

# A SOLUTION OF DYNAMIC TOTAL TRANSFER CAPABILITY BY MEANS OF TRANSIENT STABILITY CONSTRAINED OPF WITH THREE PHASE UNBALANCED FAULTS

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**Abstract** – In the deregulated electricity market, total transfer capability (TTC) is a measure of the network capability for further commercial activity above the already committed uses. This paper deals with the development of an interior point nonlinear programming methodology for evaluating Dynamic TTC. By establishing a novel method for integrating transient stability constraints into conventional steady-state TTC problem, we have proposed the dynamic TTC solution methodology using a Transient Stability Constrained OPF. In the TSCOPF based TTC formulation, we have assumed that three phase to ground faults will occur in the simulation period. In Japan, power system operator considers more severe faults conditions, like 3 phases 4 lines to ground fault with double circuits (3 $\phi$ 4LG).

In this paper, we will propose a solution of Transient Stability Constrained OPF with unbalanced faults, where three phase transmission model was considered in the dynamic simulation period. The proposed method has been implemented and tested on IEEJ WEST30 model systems.

**Keywords:** Total Transfer Capability, Optimal Power Flow, Transient Stability Problem, TSCOPF, Unbalanced Fault

## 1 INTRODUCTION

Recent years, electric power systems are experiencing an epochal revolution due to an increasingly competitive market. Also in Japan, the Electric Industry Law was amended in 1995 aiming to deregulate the wholesale electricity supply business. Under such an open transmission access environment, it is more and more important for the system operator to know how much additional power can be safely transferred across the system.

Available Transfer Capability (ATC) is the measure of the ability of interconnected electric systems to reliably transfer power from one area to another over all transmission lines or paths between those areas under specified system conditions. In nowadays deregulated market, it is a measure of the network capability for further commercial activity above the already committed uses. Until now, ATC calculation has predominantly focused on steady-state viability [1]. In the dynamic realm, evaluation of ATC including voltage stability limits has also been considered [2]. However, the integration of transient stability

constraints into ATC calculation is still a relatively new development. Especially, few OPF-based dynamic ATC algorithms are available although they are conceptually rather nice [3].

Unlike most of the linear programming (LP) methods used in Static ATC, a methodology based on primal-dual Newton interior point method for nonlinear programming problems is introduced to solve the formed Dynamic ATC optimization problem.

In this paper, we will propose a solution of Transient Stability Constrained OPF with the consideration of unbalanced faults, where three phase transmission model was implemented in the dynamic simulation period. The proposed method has been implemented and tested on IEEJ WEST30 model systems with several unbalanced fault condition. In all cases, dynamic responses obtained by our TTC results are verified by the widely-used CRIEPI's power system dynamic stability analysis program.

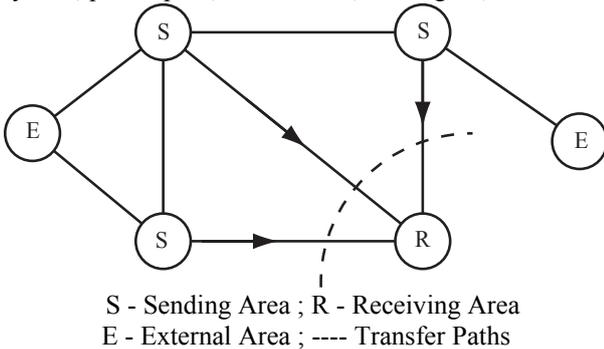
## 2 NOMENCLATURE

$\delta_i$	rotor angle of $i$ th generator
$\omega_i$	rotor speed of $i$ th generator
$\omega_0$	rated rotor speed of generators
$M_i$	moment of inertia of $i$ th generators
$D_i$	damping constant of $i$ th generator
$P_{mi}$	mechanic power input of $i$ th generator
$P_{ei}$	electric power output of $i$ th generator
$\delta_{COI}$	position of the inertial center
$a_i, b_i, c_i$	fuel cost coefficients of thermal plant $i$
$P_{gi}, Q_{ri}$	active and reactive power injection at bus $i$
$P_{li}, Q_{li}$	active and reactive power load at bus $i$
$G_{ij} + jB_{ij}$	transfer admittance between buses $i$ and $j$
$V_i e^{j\theta_i}$	magnitude and phase of voltage $\dot{V}_i$ at bus $i$
$\Delta t$	integration time-step width
$T_{max}$	maximum integration period
nb	number of buses
ng	number of active power sources
nr	number of reactive power sources
nt	number of integration time intervals
$S_G$	set of active power sources
$S_R$	set of reactive power sources

- $S_N$  set of buses
- $S_T$  set of integration steps
- $\underline{(\bullet)}$  lower limits of variables and quantities
- $\overline{(\bullet)}$  upper limits of variables and quantities

### 3 TTC MODELING

As shown in Figure 1, an interconnected power system can be divided into three kinds of areas: sending areas, receiving areas and external areas. "Area" can be defined in an arbitrary fashion. It may be an individual system, power pool, control area, sub-region, etc.



