the same problem is solved using multi-objective optimization technique, minimizing energy losses and voltage regulation.

In [13]-[15], the optimal number and location of automatic voltage regulators (VRs) are studied separately from the placement and sizing of capacitor banks problem, and aspects of power approach and energy losses are considered, outside the main problem-solving process. Finally, in the work of Safigianni and Salis in [16], the number and location of VR are determined by using a sequential algorithm. In this paper, the objective function is defined by using the VR's investment and maintenance costs, taking into account the energy loss reduction.

In brief, the optimal location of capacitor banks problem has been widely studied. However, there are only a few publications that have treated the complex problem of optimal location of VR in distribution networks even if the benefits of including VR devices are well-known, [17].

The method presented in this paper separates the original problem in two parts. Mapping of VR's number, the first part, consists in determining the optimal position of the VRs in the system, solving a multi-objective optimization problem. The second part, consists in choosing the number of VR. To do this, a decision making process is carried out through a benefit analysis decoupled from the main optimization solving-process.

The multi-objective problem of minimizing the active power losses and the voltage deviation locating voltage regulators is solved as a single-objective problem using the weighting method, because of its adaptability for solving combinatorial problems. The objective function is minimized using the genetic algorithm. The proposed method takes into account the rated power and tap constraints of VR.

3 PROBLEM FORMULATION AND PROPOSED SOLUTION

As explained in [16], the optimization problem can be separated in three subproblems: locating the voltage regulator on the network, the selecting the tap position and the necessary number of VRs.

3.1 Optimal location of voltage regulators

The optimal location problem of a VR is defined as function of two objectives, one representing power losses reduction and the other one representing voltage deviations. Both are essential to ensure the security of power supply. It is important to note that, the minimization of one of these objectives involves the diminution of the other one, but not necessarily its minimization. It is difficult to formulate the problem in terms of cost incidence of these objectives over the system operation. Because, even if the cost incidence of power losses is clear, it is not the same for keeping the voltage values at the nodes close to the rated value, [12].

The objective function to minimize is:

$$Min\ \mathrm{OF} = \mathrm{w_L} L_T + w_V V_D \tag{1}$$

The power losses and voltage deviation indices are defined using eq. 2 and 3.

$$L_T = \sum_{j=1}^{NL} i_j^2 \cdot R_j \tag{2}$$

$$V_D = \sqrt{\sum_{k=1}^{NB} (V_k - 1)^2} \tag{3}$$

$$V_{\min}^{j} \leq \Delta V^{j}_{\mathrm{Re}\,gulator} \leq V_{\max}^{j} \tag{4}$$

$$I_{VR\,j} \leq I_{max\,VR\,j} \tag{5}$$

The voltage range, and the rated current of each voltage regulator, represent the constraints of the optimization problem, as shown in eq. 4 and 5:

3.2 Selection of tap position

The determination of tap position of each VR is essential for solving the localization problem. In this kind of application tap adjustment, via successive displacement, can drive to inadequate solutions or convergence troubles. For this reason, a Newton-Raphson load flow algorithm, modeling the tap position as a state variable, is used. This improves the performance of the optimization process.

3.3 Number of voltage regulators

The number of VR becomes an important issue because of the elevated investment cost of these devices. For economical reasons, Utilities may be only interested in re-locating a fixed number of VR. In order to give more flexibility, in this paper, the optimal number of VR is decoupled from the optimal location problem.

One possible approach to evaluate economical benefit, when installing new devices on the power system, is the Net Present Value calculation. It considers investment and maintenance costs, interest rate and other economical variables. In this paper, a reliability approach of marginal benefits is considered because it is simple to use and gives good technical information to the decision making process. A marginal benefit coefficient is calculated when adding a new VR on the system. Then, decision making process is carried out, by taking into account this coefficient, to select the number of VR.

