

Building Optimal Offer Curves For An Electricity Spot Market: A Mixed-Integer Programming Approach

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Abstract— In this paper we present a mixed-integer programming approach to optimize the offer curves submitted by a power generation company to a day-ahead electricity market. Our method takes a base-offer curve as input data and proposes changes that increase the expected profit of the generation company while complying with a variety of constraints that the company may want to impose. The main advantage of this method is that, if the base-offer curve is valid, it is quite easy to guarantee that the optimized offer curve be valid for the day-ahead market of interest. A real-size numerical example in the context of the Spanish day-ahead market illustrates the potential of this approach.

Index Terms— Electricity competition, Market models, Power generation scheduling.

I. INTRODUCTION

In many wholesale electricity markets, power generation companies willing to sell the energy produced by their plants are required to submit offers to indicate the price at which they want to sell that energy. Similarly, power purchasers have to communicate the amount of energy that they are ready to buy at each price. This information is processed by a market operator, who decides the transactions that will actually take place and determines the price of energy for each time interval. Although wholesale electricity markets include mechanisms for trading electric energy within a variety of time horizons, electricity spot markets play a major role because they are considered a reference for other transactions. Therefore, in this paper we focus on the problem of selling power in an electricity spot market.

The development of strategic bidding procedures for generation companies that aim to maximize their profits in the electric marketplace has attracted the interest both from researchers and industry managers. A wide variety of approaches have been adopted. We suggest the following categorization to identify the main differences between them:

- Strategic behaviour: The company under analysis can be represented as a price taker if its decisions do not affect the market clearing price [2, 10]. In other case, the ability of the company to modify prices with its offers must be taken into account [4-6, 12].
- Production costs and technical constraints: A simplified model of the company's generation system consisting of

an aggregate cost curve (typically quadratic) can be used. A more detailed study requires defining an individual model for each generating unit including its specific cost curve and technical constraints [9]. Limited interconnection capacity with neighbouring systems is generally neglected.

- Modelling competitors and uncertainty: In a perfectly competitive setting, market clearing prices are given by the most expensive unit's variable costs, so a good estimation of the opponents' costs is essential [10]. In an oligopoly, a more complex model is required. Some modellers presume the company's profits will depend on some kind of equilibrium reached with its rivals [11, 12]. An alternative is to avoid an explicit model for competitors' behaviour and to use their historic bidding data to create future market scenarios. We follow this approach in this paper.
- Results given by the model: These can either consist of an optimal schedule for the company's generation system or take the form of optimal offer curves. An additional difference lies in the way offer curves are built. Linear [11] and quadratic functions [12] have been utilized as approximations. A more flexible alternative is to use fixed blocks of energy with variable prices or fixed price levels with variable quantities [4].
- Solution procedure: Obtaining optimal offer curves implies solving a large non-linear and non-convex optimization problem. Due to this, researchers using a mathematical programming approach have been forced to introduce important simplifying assumptions both in the modelling and in the solution process. Mixed-integer programming (MIP) has been a usual approach [4, 7]. Alternatively, genetic algorithms have become increasingly popular [11].

We propose a novel methodology to prepare optimal offer curves for a generation company operating in an electricity spot ahead market organized as a series of twenty-four hourly uniform-price multiunit double auctions by means of a MIP approach. A major drawback of previous developments in this field is that the obtained offer curves are typically not usable either because they are simplified versions of the ones actually used by generation companies (e.g. they are linear or quadratic approximations, they have less blocks than the actual ones, etc.) or because they do not take into account some of the constraints that have to be considered by the companies (e.g. they do not comply with the operation limits of the generation units, they do not take into account particular rules of the market, etc.)

The main contribution of our approach is that the offer curves we obtain can actually be used because we take valid offer curves as an initial point for the optimization process

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and then introduce modifications that maximize the expected profit of the generation company while complying with the constraints imposed by the user. This assumption has evident advantages. Firstly, solution existence is always guaranteed, given that the initial point is already a feasible solution. Additionally, it combines well with other offer-curve construction techniques that can be used to derive this initial point. Finally, it suggests modifications to the initial offer curves that may provide valuable insight for the generation company with respect to its strategy in the day-ahead market.

The paper is laid out as follows. In section II we formally describe the structure and operation of an electric generation company, and we explain in detail the dynamics of the day-ahead electricity market of interest. In section III we present a detailed formulation of our optimization model. In section IV we use this model to compute optimal offers for a real-size numerical example in the context of the Spanish day-ahead market. Finally, in section V we extract conclusions and suggest future lines of work.

II. PROBLEM DESCRIPTION

A. The electricity spot market of interest

Agents participating in a wholesale electricity market require different market mechanisms to perform their transactions in the way that best suits their business strategy. In particular, the time scope with which the agents may be willing to trade can range from several years in advance to a few minutes prior to physical delivery. The majority of wholesale electricity markets include a spot market in which electricity is traded for immediate delivery and that is considered as a reference for the rest of transactions.