**Figure 1:** A simple interconnected power system

For ATC evaluation, first a base case transfer including existing transmission commitments is chosen, then a transfer-limited case is determined. Mathematically, ATC is defined as:

$$\begin{aligned} \text{ATC} = & \text{Total Transfer Capability (TTC)} \\ & - \text{Existing Transmission Commitments (ETC)} \\ & - \text{Transmission Reliability Margin (TRM)} \\ & - \text{Capacity Benefit Margin (CBM)} \end{aligned} \quad (1)$$

TTC is defined as the amount of electric power that can be transferred over an interface or a corridor of the interconnected transmission network in a reliable manner while meeting all of a specific set of defined pre- and post-contingency system conditions. TRM is defined as that amount of transmission transfer capability necessary to ensure that the interconnected network is secure under a reasonable range of uncertainties in system conditions. CBM is defined as that amount of transmission transfer capability reserved by load serving entities to ensure access to generation from interconnected systems to meet generation reliability requirements. Since TRM and CBM are very system dependent, in the following, we address the calculation of TTC, which is at the basis of ATC evaluation.

The objective of a TTC problem is to determine the maximum real power transfers from sending areas to receiving areas through the transfer paths. And the physical and electrical characteristics of the system limiting the transfer capability include:

- **Generation limits:** Generation should not be over the rated output of each generation unit.
- **Voltage limits:** Voltages over the transmission system should be within acceptable operation ranges.

- **Thermal limits:** Constrain the amount of transfer that transmission line can be safely handled without overload.
- **Stability limits:** Voltage stability and angle stability must be maintained.

In short, the TTC is given by:

$$\text{TTC} = \text{Minimum of} \left\{ \begin{array}{l} \text{Generation Limits} \\ \text{Voltage Limits} \\ \text{Thermal Limits} \\ \text{Stability Limits} \end{array} \right\} \quad (2)$$

In most Japanese electric systems, angler stability constraints for dynamic stability are the crucial factors to determine transmission limits. On considering this point, to simplify TTC calculation, we assume that bus voltage limits are reached before the system reaches the nose point and loses voltage stability. Hence, voltage stability limits are neglected in this study. The power transfer can be formulated as the sum of power flows between the area:

$$\begin{aligned} P_T &= \sum_{i \in S_{SA}, j \in S_{RA}} P_{ij} \\ P_{ij} &= G_{ij} V_i^2 - V_i V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \end{aligned} \quad (3)$$

### 4 TTC CALCULATION BY MEANS OF TRANSIENT STABILITY CONSTRAINED OPF

#### 4.1 Transient stability model in TSCOPF

##### 4.1.1 Classical Generator Model

In this study, the classical generator model for transient stability analysis is adopted. It allows the transient electrical performance of the machine to be represented by a simple voltage source of fixed magnitude  $E'$  behind an effective reactance  $x'_d$ . This model offers considerable computational simplicity.

$$\begin{aligned} \dot{\delta}_i &= \omega_i - \omega_0 \\ M_i \dot{\omega}_i &= \omega_0 (-D_i \omega_i + P_{mi} - P_{ei}) \end{aligned} \quad (4)$$

$$i \in S_G$$

$$P_{ei} = E_i'^2 G_{ii}' + \sum_{\substack{j=1 \\ j \neq i}}^{ng} [E_i' E_j' B_{ij}' \sin \delta_{ij} + E_i' E_j' G_{ij}' \cos \delta_{ij}] \quad (5)$$

In the above equations,  $Y_{ij}' = G_{ij}' + jB_{ij}'$  is the driving point admittance ( $i=j$ ) and the transfer admittance ( $i \neq j$ ).  $Y_{ij}'$  have to be changed only in the case that there is a change in the configuration of the network because of fault or switch operation.

##### 4.1.2 Center of inertia (COI)

In describing the transient behavior of the system, it is convenient to use inertial center as a reference frame. The generators' angles with respect to COI are used to indicate whether or not the system is stable. For an  $ng$ -generator system with rotor angles  $\delta_i$  and inertia constant  $M_i$ , the position of COI is defined as:

$$\delta_{COI} = \sum_{i=1}^{ng} M_i \delta_i / \sum_{i=1}^{ng} M_i \quad (6)$$

## 4.2 Formulation of TSCOPF problem

### 4.2.1 Objective Function

$$\text{Maximize } P_T = \sum_{i \in S_{SA}, j \in S_{RA}} P_{ij} \quad (7)$$

### 4.2.2 Equality Constraints

#### a) Power flow equations:

The polar coordinate form power flow equations are used:

$$\begin{aligned} V_i \sum_{j \in i} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) + P_{li} - P_{gi} &= 0 \\ V_i \sum_{j \in i} V_j (G_{ij} \cos \theta_{ij} - B_{ij} \sin \theta_{ij}) + Q_{li} - Q_{ri} &= 0 \end{aligned} \quad (8)$$