The benefit is calculated as follows:

$$BE_k = \frac{\left(1 - {OF_k}/{OF_{O.S}}\right)}{NVR} \times 100\% \tag{6}$$

3.4 Search engine

The GA technique is used as search engine in the optimization process. This technique uses the principle of natural selection to create a set of individuals (population) that are evaluated and compared using the objective function. Holland in [18], was the pioneer in the development of this technique, and since then, it has been applied to a wide range of optimization problems.

The algorithm's structure is based on the generation of a population of individuals that represents the possible solution of the problem (generation). The individuals that have the greatest aptitude are selected in order to create a new population using cross and mutation operators (next generation). The evolutionary characteristic of this procedure allows to get or reach, the best solution, [19]-[21].

A string of binary numbers is used to represent each individual (chromosome). The string symbolizes the line where the VRs are located on the power system. This allows us to reduce the exploration universe to a 100% feasible initial population.

During GA process, the selection of individuals is done by a probabilistic tournament with uniform distribution. The principal operator interchanges genetic material using simple crossover. The uniform mutation is applied randomly to the binary string, changing one gene of the individual. Once this process is finished, the feasibility of the generated individuals is evaluated using a filter. Then, the resulting population is evaluated using the objective function and the operational constraints. Their aptitudes are compared with those of their parents. Subsequently, the individual with the highest aptitude is selected. Finally, the elitistic procedure is used to assure the conservation of the best individual.

The algorithm stops after a certain number of generations. This number is selected, considering the size of the analyzed systems.

3.5 Objective function evaluation

In order to evaluate the objective function, the Newton-Raphson load flow algorithm is used. Here, the tap position of each VR is considered as a state variable, allowing it to be automatically adjusted within the iterative process, [22].

The general formulation of the Newton-Raphson load flow algorithm is:

$$\begin{bmatrix} H & N & D_p \\ M & L & D_q \end{bmatrix}^k \cdot \begin{bmatrix} \Delta\theta \\ \Delta V/V \\ \Delta t/t \end{bmatrix}^k = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}^k \tag{7}$$

D_p and D_q, contain as many columns as voltage regulators and their coefficients ($t\partial P/\partial t$ and $t\partial Q/\partial t$) are calculated using the pi-equivalent transformer model. If one of the taps attains the maximum or minimum value allowed, the regulated node becomes a PQ node unchanging the tap position. In this work, a constant power load model is used.

Once the load flow is finished, eq. 2 and 3 are used to evaluate the power losses and voltage deviation indices.

3.6 The weighting method

Weighting the objectives to obtain non-inferior solutions is a method derived from the necessary conditions of non-inferiority developed by Kunh and Tucker, [23]. The method consists in assigning weights to the various objective functions in order to generate an equivalent single-objective optimization problem. The scalar coefficients that multiplies each objective function is called weight and can be interpreted as “the relative weight or worth” of one objective when compared to the other objectives, [24]. Usually the weights are normalized using eq. 8.

$$\sum_{J=1}^{n} W_j = 1 \tag{8}$$

A decision matrix that represent the objective functions values is calculated for each individual. As this method requires a comparable scale for all elements in the decision matrix, a normalization process is needed. Consequently, the value of each objective is divided by its highest possible value, [25]. In this work, the objective function can be rewritten as follows:

$$\mathrm{OF} = \mathrm{w_L} \frac{L_T}{L_{T\max}} + w_V \frac{V_D}{V_{D\max}} \tag{9}$$

$L_{T\max}$ and $V_{D\max}$ are maximal the power losses and voltage deviations for the initial system (non-optimized). In order to simplify the analysis, the *standard weighting coefficients* can be redefined as follows:

$$w_L' = \frac{w_L}{L_{T\,max}} \;\; ; \;\; w_V' = \frac{w_V}{V_{D\,max}} \tag{10}$$

Equations 1 and 6 are now written as function of the standard weighting coefficient as follows:

$$Min\ \mathrm{OF} = \mathrm{w_L}' L_T + w_V' V_D \tag{11}$$

$$BE_{NVR} = \frac{1 - OF_{NVR}}{NVR} \times 100\% \tag{12}$$

3.7 Location algorithm description

The input data for the proposed method are the line parameters, loads, location, and rated values of capacitors banks, number of voltage regulators to be installed, and to assign the importance values for each objective (weighting coefficients).

