

# THE SHORT-TERM UNCERTAINTY OF SYSTEM MARGINAL PRICE

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**Abstract - A novel model for computing the conditional probability of system marginal price (SMP) is proposed to evaluate SMP short-term uncertainty. In the proposed model, SMP independent influence event sources (IIEs) are assumed, whose independency is obtained by Independent Component Analysis (ICA). According to the independency of these IIEs, the complicated joint probability, which are the key puzzle for computing conditional probability (CCP), are decomposed into corresponding probability and one-event conditional probability of each IIEs. Examples show that our approach is well adapted to evaluate SMP short-term uncertainty.**

**Key Words:** *System Marginal Price, Uncertainty, Conditional Probability, Influence Event, Independent Component Analysis*

## 1 INTRODUCTION

In recent years, the deregulation of the electricity power industry becomes a trend throughout many countries, in order to enhance the efficiency of power industry. As one of the focused problems in the markets, system marginal price (SMP) of electric power is more complicated than the price in other kinds of markets, due to the specialty of electricity commodity, which need the balance of supply and demand in real time. Generally SMP is related to the supply-demand situation, the generation cost of the units, bidding strategies and other relative market factors. All these make SMP change intricately and bring to the market participants and the supervisors with many new challenges, which create the need of an efficient tool to evaluate SMP uncertainty, especially short-term uncertainty.

- For the market participants

In the competitive market environment, the market participants search their ways to improve their returns.

But SMP uncertainty makes their decision-making face great risk. Naturally SMP short-term uncertainty that can provide the probability information on SMP change in the near future, is very useful to conduce them to proper SMP forecasting and risk evaluation.

- For the supervisors

The supervisors analyze SMP changing trend, and then regulate the electricity trades to ensure the healthy and stable development of the electricity market.

In the past several years, some methods for analyzing SMP uncertainty have been studied. As a direct and mathematical tool, SMP probability has been computed by various ways, which can be classified into two mainstreams. On the Gaussian model side, SMP is assumed to be Gaussian, and SMP probability is computed according to Gaussian probability density function (PDF). On the other computation side, SMP probability is computed directly by frequency method without any assumption of SMP PDF. And most efforts have been in acquiring the uncertainty of SMP influence factors during the near trade periods.

In [1], the authors used SMP bivariate probability distribution model to introduce SMP short-term status and proposed a stochastic model by simulating the stochastic characteristics of spot market.

In [2], the authors computed SMP probability based on the bidding uncertainty by using the sequence operation theory. However this model only considered single influence factor - the bidding of the units.

On the whole, the previous methods can hardly be applied to multi-event conditional probabilities. Taking above limitation into account, this paper aims at providing a practical method to compute multi-event conditional probability for evaluating SMP short-term uncertainty more comprehensively. In the proposed model, SMP independent influence event sources (IIEs)

are assumed, whose independency is obtained by Independent Component Analysis (ICA). According to the independency of these IIESs, the complicated joint probabilities, which are the key puzzles for computing conditional probability (CCP), are decomposed into corresponding probability and one-event conditional probability of each IIES. Examples show that our approach is well adapted to evaluating SMP short-term uncertainty.

This paper includes the following parts. In section 2, the conditional probability model for evaluating SMP short-term uncertainty is defined, whose corresponding novel computation method is proposed based on the assumption of SMP IIESs. Besides, Section 3 testifies the validity of the proposed model, and compares it with SMP probability. Section 4 explains the conclusion based on case studies.

## 2 PROPOSED MODEL FOR EVALUATING SMP SHORT-TERM UNCERTAINTY

### 2.1 Model Definition

Firstly SMP multi-event conditional probability is defined as below to evaluate SMP short-term uncertainty:

$$P\left\{z_a < z \leq z_b \left| \begin{array}{l} x_{1,a} < x_1 \leq x_{1,b}, x_{2,a} < x_2 \leq x_{2,b}, \dots \\ x_{n,a} < x_n \leq x_{n,b} \end{array} \right. \right\} \quad (1)$$

where,

$z$ : the observed SMP at the future trade period, whose uncertainty is to be evaluated.