#### b) Swing equations:

By the adoption of any implicit integration rule, equation (4) can be discretized at each time step. The differential swing equations (4) can be converted to the following numerically equivalent algebraic equations using the trapezoidal rule:

$$\begin{aligned} \delta_i^t - \delta_i^{t-1} - \frac{\Delta t}{2} \left[ (\omega_i^t - \omega_0) + (\omega_i^{t-1} - \omega_0) \right] &= 0 \\ \omega_i^t - \omega_i^{t-1} - \frac{\Delta t}{2} \left[ \frac{\omega_0}{M_i} (-D_i \omega_i^t + P_{mi} - P_{ei}^t) \right. \\ \left. + \frac{\omega_0}{M_i} (-D_i \omega_i^{t-1} + P_{mi} - P_{ei}^{t-1}) \right] &= 0 \end{aligned} \quad (9)$$

$i \in S_G, t \in S_T$

where

$$P_{ei}^t = E_i'^2 G_{ii}'' + \sum_{\substack{j=1 \\ j \neq i}}^{ng} \{ E_i' E_j' B_{ij} \sin \delta_{ij}^t + E_i' E_j' G_{ij}' \cos \delta_{ij}^t \}$$

#### c) Initial-value equations:

In order to obtain the initial values of rotor angle  $\delta_i^0$  and constant voltage  $E_i'$  in the swing equations, the following initial-value equations are introduced:

$$\begin{aligned} E_i' V_{gi} \sin(\delta_i^0 - \theta_{gi}) - x_{di}' P_{gi} &= 0 \\ V_{gi}^2 - E_i' V_{gi} \cos(\delta_i^0 - \theta_{gi}) + x_{di}' Q_{ri} &= 0 \end{aligned} \quad (10)$$

### 4.2.3 Inequality Constraints

For the sake of convenience, inequality constraints are divided into two groups  $G_{uc}$  and  $G_c$ .  $G_{uc}$  group contains all the inequality constraints as that in conventional OPF, while  $G_c$  group consists of the transient stability constraints.

#### a) Inequality constraints $G_{uc}$ :

$$\begin{aligned} \underline{P}_{gi} \leq P_{gi} \leq \bar{P}_{gi} \quad i \in S_G \\ \underline{Q}_{ri} \leq Q_{ri} \leq \bar{Q}_{ri} \quad i \in S_R \\ \underline{V}_i \leq V_i \leq \bar{V}_i \quad i \in S_N \end{aligned} \quad (11)$$

#### b) Stability constraints $G_c$ :

As mentioned, generators' angles with respect to COI are used to indicate whether or not the system is

stable:

$$\begin{aligned} \underline{\delta} \leq \delta_i^0 - \delta_{COI}^0 \leq \bar{\delta} \\ \underline{\delta} \leq \delta_i^t - \delta_{COI}^t \leq \bar{\delta} \\ i \in S_G, t \in S_T \end{aligned} \quad (12)$$

### 4.3 Primal-Dual Interior Point Optimal Power Flow

Assume that  $x$  is defined as a  $n \times 1$  vector:

$$x \equiv \left[ x^{control} \mid x^{state} \right]^T \in R^n$$

Then, a dynamic TTC problem may be formulated as the following non-linear programming problem:

$$\begin{aligned} \text{minimize } f(x) \\ \text{subject to } h(x) = 0 \\ \underline{g} \leq g(x) \leq \bar{g} \end{aligned} \quad (13)$$

where  $h(x) \equiv [h_1(x), \dots, h_m(x)]^T$ ,

$g(x) \equiv [g_1(x), \dots, g_r(x)]^T$ .

By introducing slack variable vectors  $l, u \in R^r$ , system (9) can be transformed to:

$$\begin{aligned} \text{minimize } f(x) \\ \text{subject to } h(x) = 0 \\ g(x) - \underline{g} - l = 0; g(x) - \bar{g} + u = 0 \\ (l, u) \geq 0 \end{aligned} \quad (14)$$

Define a Lagrangian function associated with (10) as:

$$\begin{aligned} L(x, l, u; y, z, w, \tilde{z}, \tilde{w}) \equiv f(x) - y^T h(x) - z^T [g(x) - \underline{g} - l] \\ - w^T [g(x) - \bar{g} + u] - \tilde{z}^T l - \tilde{w}^T u \end{aligned} \quad (15)$$

where  $y \in R^m$  and  $z, w, \tilde{z}, \tilde{w} \in R^r$  are Lagrange multipliers.  $\tilde{z} = z, \tilde{w} = -w$ .