Electricity spot markets typically include several market mechanisms. The Spanish electricity spot market consists of a day-ahead market, a congestion management procedure, an adjustment market, a reserves market and a balancing mechanism, as shown in Figure 1.

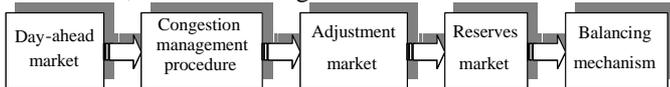


Figure 1. Sequence of the Spanish spot market mechanisms.

It is obvious that in such a spot market the main market mechanism is the day-ahead one. This does not imply the largest volume of transactions to be performed through the spot market, given that long-term bilateral contracting and OTC trading are usually also allowed.

We consider a day-ahead market constituted by twenty-four hourly auctions that take place one day in advance. This mechanism runs based on simple sell offers and purchase bids. Participants are allowed to operate as a portfolio with no limits on the number of offers or bids they wish to submit and they are allowed to adopt different strategies for different hours. Hourly double auctions are mutually independent. Consequently, although the twenty-four hourly auctions that form the day-ahead market are cleared at the same time, the results of one of these auctions are based only on the offers and bids submitted by participants for that specific hour.

In this paper a marginal or uniform pricing scheme is

assumed for all the hourly auctions. The clearing price is determined by the intersection of the aggregate offer and demand curves. In this manner, an offer is rejected if and only if its price is greater than the market-clearing price. Similarly, a bid is accepted if and only if its price is lower than the market-clearing price, as illustrated in Figure 2.

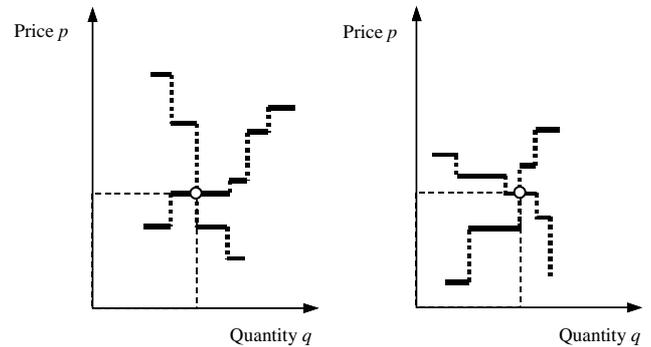


Figure 2. A representation of the market clearing process.

B. Focusing on the problem

The main contribution of this paper is to conceive the problem of obtaining an optimal offering strategy for the day-ahead market as a process of sequential refinement of an initial offering strategy. In this manner, it is easy to guarantee that the solution obtained be valid and better than the initial one. This approach contrasts with previous developments [4] in which great efforts were made to assure that the resulting offer curves be valid. These efforts resulted in oversimplified offer curves that, although valid, were not useful for real generation companies. The methodology we propose in this paper aims at improving current practices in generation companies rather than radically changing the way of doing things.

The procedure evolves as follows. The generation company constructs initial offer curves for a certain day-ahead market session using its traditional procedure. Notice that this traditional procedure used to construct offers typically prepares offer curves internalizing strategic medium-term planning guidelines that exceed the time-scope considered for our model. Consequently, generation companies prefer introducing slight changes into their offers in order to preserve that strategic planning rather than building new ones. If no medium-term guidelines are to be preserved, our model could be then directly applied to construct the offer curves. Regardless of how the initial offers are obtained, curves will consist of a collection of offer blocks where each block corresponds to a generation unit (or can be formally decomposed into sub-blocks, each one corresponding to one generation unit), even if a portfolio offering scheme is allowed in the day-ahead market. Our method then evaluates the impact of increasing and reducing the amount of energy offered in each block. For example, let us assume that quantity offered in a given block is to be increased. This may entail several effects:

- It will presumably lead to a reduction in the market-clearing price, which depends on the offers and bids submitted by the rest of agents.
- It will typically cause a rise in the energy sold by the company if the new quantity is offered at a price lower than the clearing price. It must be taken into account that

this may force out of the market other blocks offered by the company at higher prices.

III. MODEL FORMULATION

A. Modelling the rest of agents

In order to solve the problem of optimizing the offer curve of a generation company participating in a day-ahead market such as the one described, we do not need to represent in detail the portfolio of all the agents involved.

In this paper we use the concept of residual demand to represent the rest of agents. Due to uncertainty with respect to rivals' decisions, some of the input data are random parameters. We use bold characters to distinguish them. Since determining the probability distribution that better represents each of the random input data is a complicated task, and given that managing these distributions through the constraints of the model will introduce a significant mathematical effort to obtain a solution that is anyway a simplification of reality, we suggest managing uncertainty by means of scenario trees to replace that mathematical effort with a less important computational one.