$x_i (i=1,2,\dots,n)$ : SMP original influence event, having zero mean and unit variance. And  $x_1, x_2, \dots, x_n$  constitute a set of influence events, written in the vector  $\mathbf{x}=(x_1, x_2, \dots, x_n)^T \in R^n$ , which is used to introduce the short-term market information, including loads and SMPs during the near previous periods. Here the recent data are used to consider short-term effect and to avoid wrong effect of long-term market data on the model.

Generally conditional probability is computed directly by the traditional method — frequency, which uses the number ratio of the samples that satisfy the same conditions of the observed probability to the whole samples. But due to the short development of electricity market, the foregone data are often insufficient for CCP in (1).

First we convert the conditional probability as below:

$$\begin{aligned} & P\left\{z_a < z \leq z_b \left| \begin{array}{l} x_{1,a} < x_1 \leq x_{1,b}, x_{2,a} < x_2 \leq x_{2,b}, \dots \\ x_{n,a} < x_n \leq x_{n,b} \end{array} \right. \right\} \\ &= \frac{P\left\{ \begin{array}{l} x_{1,a} < x_1 \leq x_{1,b}, x_{2,a} < x_2 \leq x_{2,b} \\ \dots, x_{n,a} < x_n \leq x_{n,b} \end{array} \right\} P\{z_a < z \leq z_b\}}{P\left\{ \begin{array}{l} x_{1,a} < x_1 \leq x_{1,b}, x_{2,a} < x_2 \leq x_{2,b}, \dots \\ x_{n,a} < x_n \leq x_{n,b} \end{array} \right\}} \quad (2) \end{aligned}$$

where,

$$P\left\{ \begin{array}{l} x_{1,a} < x_1 \leq x_{1,b}, x_{2,a} < x_2 \leq x_{2,b}, \dots \\ x_{n,a} < x_n \leq x_{n,b} \end{array} \right\} \quad (3)$$

is the joint probability of  $x_1, x_2, \dots, x_n$  in their intervals without conditions;

$$P\left\{ \begin{array}{l} x_{1,a} < x_1 \leq x_{1,b}, x_{2,a} < x_2 \leq x_{2,b}, \dots \\ x_{n,a} < x_n \leq x_{n,b} \end{array} \right| z_a < z \leq z_b \quad (4)$$

is the joint probability of  $x_1, x_2, \dots, x_n$  in their intervals, under condition of  $z_a < z \leq z_b$ , also called one-event joint conditional probability.

One of the main difficulties for computing joint probabilities of (3) and (4) is the correlation among SMP original influence events. So far, there exists no model that would ideally compute those joint probabilities.

To solve that problem, we assume that  $x_1, x_2, \dots, x_n$  are the mixing results of unknown independent signal sources, among which there are not more than one Gaussian signal, as defined in (5).

$$\mathbf{x} = \mathbf{A}\mathbf{s} \quad (5)$$

where,

$\mathbf{x}$ : SMP original influence event set,  $\mathbf{x}=(x_1, x_2, \dots, x_n)^T \in R^n$ ;

$\mathbf{s}$ : unknown independent signal source set,  $\mathbf{s}=(s_1, s_2, \dots, s_n)^T \in R^n$ ;

$\mathbf{A}$ : mixing matrix.

According to FastICA[3],  $\mathbf{s}$  can be well estimated by finding the right linear combinations in (6).

$$\mathbf{y} = \mathbf{W}\mathbf{x} \quad (6)$$

where,

$\mathbf{W}$ : de-mixing constant transformation matrix. The term ' $\det \mathbf{W}$ ' is the determinant of  $\mathbf{W}$ , and  $\det \mathbf{W} \neq 0$ .