Based on the perturbed Karush-Kuhn-Tucker (KKT) optimality conditions, we have the following equations:

$$\begin{aligned} L_x \equiv \nabla f(x) - \nabla h(x)y - \nabla g(x)(z + w) &= 0 \\ L_l \equiv LZ e - \mu e &= 0 \\ L_u \equiv UWe + \mu e &= 0 \\ L_y \equiv h(x) &= 0 \\ L_z \equiv g(x) - \underline{g} - l &= 0 \\ L_w \equiv g(x) - \bar{g} + u &= 0 \\ (l, u) \geq 0, y \neq 0, z \geq 0 \text{ \& } w \leq 0 \end{aligned} \quad (16)$$

where  $L, U, Z, W \in R^{r \times r}$  are diagonal matrices with the element  $l_i, u_i, z_i$  and  $w_i$ .  $\mu > 0$  is a perturbed factor.  $e = [1, \dots, 1]^T \in R^r$ .

By applying Newton's method to the perturbed KKT equations (11), the correction equation can be expressed as:

$$\begin{aligned} \left[ \sum_{i=1}^m y_i \nabla^2 h_i(x) + \sum_{j=1}^r (z_j + w_j) \nabla^2 g_j(x) - \nabla^2 f(x) \right] \Delta x \\ + \nabla h(x) \Delta y + \nabla g(x) (\Delta z + \Delta w) = L_{x0} \\ Z \Delta l + L \Delta z = -L_{l0}^\mu \\ W \Delta u + U \Delta w = -L_{u0}^\mu \\ \nabla h(x)^T \Delta x = -L_{y0} \\ \nabla g(x)^T \Delta x - \Delta l = -L_{z0} \\ \nabla g(x)^T \Delta x + \Delta u = -L_{w0} \end{aligned} \quad (17)$$

where  $(L_{x0}, L_{l0}^\mu, L_{u0}^\mu; L_{y0}, L_{z0}, L_{w0})$  are the values at a

point of expansion and denote the residuals of the perturbed KKT equations.  $\nabla^2 f(x)$ ,  $\nabla^2 h_i(x)$  and  $\nabla^2 g_j(x)$  are Hessian matrices of  $f(x)$ ,  $h_i(x)$  and  $g_j(x)$ .

In order to handle inequality constraints efficiently, a reduced correction equation is introduced. This reduction method is very effective for Dynamic TTC problem. By eliminating  $(\Delta l, \Delta u, \Delta z, \Delta w)$  from (12), we can derive the following reduced correction equation:

$$\begin{bmatrix} H(\bullet) & J(x)^T \\ J(x) & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = - \begin{bmatrix} \psi(\bullet, \mu) \\ \phi \end{bmatrix} \quad (18)$$

where

$$H(\bullet) \equiv \begin{bmatrix} \sum_{i=1}^m y_i \nabla^2 h_i(x) + \sum_{j=1}^r (z_j + w_j) \nabla^2 g_j(x) - \nabla^2 f(x) \\ + \sum_{j=1}^r \left( \frac{w_j}{u_j} - \frac{z_j}{l_j} \right) \nabla g_j(x) \nabla g_j(x)^T \end{bmatrix}$$

$$\begin{aligned} J(x) &\equiv \nabla h(x)^T \\ \psi(\bullet, \mu) &\equiv \nabla h(x)y - \nabla f(x) \\ &\quad + \nabla g(x)[U^{-1}WL_{w0} - L^{-1}ZL_{z0} - \mu(U^{-1} - L^{-1})e] \\ \phi &\equiv h(x) \end{aligned}$$

It is obvious that the reduced correction equation has eliminated both variable inequality constraints and functional inequality constraints. The size of (18), which is determined only by the number of variables and equality constraints, is much smaller than that of (17).

#### Algorithm of the Method

**Initialization:** Set iteration counter  $k = 0$ ; define centering parameter  $\sigma \in (0, 1]$  and tolerance  $\varepsilon = 10^{-6}$ ; chose a starting point for primal variables and dual variables.

**Begin**  $k = 0, 1, \dots$ :

**Step 1:** (Test for Convergence)

Compute complementary gap:

$$CGAP_k \equiv \sum_{m=1}^r (l_m z_m - u_m w_m) \quad (19)$$

If the operating point satisfied the convergence criteria ( $CGAP_k < \varepsilon$ ), then output the optimal solution and stop. Otherwise, do Step 2 to Step 5.