In each hourly auction of the spot market, n , the amount of energy that a generation company is able to sell, q_n , depends on the clearing price, p_n . This is due to the combined effect of the demand at that price, $D_n(p_n)$, and the supply of the rest of generation companies at that price, $S_n^{rest}(p_n)$.

$$q_n = D_n(p_n) - S_n^{rest}(p_n) = R_n(p_n), \quad \forall n, \quad (1)$$

where $R_n(\cdot)$ is the residual demand faced by the company in auction n . To obtain $R_n(p_n)$, the company only needs to know the demand, $D_n(p_n)$, and the aggregate offer, $S_n(p_n)$, as it can obtain $S_n^{rest}(p_n)$ by subtracting its own offer.

$$S_n^{rest}(p_n) = S_n(p_n) - S_n^{own}(p_n), \quad \forall n. \quad (2)$$

Conversely, the clearing price can be expressed as a function of the company's sales. We refer to this function as inverse residual demand function,

$$p_n = R_n^{-1}(q_n), \quad \forall n. \quad (3)$$

Given the inverse residual demand function of the company of interest, the revenue of the company, ρ_n , can also be expressed as a function of its production,

$$\rho_n(q_n) = p_n(q_n)q_n, \quad \forall n. \quad (4)$$

In a spot market in which offer curves are expressed as stepwise functions (e.g. Spanish OMEL), inverse residual demand functions and revenue functions have vertical segments that are not easy to represent in a mathematical programming framework. To overcome this difficulty we suggest approximating these functions by means of piecewise linear functions. Figure 3 and Figure 4 illustrate how these approximations (plotted in black) work. As can be seen, the accuracy of these approximations increases with the number of steps of the residual demand curve.

We therefore assume that both inverse residual demand functions and revenue functions can be expressed as piecewise linear functions with J segments. Each segment j is defined by its lower bound, q_{jn} , and its upper bound, q_{j+1n} . We assign a binary variable u_{jn} to each segment j , such that $u_{jn} = 1$ if the company's sales in hour n are

higher than q_{jn} and $u_{jn} = 0$ in other case. We also define a continuous variable v_{jn} to represent the portion of segment j that is filled. Segment j can only be used if segment $j-1$ is fulfilled.

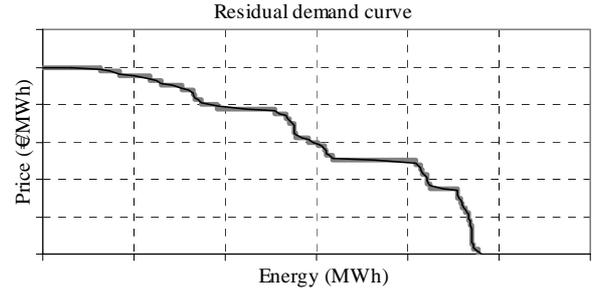


Figure 3. Approximation of a stepwise residual demand curve by means of a piecewise linear function.

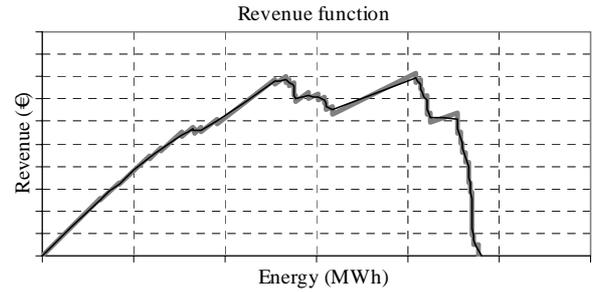


Figure 4. Approximation of a revenue function by means of a piecewise linear function.

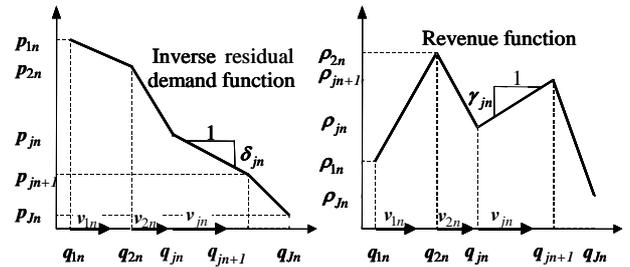


Figure 5. A piecewise linear representation of residual demand and revenues.

Making use of the slopes of these piecewise linear functions, the following expressions provide the clearing price and the company's revenues for each level of sales q_n ,

$$p_n = p_{1n} + \sum_j \delta_{jn} v_{jn}, \quad \forall n, \quad (5)$$

$$\rho_n = \rho_{1n} + \sum_j \gamma_{jn} v_{jn}, \quad \forall n, \quad (6)$$

$$q_n = \sum_j v_{jn}, \quad \forall n, \quad (7)$$

$$u_{j+1n}(q_{j+1n} - q_{jn}) \leq v_{jn} \leq u_{jn}(q_{j+1n} - q_{jn}), \quad \forall j, \forall n, \quad (8)$$

$$u_{jn} \leq u_{j-1n}, \quad \forall j, \forall n. \quad (9)$$

The construction of an offer curve should be addressed explicitly taking into account uncertainty with respect to the behaviour of the rest of agents [1]. This can be easily done by defining residual demand scenarios, as described in [4].