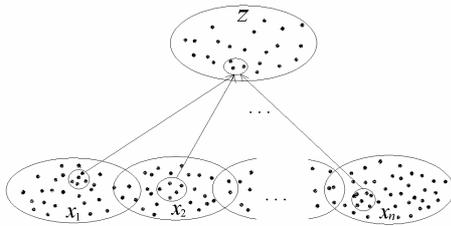
$\mathbf{y}$ : estimation of  $\mathbf{s}$ , called SMP independent influence event source (IIES) set, whose components are

statistically independent from each other. And  $\mathbf{y}=(y_1, y_2, \dots, y_n)^T \in R^n$ .

Based on that assumption, new model for computing SMP conditional probability is presented in the following section.

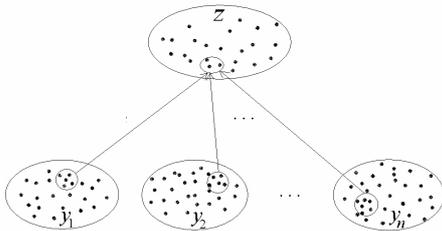
## 2.2 Building New Computation Model

In the proposed model, it is noticeable that  $x_1, x_2, \dots, x_n$  are defined as the previous loads and SMPs. The other point here is the relation of SMP, its original influence events and IIESs. As defined, any of its influence event  $x_i (i=1, \dots, n)$  happens earlier than  $z$ , then  $z$  has no ‘direct’ impact on  $x_1, x_2, \dots, x_n$ . Meanwhile,  $z$  limits the values of  $x_i (i=1, \dots, n)$  to some corresponding subspace. From the relation of  $x_i$  and  $z$  shown in Figure 1, the data of  $z$  limit all the possible data of  $x_i$  into the subset of  $R^n$ , namely  $O_X$ .



**Figure 1:** Relation of  $z$  and  $x_1, x_2, \dots, x_n$ .

Likewise, according to the independency of SMP IIESs,  $z$  limits the values of  $y_1, y_2, \dots, y_n$  to the corresponding subspace  $O_Y$ . And the relation of  $z$  and the obtained IIESs is shown in Figure 2.



**Figure 2:** Relation of  $z$  and  $y_1, y_2, \dots, y_n$ .

Now Jacobian matrix of the mapping  $\mathbf{y}=\mathbf{W}\mathbf{x}$  is

$$\mathbf{M}_{\text{Jacobian}} = \begin{vmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \dots & \frac{\partial y_1}{\partial x_n} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} & \dots & \frac{\partial y_2}{\partial x_n} \\ \vdots & \vdots & \dots & \vdots \\ \frac{\partial y_n}{\partial x_1} & \frac{\partial y_n}{\partial x_2} & \dots & \frac{\partial y_n}{\partial x_n} \end{vmatrix} = \det \mathbf{W} \quad (7)$$

And

$$p(y_1, y_2, \dots, y_n) = \frac{q(x_1, x_2, \dots, x_n)}{|\det \mathbf{W}|} \quad (8)$$

where,

$q(x_1, x_2, \dots, x_n)$ : joint probability density function of  $x_1, x_2, \dots, x_n$ ;

$p(y_1, y_2, \dots, y_n)$ : joint probability density function of  $y_1, y_2, \dots, y_n$ .

In the case of equation (8), the joint probability in (3) becomes

$$\begin{aligned} & P\{x_{1,a} < x_1 \leq x_{1,b}, x_{2,a} < x_2 \leq x_{2,b}, \dots, x_{n,a} < x_n \leq x_{n,b}\} \\ &= \int_{x_{1,b}}^{x_{1,a}} \dots \int_{x_{n,b}}^{x_{n,a}} q(x_1, x_2, \dots, x_n) dx_1 \dots dx_n \\ &= \int_{y_{1,b}}^{y_{1,a}} \dots \int_{y_{n,b}}^{y_{n,a}} p(y_1, y_2, \dots, y_n) |\det \mathbf{W}| dy_1 \dots dy_n \end{aligned} \quad (9)$$