**Step 2:** (Compute the Perturbed Factor)

$$\mu_k \equiv \sigma \frac{CGAP_k}{2r} \quad (20)$$

**Step 3:** (Compute the Perturbed Newton Correction)

Solve the reduced correction equation (18) for  $(\Delta x_k, \Delta y_k)$ , then the following equations for  $(\Delta l_k, \Delta u_k, \Delta z_k, \Delta w_k)$ :

$$\begin{cases} \Delta l_k = \nabla g(x)^T \Delta x_k + L_{z0} \\ \Delta u_k = -\nabla g(x)^T \Delta x_k - L_{w0} \\ \Delta z_k = L^{-1}(-Z\Delta l_k - LZ e + \mu e) \\ \Delta w_k = U^{-1}(-W\Delta u_k - UWe - \mu e) \end{cases} \quad (21)$$

The perturbed Newton correction  $\Delta v_k$  is:

$$\Delta v_k = [\Delta x_k, \Delta l_k, \Delta u_k, \Delta y_k, \Delta z_k, \Delta w_k]^T$$

**Step 4:** (Determine the Maximum Step Length)

Perform the ratio test to determine the maximum primal and dual step lengths that can be taken in the Newton direction:

$$\begin{aligned} \theta P_k &= \min \left\{ \min \left( \frac{-l_m}{\Delta l_m} : \Delta l_m < 0; \frac{-u_m}{\Delta u_m} : \Delta u_m < 0 \right), 1 \right\} \\ \theta D_k &= \min \left\{ \min \left( \frac{-z_m}{\Delta z_m} : \Delta z_m < 0; \frac{-w_m}{\Delta w_m} : \Delta w_m > 0 \right), 1 \right\} \quad (22) \\ (m = 1, 2, \dots, r) \end{aligned}$$

Form the step length matrix:

$$\Theta_k = 0.99 \text{diag}[\theta P_k, \dots, \theta P_k; \theta D_k, \dots, \theta D_k] \quad (23)$$

The scalar 0.99 is a safety factor to ensure that the next point will satisfy the strict non-negativity conditions imposed on the slack variables.

**Step 5:** (Update Variables)

Update the primal and dual variables by:

$$v_{k+1} = v_k + \Theta_k \Delta v_k \quad (24)$$

then return to Step 1.

**End**

#### 4.4 Three Phasor Model of Power System

Normally, a power system operates under balanced conditions. Efforts are made to ensure this desirable state. Unfortunately, under fault/emergency condition, the system may become unbalanced state. In Japan, most of the utilities consider more severe condition than 3LG-O fault, like 4LG-O with only 2 phase linked period. In order to apply this method to Japanese system, we have to incorporate three phase model.

In this paper, we assumed that all the transmission line consists of double circuit with three phase model, to consider any unbalanced fault condition.

In general, the sequence network (symmetrical components) model was used to analyze unbalanced network. In our TSCOPF model, three phasor (abc-network) model was adopted to express unbalanced power system to avoid complexity for setting up unbalanced fault conditions.

##### 4.4.1 Three Phasor Transmission Line Model

In the Japanese power system, all the transmission line and generator parameters are supplied with the sequence network based values. In order to adopt these parameters to our TSCOPF, we have to convert the

obtained data to the phasor representation using the following equations:

$$[A] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}, \quad a = \frac{-1 - j\sqrt{3}}{2} \quad (25)$$

$$[Y_{abc}] = [A] \begin{bmatrix} y_0 & 0 & 0 \\ 0 & y_1 & 0 \\ 0 & 0 & y_2 \end{bmatrix} [A]^{-1} = \frac{1}{3} \begin{pmatrix} Y_p & Y_q & Y_r \\ Y_r & Y_p & Y_q \\ Y_q & Y_r & Y_p \end{pmatrix} \quad (26)$$

$$Y_p = y_0 + y_1 + y_2, \quad Y_q = y_0 + ay_1 + a^2y_2, \quad Y_r = y_0 + a^2y_1 + ay_2$$

Figure 2 shows the system admittance matrix  $Y$  with double circuit in each node/lines. The size of  $Y$  matrix was  $6n \times 6n$  ( $n$ : the number of nodes). In this figure, heavy boarded parts  correspond to the mutual admittance between different routed transmission lines and double bordered parts  correspond to the mutual admittance between the same routed circuits.

		bus1		bus n	
		abc	a'b'c'	abc	a'b'c'
bus 1	a	$Y_{11}^{abc}$	$M_{11}$	$Y_{1n}^{abc}$	$M_{1n}$
	b	$Y_{11}^{abc}$	$M_{11}$	$Y_{1n}^{abc}$	$M_{1n}$
	c	$Y_{11}^{abc}$	$M_{11}$	$Y_{1n}^{abc}$	$M_{1n}$
bus n	a	$M_{n1}$	$Y_{n1}^{a'b'c'}$	$M_{nn}$	$Y_{nn}^{a'b'c'}$
	b	$M_{n1}$	$Y_{n1}^{a'b'c'}$	$M_{nn}$	$Y_{nn}^{a'b'c'}$
	c	$M_{n1}$	$Y_{n1}^{a'b'c'}$	$M_{nn}$	$Y_{nn}^{a'b'c'}$