B. Modelling the company's offer

We assume that the company has already constructed a base-offer curve for each hourly auction in the spot market so that we only introduce slight changes in order to obtain some new but very similar offer curves that increase the company's profit.

The base-offer curve constructed by the company for the hourly auction n , $S_n^0(p_n)$, is modelled as a stepwise increasing function consisting of I steps. Each step, i , is defined by a price, p_{ni} , and an amount of energy, q_{ni}^0 , that the company is willing to sell if the clearing price, p_n , levels or exceeds the value p_{ni} . These steps are sorted by increasing-price order, and they are consecutively placed in the offer curve so that the energy accumulated up to each step of the resulting curve, q_n , is the total energy that the company would be willing to sell if the clearing price, p_n , were equal to p_{ni} .

Our purpose is to introduce changes into the company's base-offer curve. We will model changes corresponding to each step i by means of a variable, q_{ni}^a , which represents the quantity of energy that should be added (if positive) or subtracted (if negative) to the original step in order to construct the optimized curve, as illustrated in Figure 6. The resulting curve, $S_n(p_n)$, will be the one that the company should submit to the market operator.

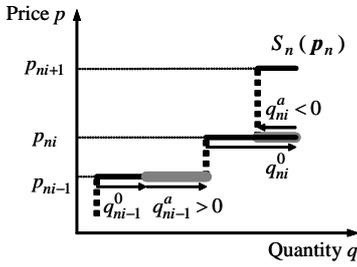


Figure 6. A representation of a modified company's offer curve.

In the Spanish day-ahead market, generation companies are required to offer all the available energy of each generation unit in each hourly auction. Notice that generation companies are allowed to perform transactions not only in the day-ahead market but also through bilateral contracts and OTC trading, so each unit's available energy does not necessarily fit the real energy that the generation unit is physically able to produce. Nevertheless, this consideration is supposed to have been previously taken into account within the initial curves and we represent this constraint by forcing the sum of the changes in the amounts offered at different prices for a given generation unit to be equal to zero,

$$\sum_{i \in g} q_{ni}^a = 0, \quad \forall n. \quad (10)$$

Additionally, assuming that initial offers satisfy generation units' ramp constraints, we maintain smoothness over load profiles keeping the added amount of energy for every step within some certain limits, \underline{q}_i^a and \bar{q}_i^a ,

$$q_i^a \leq \bar{q}_i^a \leq \underline{q}_i^a, \quad \forall i, \forall n. \quad (11)$$

Finally, we should not forget that short-term operation must generally respond to some medium-term planning guidelines. Some of these may imply that a certain unit g produces a specified amount of energy during the day of study (e.g. a hydro plant that must release a given amount of water). This is introduced by forcing the changes in the amounts offered for that unit during the day to sum zero,

$$\sum_n q_{ni}^a = 0, \quad \forall i \in \hat{I}_g, \quad (12)$$

where \hat{I}_g represents the subset of steps of the offer curve of unit g for which the daily energy must remain unchanged.

Summarizing, our model is conceived to modify a given

family of offer curves by introducing changes in the offer blocks. The changes introduced for the blocks corresponding to the same unit can be limited both within the hour and within the day.

C. Modelling the market clearing process

The day-ahead market considered in this paper is constituted by N hourly independent auctions, so that the clearing process in a given hour n is based only on the offers and bids submitted by participants for that specific hour. As discussed in III.A, the energy sold by the company is determined by the intersection of the company's offer curve, $S_n(p_n)$, and the residual demand curve, $R_n(p_n)$. This intersection is given by a clearing price, p_n^* such that $R_n(p_n^*) = S_n(p_n^*)$.

We consider two cases when calculating the intersection of both curves, as illustrated in Figure 7 and Figure 8.

The case illustrated in Figure 7 occurs when the residual demand curve intersects the vertical segment of a given step i of the company's offer curve. In this case the energy sold by the company is the energy accumulated from the first step to step $i-1$. There are no partially cleared steps and the clearing price p_n^* is determined by the residual demand curve.

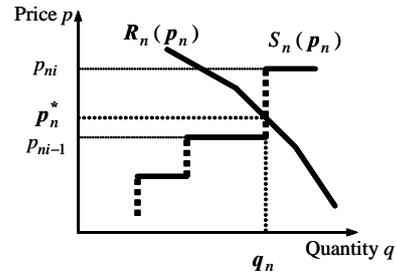


Figure 7. Market clearing with a vertical intersection.