Considering the independency of SMP IIESs, that is,

$$p(y_1, y_2, \dots, y_n) = p_1(y_1)p_2(y_2)\dots p_n(y_n) \quad (10)$$

We deduce

$$\begin{aligned} & P\{x_{1,a} < x_1 \leq x_{1,b}, x_{2,a} < x_2 \leq x_{2,b}, \dots, x_{n,a} < x_n \leq x_{n,b}\} \\ &= |\det \mathbf{W}| \int_{y_{1,b}}^{y_{1,a}} p_1(y_1) dy_1 \int_{y_{2,b}}^{y_{2,a}} p_2(y_2) dy_2 \dots \int_{y_{n,b}}^{y_{n,a}} p_n(y_n) dy_n \\ &= |\det \mathbf{W}| P\{y_{1,a} < y_1 \leq y_{1,b}\} \cdot P\{y_{2,a} < y_2 \leq y_{2,b}\} \\ & \quad \dots P\{y_{n,a} < y_n \leq y_{n,b}\} \\ &= |\det \mathbf{W}| \prod_{i=1}^n P\{y_{i,a} < y_i \leq y_{i,b}\} \end{aligned} \quad (11)$$

where,

$p_i(y_i)$ : probability density function of  $y_i, i=1, 2, \dots, n$ .

Note that we transform the subset of  $\mathbf{x}$  under condition of  $z$  by the same matrix  $\mathbf{W}$ . Since the whole dataset of  $y_i (i=1, 2, \dots, n)$  is independent from that of  $y_j (i \neq j)$ , the subset of  $y_i$  under condition of  $z$  is also independent from the subset of  $y_j (i \neq j)$ . Thus, the following equation is true.

$$\begin{aligned}
& P \left\{ \begin{array}{l} x_{1,a} < x_1 \leq x_{1,b}, x_{2,a} < x_2 \leq x_{2,b}, \dots, \\ x_{n,a} < x_n \leq x_{n,b} | z_a < z \leq z_b \end{array} \right\} \\
&= |\det \mathbf{W}| \int_{y_{1,b}}^{y_{1,a}} p_1(y_1 | z_a < z \leq z_b) dy_1 \int_{y_{2,b}}^{y_{2,a}} p_2(y_2 | z_a < z \leq z_b) dy_2 \\
&\dots \int_{y_{n,b}}^{y_{n,a}} p_n(y_n | z_a < z \leq z_b) dy_n \\
&= |\det \mathbf{W}| P \{ y_{1,a} < y_1 \leq y_{1,b} | z_a < z \leq z_b \} \\
&\cdot P \{ y_{2,a} < y_2 \leq y_{2,b} | z_a < z \leq z_b \} \\
&\dots P \{ y_{n,a} < y_n \leq y_{n,b} | z_a < z \leq z_b \} \\
&= |\det \mathbf{W}| \prod_{i=1}^n P \{ y_{i,a} < y_i \leq y_{i,b} | z_a < z \leq z_b \} \tag{12}
\end{aligned}$$

where,

$$p_i(y_i | z_a < z \leq z_b) \quad :i=1,2,\dots,n, \text{ probability density}$$

function of  $y_i$  under condition of  $z$ .