**Figure 2:**  $Y$  matrix considering abc-phasors

In our formulation, we assumed that the mutual admittance between the different route would be negligibly small. On the other hand, the mutual admittance between the same nodes can be calculated with the following simplified equations:

$$Z_0 = \frac{Z_0 - Z_m}{L} + Z_m \quad (27)$$

where  $Z_0$ : zero-sequence admittance for each line,

$Z_m$ : zero-sequence mutual admittance between lines,

$L$ : the number of lines.

The above mutual admittance should be converted to phasor representation. Fortunately, the mutual admittance would appear only in diagonal block of the system admittance matrix  $Y$ .

#### 4.4.2 Reduction of admittance matrix from double to single circuit model

In order to reduce the admittance matrix from double to single circuit, we will simply add  $Y^{abc}$  with  $Y^{a'b'c'}$  to obtain reduced matrix. This desirable characteristic comes from the mutual admittance model given by the equation (25).

		bus1	...	bus n
bus1	a	$Y_{11}^{abc} + Y_{11}^{a'b'c'}$	...	$Y_{1n}^{abc} + Y_{1n}^{a'b'c'}$
	b	$Y_{11}^{abc} + Y_{11}^{a'b'c'}$	...	$Y_{1n}^{abc} + Y_{1n}^{a'b'c'}$
bus n	a	$Y_{n1}^{abc} + Y_{n1}^{a'b'c'}$	...	$Y_{nn}^{abc} + Y_{nn}^{a'b'c'}$
	b	$Y_{n1}^{abc} + Y_{n1}^{a'b'c'}$	...	$Y_{nn}^{abc} + Y_{nn}^{a'b'c'}$

**Figure 3:** Reduced  $Y$  matrix from double to single circuit model

The size of obtained reduced  $Y$  matrix was  $3n \times 3n$ , which is the 1/4 of the original matrix.

#### 4.4.3 Electric Generator Output $P_e$ from Three Phasor model

For the single phasor model, the electric generator output  $P_e$  can be calculated from equation (5). However, for the three phasor model, we have to calculate the electric outputs of generator separately for each phasor.

$$\dot{I}_G = Y_{E'} \dot{E}'_i \quad (28)$$

$$Y_{E'} = Y_G - [-Y_G \quad 0] \begin{bmatrix} Y_G + Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL} \end{bmatrix}^{-1} \begin{bmatrix} -Y_G \\ 0 \end{bmatrix} \quad (29)$$

$$\dot{E}'_{ia} = \dot{E}'_{ia'} = a^2 \dot{E}'_{ib} = a^2 \dot{E}'_{ib'} = a \dot{E}'_{ic} = a \dot{E}'_{ic'} = \frac{1}{\sqrt{3}} \dot{E}'_i \quad (30)$$

Electrical output of each generator is the sum of product of each phasor voltage and current. Equation (28)~(30) shows the electric output for each phasor, where the admittance  $Y_{E'}$  in these equations can be obtained from the equation .

$$P_{ei}^a = \sum_{j=1}^{ng} \frac{1}{3} V_i V_j \left\{ G_{ij}^{aa} \cos \delta_{ij} + B_{ij}^{aa} \sin \delta_{ij} + G_{ij}^{ab} \cos \left( \delta_{ij} + \frac{2}{3} \pi \right) + B_{ij}^{ab} \sin \left( \delta_{ij} + \frac{2}{3} \pi \right) + G_{ij}^{ac} \cos \left( \delta_{ij} - \frac{2}{3} \pi \right) + B_{ij}^{ac} \sin \left( \delta_{ij} - \frac{2}{3} \pi \right) \right\} \quad (31)$$

$$P_{ei}^b = \sum_{j=1}^{ng} \frac{1}{3} V_i V_j \left\{ G_{ij}^{ab} \cos \left( \delta_{ij} - \frac{2}{3} \pi \right) + B_{ij}^{ab} \sin \left( \delta_{ij} - \frac{2}{3} \pi \right) + G_{ij}^{bb} \cos \delta_{ij} + B_{ij}^{bb} \sin \delta_{ij} + G_{ij}^{bc} \cos \left( \delta_{ij} + \frac{2}{3} \pi \right) + B_{ij}^{bc} \sin \left( \delta_{ij} + \frac{2}{3} \pi \right) \right\} \quad (32)$$

$$P_{ei}^c = \sum_{j=1}^{ng} \frac{1}{3} V_i V_j \left\{ G_{ij}^{ac} \cos \left( \delta_{ij} + \frac{2}{3} \pi \right) + B_{ij}^{ac} \sin \left( \delta_{ij} + \frac{2}{3} \pi \right) + G_{ij}^{bc} \cos \left( \delta_{ij} - \frac{2}{3} \pi \right) + B_{ij}^{bc} \sin \left( \delta_{ij} - \frac{2}{3} \pi \right) + G_{ij}^{cc} \cos \delta_{ij} + B_{ij}^{cc} \sin \delta_{ij} \right\} \quad (33)$$