We can model the intersection by means of two constraints.

$$p_n^* \leq y_{ni} M^P + (1 - y_{ni}) p_{ni}, \quad \forall i, \forall n, \quad (13)$$

$$p_n^* \geq y_{ni} p_{ni}, \quad \forall i, \forall n. \quad (14)$$

where y_{ni} is a binary variable that indicates if a certain step i has been accepted in the hourly auction n , and M^P represents the biggest possible price for this model. Then, the amount of energy sold by the company can be simply calculated as

$$q_n = \sum_{\forall i} y_{ni} q_{ni}, \quad \forall n. \quad (15)$$

Figure 8 depicts the other possible case, when the residual demand curve intersects the horizontal segment of a given step i of the company's offer, so that the clearing price is determined by the price of step i . Unfortunately, determining the amount of energy sold by the company in this case requires managing a partially accepted step.

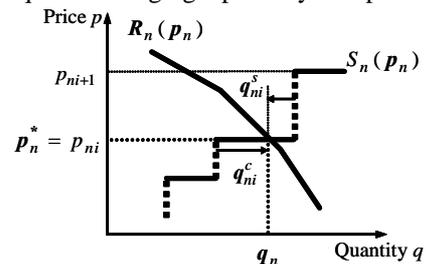


Figure 8. Market clearing with a horizontal intersection.

At this point we are forced to redefine each step of the company's offer curve introducing two new variables. As shown in Figure 8, the first variable represents the amount of energy of the i -th step sold by the company due to the clearing of auction n , q_{ni}^c . The second variable represents the non-accepted or slack energy in each step, q_{ni}^s .

Consequently, in order to determine the energy sold by the company and the clearing price in each auction, n , we must add a number of constraints,

$$q_{ni}^c + q_{ni}^s = q_{ni}^o + q_{ni}^a, \quad \forall i, \forall n, \quad (16)$$

$$q_{ni}^c \geq 0, q_{ni}^s \geq 0, \quad \forall i, \forall n. \quad (17)$$

$$q_{ni}^c \leq y_{ni} M^q, \quad \forall i, \forall n, \quad (18)$$

where M^q represents the biggest possible amount of energy for this model.

We also define a binary variable z_{ni} that indicates if a certain step i has been partially cleared in auction n ,

$$z_{ni} \leq y_{ni}, \quad \forall i, \forall n. \quad (19)$$

$$p_n^* - p_{ni} \leq (1 - z_{ni}) M^p + (y_{ni} - z_{ni}) M^p, \quad \forall i, \forall n. \quad (20)$$

$$q_{ni}^s \leq (1 - y_{ni}) M^q + z_{ni} M^q, \quad \forall i, \forall n, \quad (21)$$

Given this extended formulation, constraint (15) must be replaced by

$$q_n = \sum_i q_{ni}^c, \quad \forall n. \quad (22)$$

To summarize, Figure 9 depicts the intersection of the residual demand curve and the company's offer curve, at step i , showing the most meaningful variables previously defined.

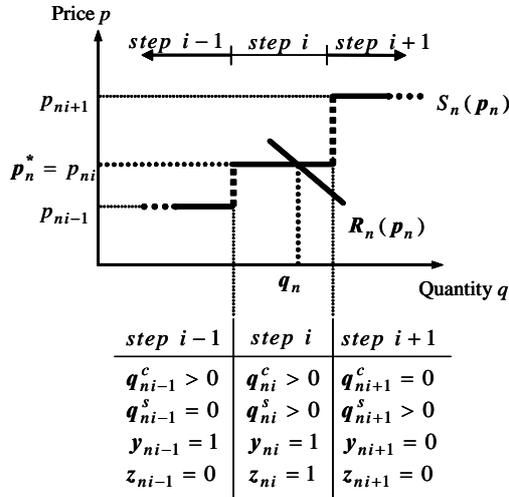


Figure 9. Market clearing for a horizontal-intersection.

D. Modelling the operation of generation units

We assume that, when preparing the offers for the day-ahead market, the generation company has already decided a unit commitment schedule. Under this assumption we can approximate the production costs of thermal unit t in each hour n as a linear function,

$$c_n^t = o^t q_n^t + f^t (\beta^t u_n^t + \alpha^t q_n^t / k^t), \quad \forall t, \forall n, \quad (23)$$

where o^t are unit t 's variable O&M costs, in €/MWh, q_n^t is the net output of unit t , in MW, f^t is the fuel cost, in €/Tcal, β^t is the independent term of the heat rate function, in Tcal/h, $u_n^t \in \{0,1\}$ is the commitment state (on/off) for unit t in hour n , α^t is the linear term of the heat rate

function, in Tcal/MWh, and k^t is the self-consumption coefficient of unit t , in p.u. Given that commitment decisions have already been made, u_n^t enters as input data [3]. Thermal units also have a gross maximum capacity \bar{q}^t , in MW, a gross minimum stable output \underline{q}^t , in MW, and a ramp-rate limit, l_t , in MW/h,