By substituting (11) and (12) into (2), new model for CCP is derived to be

$$\begin{aligned}
& P \left\{ z_a < z \leq z_b \left| \begin{array}{l} x_{1,a} < x_1 \leq x_{1,b}, x_{2,a} < x_2 \leq x_{2,b}, \dots, \\ x_{n,a} < x_n \leq x_{n,b} \end{array} \right. \right\} \\
&= \frac{\prod_{i=1}^n P \{ y_{i,a} < y_i \leq y_{i,b} | z_a < z \leq z_b \}}{\prod_{i=1}^n P \{ y_{i,a} < y_i \leq y_{i,b} \}} P \{ z_a < z \leq z_b \} \tag{13}
\end{aligned}$$

where,

$P \{ y_{i,a} < y_i \leq y_{i,b} | z_a < z \leq z_b \}$ : conditional probability of  $y_i$ ? ( $y_{i,a}, y_{i,b}$ ) when  $z_a < z \leq z_b$ .

$$P \{ y_{i,a} < y_i \leq y_{i,b} \}: \text{probability of } y_i \in (y_{i,a}, y_{i,b}).$$

Obviously, we have solved the problem of computing SMP conditional probability successfully by the decomposed computation model in (13).

Since above derivation is based on the assumption of SMP IIESs, we will introduce how to mine these IIESs as follows.

### 2.3 Mining SMP IIESs by ICA

In recent years, independent component analysis has been an efficient technique for Blind Source Separation, which is typically to recover unknown independent sources from the received mixing signals whose method of mixing is unknown [4]. Aapo Hyvärinen proposed a

rapid and simple algorithm of ICA, named FastICA, which has been widely applied for its appealing convergence properties [5].

In the present model,  $\mathbf{x}$  and  $\mathbf{y}$  are firstly standardized to have zero mean and unit variance. According ICA theory, negentropy  $J(\mathbf{y})$  and mutual information  $I(y_1, y_2, \dots, y_n)$  are defined individually [6,7].

$$J(\mathbf{y}) = H(y_{gauss}) - H(\mathbf{y}) \tag{14}$$

$$I(y_1, y_2, \dots, y_n) = J(\mathbf{y}) - \sum_{i=1}^n J(y_i) \tag{15}$$

where,  $y_{gauss}$  is a Gaussian random variable of the same covariance matrix as  $\mathbf{y}$ .

For random SMP IIES  $y_i = \mathbf{w}_i^T \mathbf{x}$ , non-quadratic function  $G$  is used to form the simpler approximations of above negentropy. Then the optimization objective is in the following general form [6,7]:

$$\begin{aligned}
& \text{maximize } \sum_{i=1}^n \left\{ E[G(\mathbf{w}_i^T \mathbf{x})] - E[G(y_{gauss})] \right\}^2 \\
& \text{under constraint } E[\mathbf{w}_i^T \mathbf{x} \mathbf{w}_j^T \mathbf{x}] = d_{ij} \tag{16} \\
& \text{wrt. } \mathbf{w}_i, i=1,2,\dots,n
\end{aligned}$$

where,

$\mathbf{w}_i$  is the  $i$ -th row of  $\mathbf{W}$ .

$$\text{Practically } G(u) = \frac{1}{a} \log \cosh au.$$

And FastICA seeks independent components one by one. After  $\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_n$  are found,  $y_1, y_2, \dots, y_n$  are obtained.

### 2.4 Computing the Relative Probabilities and Conditional Probabilities

According to the proposed model in (13), we decompose the joint probabilities in (3) and (4) into  $2n+1$  probabilities and one-event conditional probabilities of SMP IIESs. Here the corresponding probabilities are computed as follows,

$$\begin{aligned}
& P \{ y_{i,a} < y_i \leq y_{i,b} | z_a < z \leq z_b \} \\
&= \int_{y_{i,a}}^{y_{i,b}} p_i(y_i | z_a < z \leq z_b) dy_i \\
&= \sum_{k=1}^{t-1} f_{i,k}(y_{i,k} | z_a < z \leq z_b) (y_{i,k+1} - y_{i,k}) \tag{17} \\
& \text{wrt. } y_{i,1} < y_{i,2} < \dots < y_{i,k} < y_{i,k+1} < \dots < y_{i,t} \\
& \quad y_{i,a} = y_{i,1} \quad y_{i,t} = y_{i,b}
\end{aligned}$$

where

$i: 1, 2, \dots, n;$

$t$ : interval number of  $y_i$ .