## 5 SIMULATION RESULT

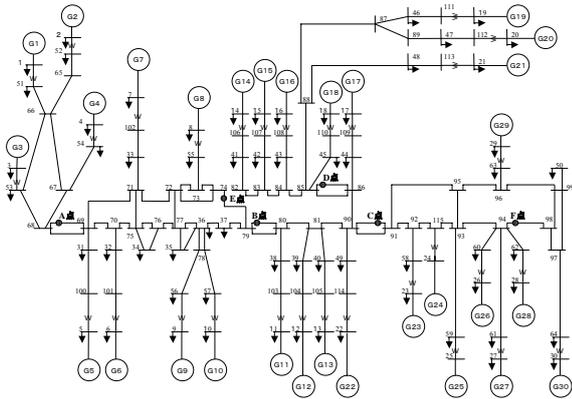
### 5.1 Simulation condition

#### 5.1.1 IEEJ WEST 30 Model

In order to help other researchers to crosscheck the results, in this paper we present the results of public domain systems --- IEEJ WEST30 model system [10].

	IEEJ WEST30
No. node	115
No. branch	124
No. generator	30 (No 30 was fixed)
No. transformer	30

**Table 1:** System Outline of the IEEJ WEST30 System



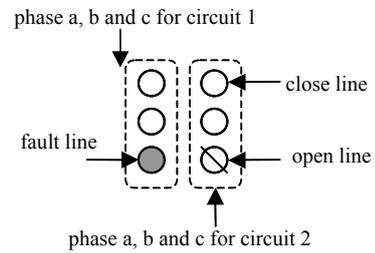
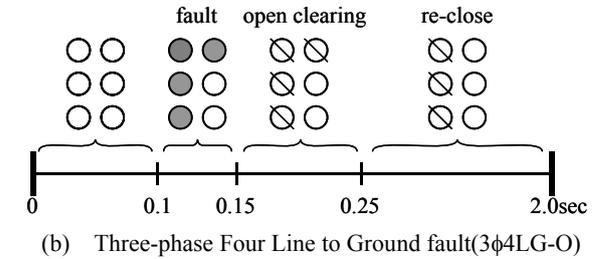
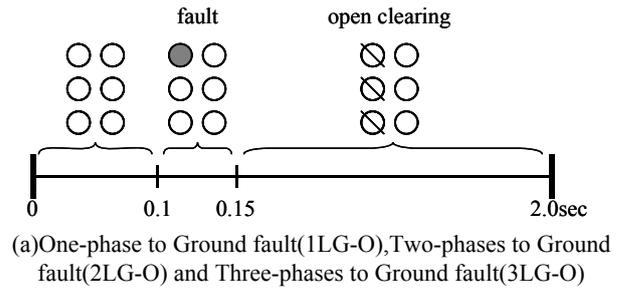
**Figure 4:** IEEJ WEST30 Power System model

In this simulation, the transient stability constraint was defined as the maximum relative rotor angle with respect to center of inertia (COI) should be in the specified limit. The integration step-width  $\Delta t$  is fixed to be 0.01 [sec] and the maximum integration period  $T_{max}$  is set to be 2.0 [sec]. It is possible to increase the number of integration steps, however the solution time will almost proportion to the number of the step. We assumed that the TTC would be power transfer between node 90 - 91, and a fault occurs at the line near node 90.

#### 5.1.2 Fault Conditions

The following types of contingency are considered :

- (1) A fault occurs at 0.1 [sec] then the CB open the faulted circuit at 0.15 [sec]. This was applied to 1LG-O, 2LG-O and 3LG-O fault conditions (Figure 4(a)).
- (2) A fault occurs at 0.10 [sec] then the CB open only the faulted lines at 0.15 [sec] to clear the faults. At 0.20 [sec], only one circuit will be re-closed for recovering balanced operation. This was applied to 3 $\phi$ 4LG-O ( Figure 4(b) ).