The algorithm begins with the evaluation of the aptitude of the original system for determining the standard weighting coefficients. Then, the initial population for the GA is created, in order to start the evolutionary process.

The result giving by the algorithm are VRs location and tap position.

Figure 1, shows the block diagram for the proposed method.

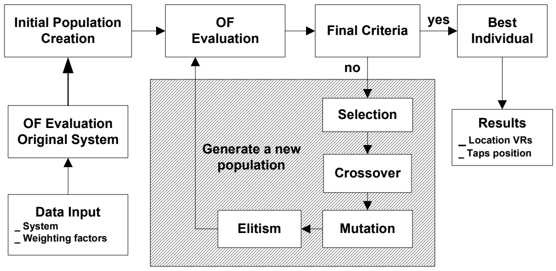

Figure 1: Block diagram for the optimal placement algorithm.

4 APPLICATIONS

The algorithm proposed in this paper was developed in MATLAB. To let the GA have a large searching space, the selection, crossover and mutation probabilities were set at 90%, 80% and 70%, respectively. The generation of individuals' number are specified, according to the system's dimension.

Test system

The proposed method is applied to a radial test system with 16 lines and 17 nodes, shown in fig. 2. A summary of the test system is given in appendix I.

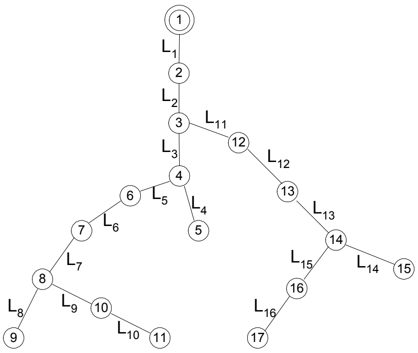

Figure 2: Test system

The results of the optimization process using 1 VR on the test system are shown in table 1. A sensibility analysis has been done, in order to evaluate the influence of the weighting coefficients in the final solution. The number of individuals was five. The process was stopped after 20 generations.

w_L, w_v	1 – 0	0 – 1	0.5 – 0.5
OF	0.9729	0.5342	0.7536
Location	2	2	2
Tap position	0.963	0.963	0.963

Table 1: Test system 1 VR

Considering three possible cases for the weighting coefficients: losses minimization (1-0), voltage deviation minimization (0-1) and objectives with the same relative importance, the optimization process gave the same results, locating the VR at the end of line 2.

The problem was solved for 2 VRs using the same weighting coefficients, result are shown in table 2.

w_L, w_v	1 – 0	0 – 1	0.5 – 0.5
OF	0.9650	0.3421	0.6567
Location	2 – 6	2 – 7	2 – 5
Tap position	t_1: 0.942 t_2: 0.995	t_1: 0.942 t_2: 0.969	t_1: 0.962 t_2: 0.975

Table 2: Test system results: 2 VR

For all the three cases, the optimization algorithm gave different results. However, the VR located at the end of line 2 is repeated.

Considering the case in which both weighting coefficients are equal, the evolution of the best individual, i.e. minimum Objective Function value (y-axis), for each generation of the GA (x-axis), is shown in fig. 3. The solution is achieved after 8 generations.

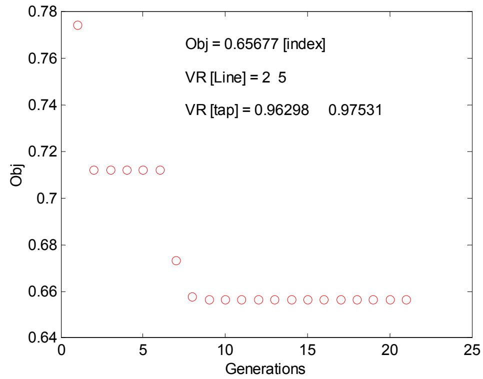

Figure 3: Best individual v/s generation number.