$p_i(y_i | z_a < z \leq z_b)$  : conditional probability density

function of  $y_i$  when  $z_a < z \leq z_b$  ;

$f_{i,k}(y_{i,k} | z_a < z \leq z_b)$  : conditional probability density of

$y_{i,k}$  when  $z_a < z \leq z_b$  .

$$\begin{aligned}
 & P\{y_{i,a} < y_i \leq y_{i,b}\} \\
 &= \int_{y_{i,a}}^{y_{i,b}} p_i(y_i) dy_i \\
 &= \sum_{k=1}^{t-1} f_{i,k}(y_{i,k}) (y_{i,k+1} - y_{i,k}) \quad (18) \\
 & \text{wrt. } y_{i,1} < y_{i,2} < \dots < y_{i,k} < y_{i,k+1} < \dots < y_{i,t} \\
 & \quad y_{i,a} = y_{i,1} \quad y_{i,t} = y_{i,b}
 \end{aligned}$$

where,

$i: 1, 2, \dots, n;$

$t$ : interval number of  $y_i$ .

$p_i(y_i)$  : conditional probability density function of  $y_i$ ;

$f_{i,k}(y_{i,k})$  : conditional probability density of  $y_{i,k}$ .

By utilizing the dataset of every SMP IIES and its subset under condition of  $z$ , we compute  $f_{i,k}(y_{i,k} | z_a < z \leq z_b)$  and  $f_{i,k}(y_{i,k})$ ,  $k=1, 2, \dots, t$ , based on kernel density estimator that can deal with various distributions.

Finally we put the computational results of  $P\{y_{i,a} < y_i \leq y_{i,b} | z_a \leq z \leq z_b\}$  and  $P\{y_{i,a} < y_i \leq y_{i,b}\}$ ,  $i=1, 2, \dots, n$  into (13), and obtain the solution of SMP conditional probability.

### 3 CASE STUDIES

#### 3.1 Examples

The approach presented above has been tested by the real data of New England market. In this test, we evaluate the short-term uncertainty of forecasted SMP, which is an important and unsolved problem in the research of SMP forecasting. We take the conditional probability of forecasted SMP at 15:00 on Dec. 26<sup>th</sup> 2002 for example. Here SMP conditional probability is defined as:

$$P\left\{z_a < z \leq z_b \left| \begin{array}{l} x_{1,a} < x_1 \leq x_{1,b}, x_{2,a} < x_2 \leq x_{2,b}, \dots, \\ x_{7,a} < x_7 \leq x_{7,b} \end{array} \right. \right\} \quad (19)$$

where,

- $z$ : forecasted SMP;
- $x_1$ : forecasted load of the observed trade period;
- $x_2, x_3, x_4$ : loads at 15:00 of preceding three days;
- $x_5, x_6, x_7$ : SMPs at 15:00 of preceding three days.

Then an interesting question is on earth what is the advantage of the proposed method with comparison to traditional frequency method. Our experiments show that the number of historical samples that meet the conditions in (19), is so few, even none. Considering the insufficiency of traditional methods, we use the proposed method for CCP. First the samples of  $x$  and  $x$  under condition of  $z$  are plotted in Figure 3 and Figure 4. After that, SMP IIES vector  $y$  and  $y$  under condition of  $z$  are mined and shown in Figure 5 and Figure 6 respectively.