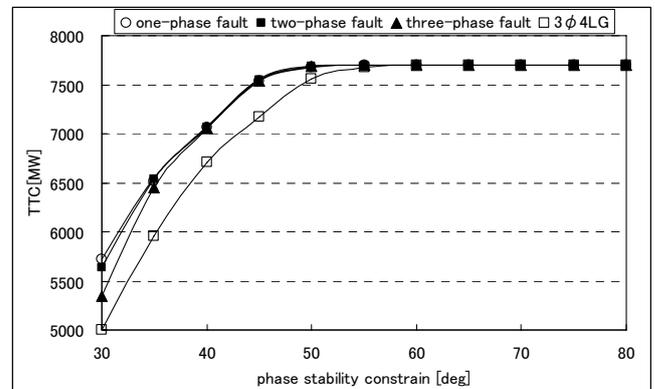
**Figure 5:** Transition of the fault condition

### 5.2 Simulation results

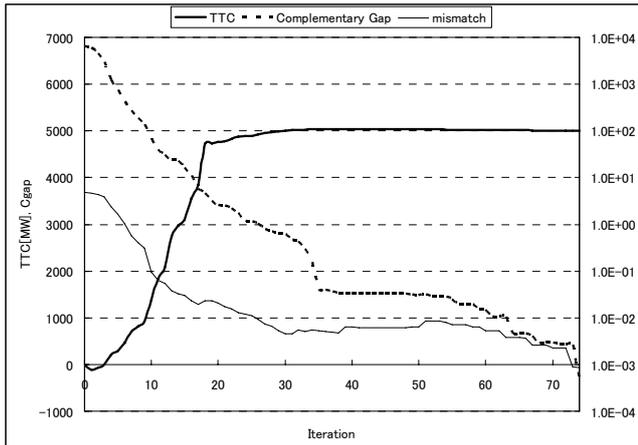
Figure 6 show the result of TTC calculation with the different rotor angle limits with respect to COI.

In this simulation, the most severe condition was 3 $\phi$ 4LG, because only two lines are connected between the nodes. The stability limit more than 60 [degree] gives the same TTC value, which is also the same as the Static TTC.

It took about a few minutes to solve one TTC problem for this IEEJ WEST30 system using Sun Blade 1500 Workstation. With the severe stability constraints, it requires more than a hundred iterations with several minutes of solution time.



**Figure 6:** Result of TTC simulations with different angle constraints for each fault condition



**Figure 7:** Convergence characteristics for the proposed method

Node	PGmin	PGmax	Generator output[GW]		
			70deg	50deg	30deg
1	2.50	8.50	2.50	2.50	3.91
2	4.70	4.70	4.70	4.70	4.70
3	0.48	3.10	0.48	0.48	0.48
4	0.59	3.80	2.14	0.59	2.34
5	0.45	1.80	1.80	0.45	0.45
6	0.86	3.40	2.68	1.93	0.86
7	0.30	1.20	1.20	1.20	0.30
8	2.30	2.30	2.30	2.30	2.30
9	1.20	4.80	2.29	1.87	1.74
10	1.33	5.30	3.07	5.30	4.16
11	1.14	5.30	3.19	4.50	4.88
12	1.22	5.50	3.87	3.91	3.96
13	1.60	7.50	5.88	6.56	6.40
14	2.30	2.30	2.30	2.30	2.30
15	1.55	6.20	2.45	1.68	1.55
16	3.50	3.50	3.50	3.50	3.50
17	0.88	4.30	3.73	4.17	3.75
18	0.86	3.40	2.09	1.92	0.86
19	0.50	2.00	0.97	0.94	0.50
20	1.55	6.20	3.20	3.08	2.72
21	0.33	1.30	0.58	0.58	0.33
22	0.50	2.30	1.86	2.23	2.06
23	2.14	8.60	2.14	2.14	2.72
24	0.75	0.75	0.75	0.75	0.75
25	0.45	0.45	0.45	0.45	0.45
26	0.59	2.40	0.59	0.59	0.81
27	0.63	2.60	0.63	0.86	1.22
28	1.31	5.30	1.63	1.62	2.28
29	0.82	3.40	1.01	0.92	1.60

**Table 2:** Change of Generator Output

Figure 7 show the convergence characteristics for the proposed method. It converge after 75 iterations. TTC was obtained at 35th iteration, while 40 iterations are required to reduce complimentary gap and mismatch.

For IEEJ WEST30 System, it took about 2 min to solve one TTC calculation, using 2.8 GHz Pentium 4 personal computer. The solution time will increase proportion to the number of generator and simulation time period.

## 6 CONCLUDING REMAKS

In this paper, first, a novel method for integrating transient stability constraints into TTC problem was presented. Then, the dynamic TTC problem with three phasor model was successfully formulated using TSCOPF method.

The effectiveness of the dynamic TTC formulation and the solution algorithm was demonstrated on the IEEJ WEST30 model systems.

For the future work, the generator shedding model will be implemented on the proposed method.

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