$$\underline{q}^t k^t u_n^t \leq q_n^t \leq \bar{q}^t k^t u_n^t, \quad \forall t, \forall n, \quad (24)$$

$$-l^t \leq q_n^t - q_{n-1}^t \leq l^t, \quad \forall t, \forall n, \quad (25)$$

Our model manages hydro reserves in an aggregate manner by integrating hydro plants located in the same river basin into an equivalent hydro unit, h . The detail of the hydro network can be considered in a subsequent decision stage in order to derive a more precise hydro schedule. We consider a constant energy/flow ratio for each equivalent hydro unit and express hydro reserves in terms of stored energy, in MWh. Equivalent units can also operate in pumping mode. The state of each equivalent reservoir h is evaluated as follows,

$$w_n^h = w_{n-1}^h - q_n^h / k^h + i_n^h - s_n^h + \eta^h b_n^h, \quad \forall h, \forall n, \quad (26)$$

where w_n^h is the energy stored by unit h at the end of hour n in MWh, q_n^h is its net output in hour n in MW, k^h is its self consumption coefficient, in p.u., i_n^h are the net inflows it receives in hour n , in MWh, s_n^h is the energy spilt in MWh, b_n^h is the energy pumped in hour n in MWh and η^h is the performance of the pump-turbine cycle, in p.u.

Each hydro unit has maximum generation and pumping capacities, \bar{q}^h , \bar{b}^h , in MW. Its reservoir has maximum and minimum operating levels, \bar{w}^h , \underline{w}^h , in MWh,

$$0 \leq q_n^h \leq k^h \bar{q}^h, \quad \forall h, \forall n, \quad (27)$$

$$0 \leq b_n^h \leq \bar{b}^h, \quad \forall h, \forall n, \quad (28)$$

$$0 \leq s_n^h, \quad \forall h, \forall n, \quad (29)$$

$$\underline{w}^h \leq w_n^h \leq \bar{w}^h, \quad \forall h, \forall n. \quad (30)$$

In this paper we assume that unit h has a certain amount of energy, w_0^h , available for the planning horizon.

E. Modelling the problem's objective function

The objective of a risk-neutral generation company operating in a day-ahead market is the maximization of its expected profit. In the context of this paper, the company's profit, P , is a random variable calculated as follows:

$$P = \sum_n [\rho_n(q_n) - \sum_t c_n^t(q_n^t)] \quad (31)$$

IV. NUMERICAL APPLICATION TO THE SPANISH ELECTRICITY MARKET

We have implemented the proposed model in GAMS. In this section we present numerical results for an example in the context of the Spanish day-ahead electricity market.

We address the problem of a fictitious power generation company (Table I) owning a number of randomly chosen generation units present in the Spanish power system.

TOTAL	Nuclear	Hydro	Pumping	Fuel	Gas	Coal	CCGT	
18.0913	3.2033	7.571	0.628	3.018	0.174	1.217	2.28	[GW]
100	17.71	41.85	3.47	16.68	0.96	6.73	12.60	[%]

Although our model is intended to solve a stochastic problem, in this case we apply it to a deterministic example to better illustrate the dynamics of the solution process. Introducing uncertainty in the input data should not render qualitative changes to the conclusions. We have chosen a certain day of the past (September 17th 2004) and we have constructed the base-offer curves for our fictitious generation company by aggregating the offers corresponding to the selected units (OMEL publishes the offers and bids submitted by each agent with a three-month delay [8]). Residual demand curves are obtained as described in III.A.

The resulting mixed-integer programming problem has the structure shown in Table II.

Number of Equations	25632
Number of Variables	17765
Number of Binary Variables	8135

We have solved a number of instances of this problem with CPLEX 9.0 in a PC P-IV 2.8GHz 512MB RAM. We explain each of these instances in further detail.

A. Simulation of the market clearing price

We have firstly solved the problem forcing the model to introduce no modifications into the original company's offer curves, so that the optimization runs just as a simulation of the market clearing process. The execution takes 4.175 s and results provide us an expected benefit (Table III), an hourly market-clearing price and an hourly amount of energy sold by the company (Figure 10). These results mimic the actual outcome of the Spanish day-market [8].

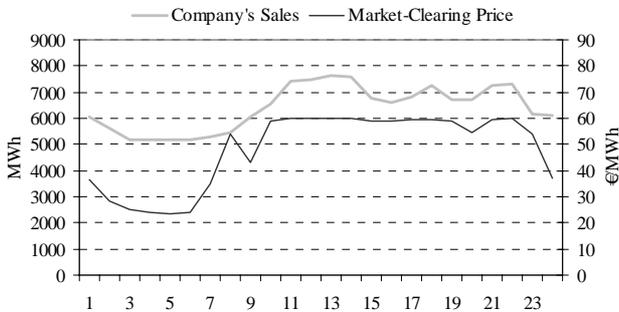


Figure 10. Hourly prices and energy sold by the company for the simulation of the market clearing process.