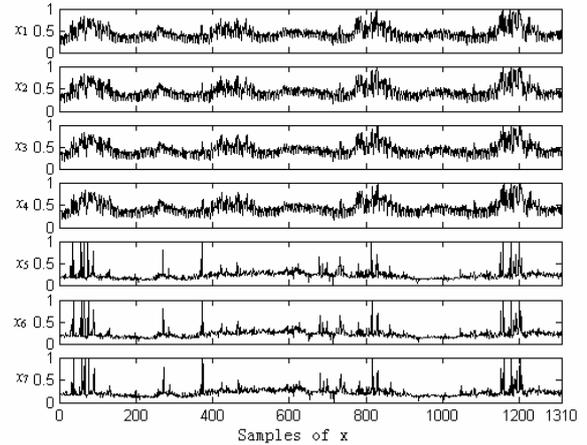


Figure 3: Samples of SMP original influence events

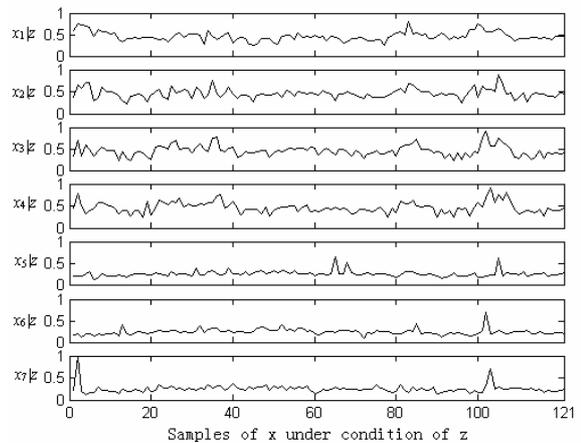


Figure 4: Samples of SMP original influence events under condition of  $z$

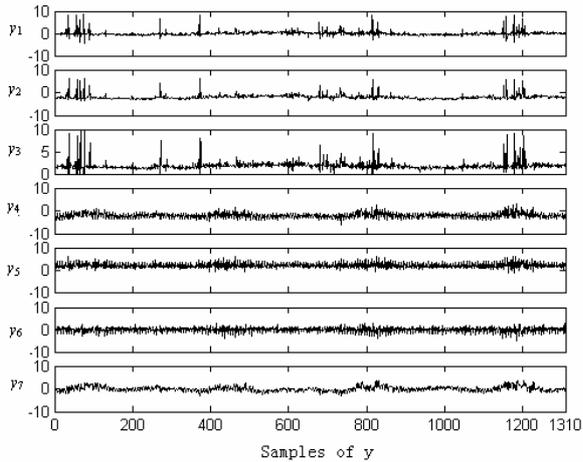


Figure 5: Samples of SMP IIESs

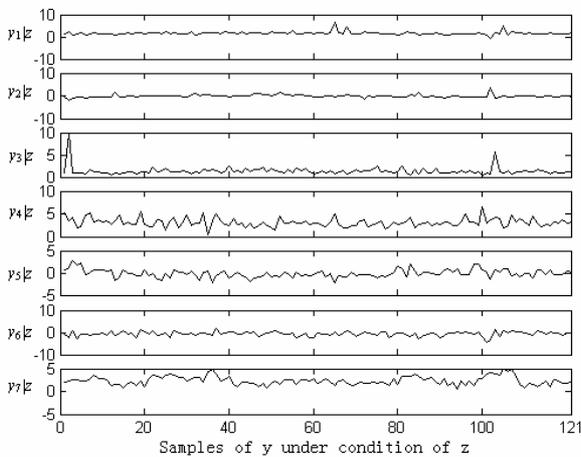


Figure 6: Samples of SMP IIESs under condition of  $z$

Besides, the kurtoses of SMP IIESs defined in (20) are listed in TABLE 1, which proved the validity of the assumption on SMP IIESs.

$$C_{kurt}(y) = \frac{E[y - E(y)]^4}{\{E[y - E(y)]^2\}^2} - 3 = \begin{cases} > 0, & \text{Super-Gauss} \\ = 0, & \text{Gauss} \\ < 0, & \text{Sub-Gauss} \end{cases} \quad (20)$$

IIES	$C_{kurt}$	IIES	$C_{kurt}$
$y_1$	32.0021	$y_5$	1.4543
$y_2$	32.1549	$y_6$	1.7246
$y_3$	24.7970	$y_7$	0.4620
$y_4$	1.9355		

TABLE 1: KURTOSSES OF SMP IIESs.