A summary of results, to locate 3 and 4 VRs, is given in appendix II.

In order to compare the results of the GA, an exhaustive search was performed for 1 and 2 VRs. This analysis confirms that the results given by the GA process correspond to the optimal solution.

The results of the exhaustive search for 2 VRs and weighting coefficients of 0.5, are shown in fig 4.

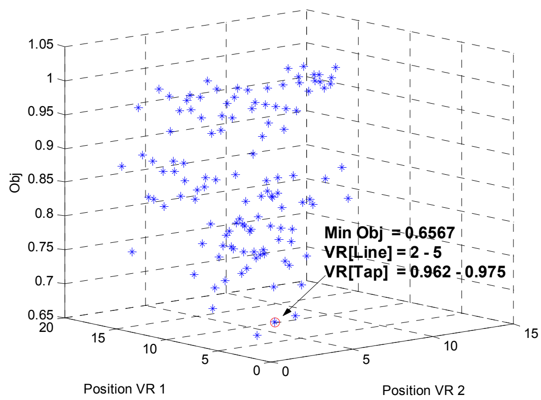

Figure 4: Test system: Exhaustive search for 2 VRs

The benefit analysis for this system is done using eq. 12. The results of this analysis is shown in table 3.

w_L, w_v	1 – 0	0 – 1	0.5 – 0.5
1 VR	2.71	46.57	24.64
2 VR	1.74	32.89	17.16
3 VR	1.35	24.99	12.58
4 VR	1.01	19.32	10.06

Table 3: Benefit index for test system (%)

The influence of increasing the number of VRs on the system, is more important on the voltages deviation index than on the energy losses reduction. In the case of 1 VR, the voltage deviation index reach a reduction of about 46%, with respect to the original system. If only the energy losses index is considered, a reduction of 2.7% is obtained. However, 2.7% of losses reduction is economically interesting.

In this test system, it is clear that the highest benefit is given with 1 VR on the system. Nevertheless, the final decision of how many VRs depends on the decision maker expectations.

Real system

A real system of 229 nodes is analyzed. The system's line diagram, specifications, and results are detailed in [13]. The method proposed, in this reference, gave as result the location of 1 VR at the end of the line 36. This result is used to compare the performance of the proposed method.

The evolution of the best individual, i.e. minimum Objective Function value (y-axis), for each generation of the GA (x-axis), for 1 and 3 VRs are shown in fig. 5 and 6, respectively.

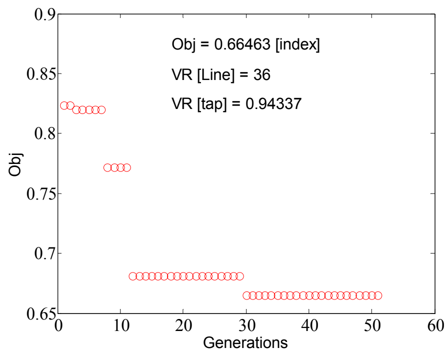

Figure 5: Best individual evolution for the real system with 1 VR

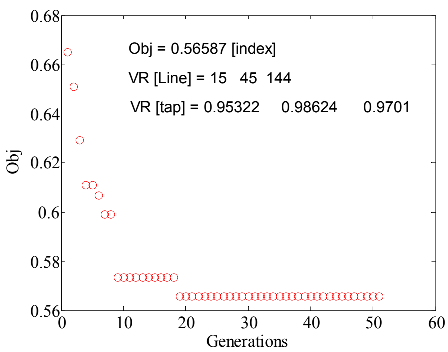

Figure 6: Best individual evolution for the real system with 3 VRs

In table 4, a summary of results obtained for 1 to 4 VRs location considering $w_L = w_V = 0.5$, are shown (objective function value, tap positions).