Total Profit [M€]	2.7584
Total Accepted Quantity [GWh]	153.607
Weighted Average Price [€/MWh]	49.21

B. Optimization of a hydraulic unit without preserving smoothness

In this case we allow the model to modify the offer blocks corresponding to a hydraulic unit. The original unit's offers for hours 6 and 23 are represented in Figure 11.

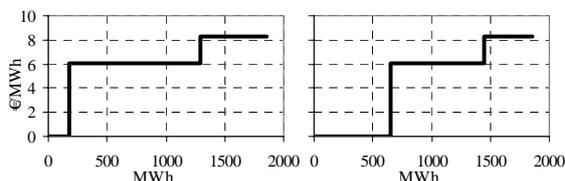


Figure 11. Offer curves for the hydro unit in hours 6 and 23.

For the sake of simplicity we only allow modifications in the lowest-price and the highest-price offer blocks of each curve. Additionally, we relax constraint (11) considering $q_i^a = -M^q$ and $q_i^a = M^q$, so that no limitations are imposed about load-profile smoothness.

60.826 s of execution yield the results shown in Table IV.

Total Profit [M€]	2.7985
Total Accepted Quantity [GWh]	154.957
Weighted Average Price [€/MWh]	49.06

Figure 12 and Figure 13 depict the optimization results and compare them with the simulation results. Figure 14 shows the new offer curves for the hydro unit in two different hours.

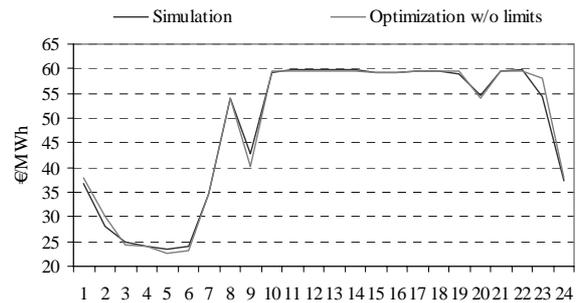


Figure 12. Hourly prices obtained by the model optimization.

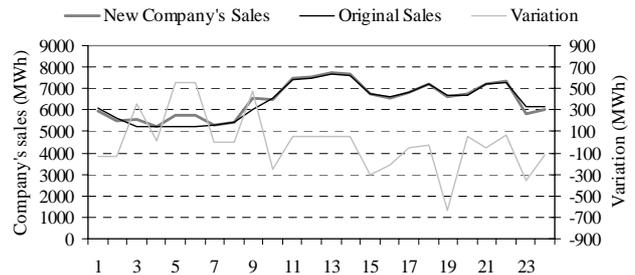


Figure 13. Hourly modifications proposed by the model optimization.

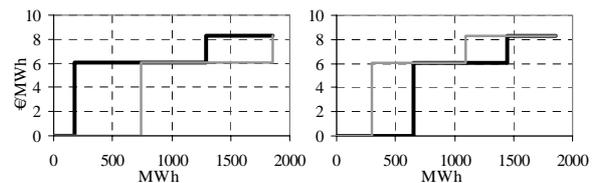


Figure 14. New offer curves for the hydro unit in hours 6 and 23 (in grey).

This comparison reveals that the model is suggesting three different types of modifications in the company's offering strategy.

- Firstly, the model suggests offering more energy at a low price in certain hours when the residual demand is able to absorb an increment of production without forcing the clearing price downwards (e.g. hour 5).
- Secondly, the solution suggests offering less energy at a low price in certain hours when this reduction of production is compensated by an increase of the company's production due to the acceptance of other previously non-accepted offers submitted by the company without forcing the clearing price upwards (e.g. hour 19).

- Finally, the model suggests reducing the amount of energy offered at low prices in hours when such a reduction induces a sharp rise in the clearing price (e.g. hour 23).

To summarize, the model yields an offering strategy in which the company produces more energy while almost maintaining energy prices, so that the expected profit increases and the weighted average price decreases (notice that we have explicitly forced our model to introduce no changes in the daily total amount of energy offered by the hydro unit).

C. Optimization of a hydraulic unit preserving smoothness

In this case we allow the model to modify the offer blocks corresponding to a hydraulic unit again, but imposing a load-profile smoothness constraint by considering $q_i^a = -100MWh$ and $\bar{q}_i^a = 100MWh$ in (11). After 6.399 s of execution, we obtain the results shown in Table V.