By the proposed model, the relative probabilities and conditional probabilities of SMP IIESs are computed and

listed in TABLE 2, when SMP is forecasted to be 43\$/MWh.

IIES	$P\left\{ \begin{matrix} y_{i,a} < y_i \leq y_{i,b} \\  z_a \leq z \leq z_b \end{matrix} \right\}$	Number of Samples	$P\{y_{i,a} < y_i \leq y_{i,b}\}$	Number of Samples
$y_1$	0.53354	10	0.28539	57
$y_2$	0.68326	10	0.52931	69
$y_3$	0.36343	5	0.20471	51
$y_4$	0.46135	20	0.30562	161
$y_5$	0.55456	23	0.52653	154
$y_6$	0.60901	26	0.63071	210
$y_7$	0.44168	23	0.34596	170

TABLE 2: PROBABILITIES AND CONDITIONAL PROBABILITIES OF SMP IIESs WHEN SMP IS FORECASTED TO BE 43\$/MWh.

Finally, we obtain SMP conditional probability:

$$P \left\{ \begin{matrix} 16200 < x_1 \leq 17000, \\ 14600 < x_2 \leq 15400, \\ 14600 < x_3 \leq 15400, \\ 15400 < x_4 \leq 16200, \\ 44 < x_5 \leq 48, \\ 40 < x_6 \leq 44, \\ 44 < x_7 \leq 48 \end{matrix} \right\} = 0.775619$$

### 3.2 Discussion

TABLE 2 tells us another fact that SMP conditional probabilities are compared with its probabilities to verify the representation ability of short-term uncertainty. Here TABLE 3 lists the relative probabilities and conditional probabilities of different forecasted SMPs at 15:00 on Dec. 26<sup>th</sup>, with the actual SMP being 41.9\$/MWh.

Forecasted SMP (\$/MWh)	$P\{z_a \leq z \leq z_b\}$	$P\left\{ \begin{matrix} z_a < z \leq z_b \\ \left. \begin{matrix} x_{1,a} < x_1 \leq x_{1,b}, \\ x_{2,a} < x_2 \leq x_{2,b}, \dots, \\ x_{n,a} < x_n \leq x_{n,b} \end{matrix} \right\} \right\}$
51.2	0.04809	0.24
43.0	0.092366	0.775619
37.0	0.09313	0.203637

TABLE 3: PROBABILITIES AND CONDITIONAL PROBABILITIES OF DIFFERENT FORECASTED SMPs WHEN ACTUAL SMP IS 41.9\$/MWh

It is obvious in TABLE 3 that the probabilities and conditional probabilities of the forecasted SMPs are different. As the accuracy of forecasted SMP increases, its conditional probability becomes more. For actual SMP, the closest forecast value is 43\$/MWh, whose conditional probability is the most. Contrastively, the conditional probabilities of 37\$/MWh and 51.2\$/MWh are so low, which reflects that these two forecasted SMPs are a little far from the real SMP.

On the contrary, SMP probability data have not shown that trend. In detail, the probabilities of forecasted SMPs being 43\$/MWh and 37\$/MWh are adjacent, though these SMPs differ in their forecast errors. So above analysis explains that conditional probability can reflect the short-term uncertainty of the observed SMP to some extent.

#### 4 CONCLUSIONS

As a reasonable and efficient tool, SMP conditional probability is presented to evaluate SMP short-term uncertainty by considering short-term market information.

For the complexity of CCP, this paper breaks through the old traditional computation methods and invents a new way that decomposes the impalpable joint probabilities by seeking latent SMP independent influence event sources. Finally, the method is proved to be applicable and efficient.

Moreover, the proposed model takes the public information, such as previous SMPs and loads, as the influence events of the observed SMP. On the basis of such information, the model considers more comprehensive market status and has a wide application scope.

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