To verify the exact position for 1 VR with equal weighting coefficients, an exhaustive search process is performed.

NVR	1	2	3	4
OF	0.6646	0.6201	0.5658	0.5593
Location	36	15 – 140	15 – 45 – 144	13 – 36 – 96 – 166
Position t1: t2	t_1: 0.9433	t_1: 0.9532 t_2: 0.9797	t_1: 0.9532 t_2: 0.9862 t_3: 0.9701	t_1: 0.9631 t_2: 0.9780 t_3: 0.9866 t_4:0.9733

Table 4: Real system's results

In fig. 7 the objective function v/s the VR location, using the exhaustive search method for 1 VR location, is depicted. The arrow indicates the optimal solution. The VR is located at the end of line 36. In fig. 8 the position of the VR is depicted v/s the energy losses and voltage deviation indices.

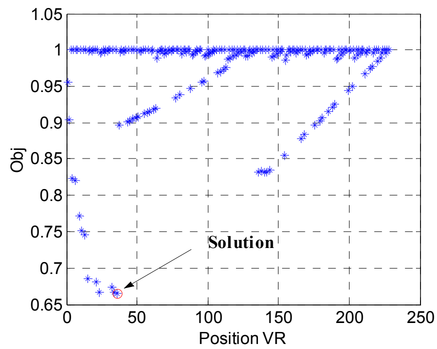

Figure 7: Exhaustive search: Objective function evaluation for the real system with 1 VR

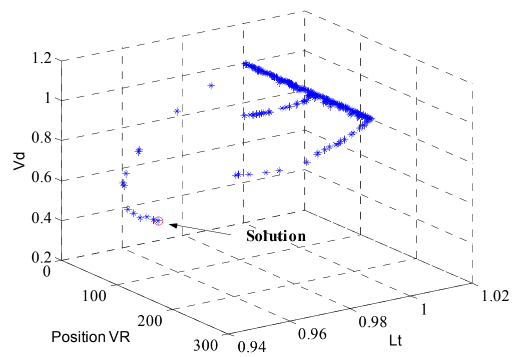

Figure 8: Exhaustive search: Energy losses and voltage deviation indices evaluation for the real system with 1 VR .

The benefit analysis for this system is shown in table 5.

w_p, w_v	0.5 – 0.5
1 VR	33.54
2 VR	18.99
3 VR	14.47
4 VR	11.01

Table 5: Benefits indices for the real system (%)

From table 5, the use of 2 VRs improves the value of the objective function about 20% with respect to the original system. Once again, the total amount of VRs depends on the decision maker criterion.

5 CONCLUSION

In this work, the optimal location of voltage regulators in an energy distribution system is studied. The multi-objective problem is formulated as a weighted-objective optimization problem using the GA technique. Constraints, such as the maximal deviation of tap position, and the standardized nominal values of VRs, are considered. The objective function is expressed as a function of energy losses and voltage deviation indices. A tap position state variable Newton-Raphson Load Flow algorithm allows us to calculate the tap position of each voltage regulator directly avoiding the convergence problems without making any simplification in system modeling. Because of its simplicity, the number of VRs is determined by using the benefits index as the decision making criterion.

The method's performance is evaluated with a simple 17-nodes test system, and with a 229-node real system. In both cases, the procedure gave important information to the decision maker, for finding the number, and the optimal location of voltage regulators on the system.