Total Profit [M€]	2.783
Total Accepted Quantity [GWh]	154.086
Weighted Average Price [€/MWh]	49.21

Since we have restricted the modifications that the model can introduce in the company's offer curves by introducing a new constraint, the differences observed with respect the first case are less significant, although our model still obtains a greater expected profit, as well as a larger amount of energy sales for the day of interest. Essentially, the solution is an intermediate point between the first and the second case, as illustrated in Figure 15 and Figure 16.

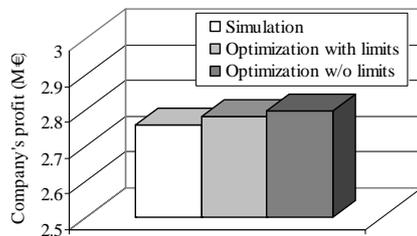


Figure 15. A comparison of the company's expected sales obtained for the three cases.

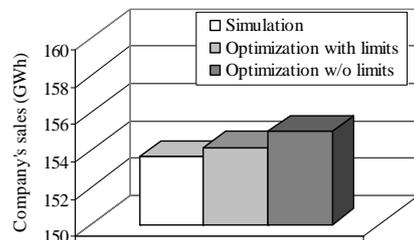


Figure 16. A comparison of expected profits obtained for the three cases.

V. CONCLUSIONS

In this paper we have introduced a novel methodology to build optimal offer curves for a generation company operating in an electricity spot market. Our aim is to optimize pre-existing offer curves rather than to build new optimal ones. This presents a number of advantages when compared with former proposals:

- Our model can be integrated in the decision-making procedure followed by a company when preparing its offer curves. This integration would consist of implementing a new phase at the end of the process of preparing offers that would refine the original curves.
- Given that the model takes actual offer curves prepared by the company as input data, modifications introduced to obtain the optimal curves will always provide a feasible solution, which is at least as profitable as the original one. Moreover, the results of our model suggest qualitative improvements that show company how to improve the construction of offer curves.

The results obtained for a numerical example indicate that slight modifications in the company's original offer curves yield an increment of both company's expected profit and production, while not significantly influencing the market-clearing results. This confirms the validity of our approach.

REFERENCES

- [1] E.J. Anderson and A.B. Philpott, "Optimal Offer Construction in Electricity Markets," *Mathematics of Operations Research*, 27, 82-100, 2002.
- [2] J.M. Arroyo and A.J. Conejo, "Optimal Response of a Thermal Unit to an Electricity Spot Market," *IEEE Transactions on Power Systems*, 15, 1098-1104, 2000.
- [3] Á. Baíllo, M. Ventosa, A. Ramos, M. Rivier, and A. Canseco, "Strategic Unit Commitment for Generation in Deregulated Electricity Markets," in *The Next Generation of Electric Power Unit Commitment Models*, B.F. Hobbs, M.H. Rothkopf, R.P. O'Neill, and H.-P. Chao, eds., 1st ed. Kluwer Academic Publishers, Boston, 2001, 227-248.
- [4] Á. Baíllo, M. Ventosa, M. Rivier, and A. Ramos, "Optimal Offering Strategies for Generation Companies Operating in Electricity Spot Markets," *IEEE Transactions on Power Systems*, 19, 745-753, 2004.
- [5] S. de la Torre, J.M. Arroyo, A. Conejo, and J. Contreras, "Price-Maker Self-Scheduling in a Pool-Based Electricity Market: A Mixed Integer LP Approach," *IEEE Transactions on Power Systems*, 17, 1037-1042, 2002.
- [6] X. Guan, E. Ni, Lu, and Y.-C. Ho, "Optimization-based Bidding Strategies for Deregulated Electric Power Markets," in *The Next Generation of Electric Power Unit Commitment Models*, B.F. Hobbs, M.H. Rothkopf, R.P. O'Neill, and H.-P. Chao, eds., 1st ed. Kluwer Academic Publishers, Boston, 2001, 249-270.
- [7] M.P. Nowak, R. Nürnberg, W. Römisch, R. Schultz, and M. Westphalen, "Stochastic Programming for Power Production and Trading under Uncertainty," in *Mathematics -- Key Technology for the Future*, 2003, 623-636.
- [8] OMEL, "http://www.omel.es."
- [9] G.B. Shrestha, S. Kai, and L. Goel, "Strategic Self-Dispatch Considering Ramping Costs in Deregulated Power Markets," *IEEE Transactions on Power Systems*, 19, 1575-1581, 2004.
- [10] J. Valenzuela and M. Mazumdar, "Unit Commitment based on Hourly Spot Prices in the Electric Power Industry," *Operations Research*, 51, 880-893, 2003.
- [11] F. Wen and A.K. David, "Optimal Bidding Strategies and Modeling of Imperfect Information Among Competitive Generators," *IEEE Transactions on Power Systems*, 16, 15-21, 2001.
- [12] D. Zhang, Y. Wang, and P.B. Luh, "Optimization Based Bidding Strategies in the Deregulated Market," *IEEE Transactions on Power Systems*, 15, 981-986, 2000.