6 APPENDIX I

The line and loads data for the test system are:

in	out	R(º/1)	X(º/1)	MW	MVAr
1	2	0.05	0.05	0.8	0.6
2	3	0.11	0.11	0.8	0.6
3	4	0.15	0.11	0.8	0.6
4	5	0.08	0.11	0.8	0.64
4	6	0.11	0.11	1.2	0.16
6	7	0.04	0.04	0.8	-0.16
7	8	0.80	0.11	0.6	0.48
8	9	0.075	0.10	1.6	1.08
8	10	0.09	0.18	2.0	0.72
10	11	0.04	0.04	0.4	0.36
3	12	0.11	0.11	0.24	-0.20
12	13	0.04	0.04	1.8	0.80
13	14	0.09	0.12	0.4	0.36
14	15	0.11	0.11	0.4	-0.44
14	16	0.08	0.11	0.4	0.36
16	17	0.04	0.04	0.84	-0.32

Table 6: Test System line and loads data

The rated value for the VR are: 5, 10 and 15 MVA with a tap variation range of ±10%.

7 APPENDIX II

w_p, w_v	1 – 0	0 – 1	0.5 – 0.5
OF	0.9594	0.2502	0.6224
Location	1 – 2 – 6	2 – 3 – 7	2 – 6 – 7
Tap position	t_1: 0.983 t_2: 0.973 t_3: 0.972	t_1: 0.926 t_2: 0.980 t_3: 0.962	t_1: 0.942 t_2: 0.995 t_3: 0.971

Table 7: Test system's results using 3 VR

w_p, w_v	1 – 0	0 – 1	0.5 – 0.5
OF	0.9595	0.2271	0.5973
Location	1–2–6–16	2–3–6–7	2–3–6–7
Tap position	t_1: 0.983 t_2: 0.973 t_3: 0.972 t_4: 0.989	t_1: 0.926 t_2: 0.980 t_3: 0.989 t_4: 0.971	t_1: 0.926 t_2: 0.980 t_3: 0.989 t_4: 0.971

Table 8: Tests system's results using 4 VR

8 ACKNOWLEDGMENT

The authors would like to thank to the University of Concepción, Chile and the National Commission for the research science and technology of Chile (CONICYT), for their support in this research. Mr. J. Mendoza is schistose of CONICYT for his doctoral studies.

REFERENCES

[1] Borka Milosevic, Miroslav Begovic, "Capacitor Placement for Concervative Voltage Reduction on Distribution Feeders," *IEEE Trans. on Power Delivery*, vol. 19, Nº3 pp. 1360–1367, July 2004.

[2] H. Vu, P. Pruvot, C. Launay, Y. Harmand, "An Improved Voltage Control on Large-Scale Power Systems" *IEEE Trans. on Power Systems*, vol. 11, Nº3 pp. 1295–1303, August 1996.

[3] Show-Kang Chang, George Marks, Ken Kato, "Optimal Real Time Voltage Control" *IEEE Trans. on Power Systems*, vol. 5, Nº3 pp. 750–758, August 1990.

[4] H. N. Ng, M. M. A. Salama, A. Y. Chikhani, "Classification of Capacitor Allocation Techniques" *IEEE Trans. on Power Delivery*, vol. 15, Nº1 pp. 387–392, January 2000.

[5] M. A. S. Masoum, A. Jafarian, M. Ladjevardi, E. F. Fuchs, W. N Grady, "Fuzzy Approach for Optimal Placement and Sizing of capacitor Banks in the Presence of Harmonic" *IEEE Trans. on Power Delivery*, vol. 16, Nº2 pp. 822–829, April 2004.

[6] Benemar Alencar de Souza, Helton do Nascimento Alves, Helvio Alves Ferreira, "Microgenetic Algorithms and Fuzzy Logic Applied to the Optimal Placement of Capacitor Banks in Distribution networks" *IEEE Trans. on Power Systems*, vol. 19, Nº2 pp. 942–947, May 2004.

[7] M. A. S. Masoum, M. Ladjevardi, A. Jafarian, E. Fuchs, "Optimal Placement, Replacement and Sizing of Capacitor banks in distorted Distribution Networks by Genetic Algorithms" *IEEE Trans. on Power Delivery*, vol. 19, Nº4 pp. 1794–1801, October 2004.

[8] J. Chiou, C. Chang, C. Su, "Ant Direction Hybrid Differential Evolution for Solving Large Capacitor Placement Problems" *IEEE Trans. on Power Systems*, vol. 19, Nº4 pp. 1794–1800, November 2004.

[9] C. J. Bridenbaugh. D. A. DiMascio, R. D'Aquila, "Voltage Control Improvement Through capacitor and Transformer tap Optimization" *IEEE Trans. on Power Systems*, vol. 7, Nº1 pp. 222–226, February 1992.

[10] Z. Gu, D. T. Rizy, "Neuronal Network For Combined Control of capacitor banks and Voltage Regulators in Distribution Systems" *IEEE Trans. on Power Delivery*, vol. 11, Nº4 pp.1921–1928, October 1996.

[11] I. Roytelman, V. Ganesan, “Modeling of Local Controllers in Distribution Network Application” *IEEE Trans. on Power Delivery*, vol. 15, Nº4 pp.1232–1237, October 2000.

[12] A. Augugliaro, L. Dusonchet, S. Favazza, E. Riva, “Voltage Regulation and Power Losses Minimization in Automated Distribution Networks by an Evolutionary Multiobjective Approach” *IEEE Trans. on Power Delivery*, vol. 19, Nº3 pp.1516–11527, August 2004.

[13] S. Civanlar, J. J. Grainger, “Volt/Var Control on Distribution Systems with lateral Branches Using Shunt Capacitors and Voltage Regulators, Part I The Overall Problems”, *IEEE Transactions on Power Apparatus and Systems,* Vol. PAS-104, Nº1, November 1985.

[14] S. Civanlar, J. J. Grainger, “Volt/Var Control on Distribution Systems with lateral Branches Using Shunt Capacitors and Voltage Regulators, Part II The Solution Method”, *IEEE Transactions on Power Apparatus and Systems,* Vol. PAS-104, Nº1, November 1985.

[15] S. Civanlar, J. J. Grainger, “Volt/Var Control on Distribution Systems with lateral Branches Using Shunt Capacitors and Voltage Regulators, Part III The Numerical Results”, *IEEE Transactions on Power Apparatus and Systems,* Vol. PAS-104, Nº1, November 1985.

[16] A. Safigianni, G. Salis, “Optimum Voltage Regulator Placement in radial Power Distribution Network”, *IEEE Trans. on Power Systems*, vol. 15, Nº2 pp. 879–886, May 2000.

[17] J. F. Aldrich, H. H. Happ, “Benefits of Voltage Scheduling in Power Systems,” *IEEE Trans. on Power Apparatus and Systems*, vol. PAS-99, Nº5, pp. 1701-1711, Sep/Oct 1980.

[18] J. H. Holland, Adaptive in nature and artificial systems. Ann Arbor, MI: Univ. Michigan Press, 1975.

[19] D. GOLDBERG. “Genetic Algorithms in Search, Optimization and Machine Learning”. Addison-Wesley Publishing Company, 1989.

[20] Z. MICHALEWICZ. “Genetic Algorithms + Data Structures = Evolution Programs”. Springer-Verlag, 1992.

[21] M. MITCHELL. “An Introduction to Genetic Algorithms”. MIT Press, 1998.

[22] N. M. Peterson, W. S. Meyer, “Automatic Adjustment of transformer and Phase-Shifter taps in the Newton Power Flow,” *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-90 Nº1, pp. 103-106, January/February 1971.

[23] J. Cohon. “Multiobjective Programming and Planning”, New York, NY. Academic Press, 1978.

[24] A. Goicoechea, D. Hansen, L. Duckstein. “Multiobjective Decision Analysis With Engineering And Business Applications”, New York, NY.: Wiley, 1982.

[25] Ching-Lai Hwang, Kwangsun Yoon. “Multiple Attribute Decision Making, Methods and Applications”. Springer-Verlag.