

# UNIT COMMITMENT WITH PROBABILISTIC SPINNING RESERVE ASSESSMENT USING SIMULATED ANNEALING

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**Abstract** — This paper proposes a method for the probabilistic spinning reserve assessment, incorporated in a simulated annealing algorithm for the solution of the unit commitment problem. Simulated annealing is used for the scheduling of the on/off status of the generating units, while the economic load dispatch is performed through a quadratic programming routine. The evaluation of the required spinning reserve capacity is performed by using the expected unserved energy reliability index. The proposed method ensures that the availability of the generating units is taken into consideration in the solution of the daily unit commitment problem. In this way, the required spinning reserve capacity is effectively assessed, resulting to near optimal unit commitment solutions, which provide a reasonable level of reliability. Numerical simulations have proved the effectiveness of the proposed model.

**Keywords:** *Unit commitment, probabilistic reserve assessment, simulated annealing*

## 1 INTRODUCTION

Unit commitment (UC) plays major role in the daily operation planning of the power systems. In the new era of liberalization, system operators need to perform many UC studies, in order to assess the spinning reserve capacity required to operate the system as securely as possible, even in the presence of contingencies. The objective of the UC problem is the minimization of the total operating cost of the generating units during the scheduling horizon, subject to many system and unit constraints. The overall problem can be divided into two sub-problems: the mixed-integer nonlinear programming problem of scheduling the on/off status of the generating units for every hour of the dispatch period (usually 24 hours) and the quadratic programming problem of dispatching the forecasted load among them. The simultaneous solution of both problems is a very complicated procedure, the difficulty of which grows proportionally to the number of units and constraints taken into consideration.

The evaluation of the system spinning reserve is usually based on deterministic criteria. According to the most common deterministic criterion, the reserve should be at least equal to the capacity of the largest unit, or to a specific percentage of the system load. The basic disadvantage of the deterministic approach is that it does not reflect the stochastic nature of the system

components. On the contrary, the probabilistic techniques can provide a more realistic evaluation of the spinning reserve requirements by incorporating various system uncertainties [1]-[3].

During the last few years, simulated annealing (SA) has been widely used for the solution of the UC problem [4]-[6]. SA is a powerful optimization technique, proposed by Kirkpatrick, Gelatt and Vecchi in 1983 [7], which is useful for solving difficult combinatorial optimization problems without a specific structure. This technique takes advantage of the analogy between the minimization of the objective function of an optimization problem and the slow procedure of gradually cooling a metal, until it reaches its “freezing” point, where the energy of the system has acquired the globally minimal value. The SA method attracts much attention, because of its ability to come up with very good solutions, while it can easily deal with difficult nonlinear constraints.

In this paper, a new method for incorporating the probabilistic spinning reserve assessment in the solution of the UC problem is presented. A robust SA algorithm is used for solving the UC problem, while the evaluation of the required spinning reserve capacity is performed by using the expected unserved energy (EUE) reliability index. This index expresses the expected energy that will not be served by the generation system. For each candidate solution provided by the SA algorithm, the EUE of the dispatch period is calculated. In case the calculated unserved energy exceeds a given maximum allowed limit, a quadratic penalty term is included in the operating cost of the current solution. The use of a dynamic penalty function allows the solutions which violate the above reliability constraint to evolve into feasible solutions. The above procedure ensures that the required spinning reserve capacity is effectively scheduled, resulting to near optimal unit commitment solutions, which provide a reasonable level of reliability.

The proposed algorithm is significantly fast, while the quality of the obtained results is very high. The effectiveness of the model has been tested on a system found in the literature. Furthermore, a sensitivity analysis was performed in order to examine the effect of various reliability parameters on the solution of the UC problem.

## 2 UNIT COMMITMENT PROBLEM

The objective of the UC problem is the minimization of the total operating cost of the generating units during the scheduling horizon subject to several system and unit constraints.

### 2.1 Objective Function

The overall objective function is composed of two parts: the fuel cost and the start-up cost. The fuel cost  $FC_{i,t}$  of unit  $i$  at hour  $t$  is expressed as a second-order polynomial:

$$FC_{i,t}(P_{i,t}) = A_i P_{i,t}^2 + B_i P_{i,t} + C_i \quad (1)$$

where  $P_{i,t}$  is the power generation of unit  $i$  at hour  $t$  and  $A_i, B_i, C_i$  are the cost coefficients of unit  $i$ .

The start-up cost depends on the number of hours during which the unit has been off. The most commonly used functions, which describe the start-up cost, are the exponential and the two-step functions.

According to the exponential function, the start-up cost  $STRT_{i,t}$  of unit  $i$  at hour  $t$  is given by:

$$STRT_{i,t} = b_{0,i} \left( 1 - e^{-\frac{\max(0, -Toff_{i,t-1})}{k_i}} \right) + b_{1,i} \quad (2)$$

where  $b_{0,i}, b_{1,i}, k_i$  are the start-up cost coefficients of unit  $i$  and  $Toff_{i,t}$  is a negative integer indicating the consecutive time that unit  $i$  has been off at hour  $t$ .

On the other hand, the two-step function can be written as:

$$STRT_{i,t} = \begin{cases} S_{h,i}, & \text{if } -Toff_{i,t-1} \leq t_{cold,i} \\ S_{c,i}, & \text{otherwise} \end{cases} \quad (3)$$

where  $t_{cold,i}$  is the number of hours that it takes for the boiler of unit  $i$  to cool down, while  $S_{h,i}$  and  $S_{c,i}$  are the start-up costs of unit  $i$  incurred for a hot and cold start, respectively.

Consequently, the UC objective function is given by the minimization of the following cost function:

$$TC = \sum_{t=1}^T \sum_{i=1}^{NG} [ST_{i,t} FC_{i,t}(P_{i,t}) + ST_{i,t}(1 - ST_{i,t-1}) STRT_{i,t}] \quad (4)$$

where  $TC$  is the total cost of the system,  $T$  is the dispatch period under consideration,  $NG$  is the number of generating units and  $ST_{i,t}$  represents the status of unit  $i$  at hour  $t$  (1 if the unit is on and 0 if the unit is off).

### 2.2 Constraints

The minimization of the objective function is subject to a number of system and unit constraints.

The system power balance is given by:

$$\sum_{i=1}^{NG} P_{i,t} = LOAD_t, \quad t \in [1, T] \quad (5)$$

where  $LOAD_t$  is the system demand at hour  $t$ .

The power produced by each unit must be within certain limits, as indicated below:

$$ST_{i,t} P \min_i \leq P_{i,t} \leq ST_{i,t} P \max_i, \quad i \in NG; t \in [1, T] \quad (6)$$

where  $P \min_i$  and  $P \max_i$  are the minimum and maximum power output of unit  $i$ , respectively.

As already mentioned, spinning reserve requirements can be taken into account by using either deterministic criteria or probabilistic techniques. In the first case, these requirements can be specified in terms of excess megawatt capacity, which is expressed by:

$$\sum_{i=1}^{NG} ST_{i,t} P \max_i \geq LOAD_t + RSRV_t, \quad t \in [1, T] \quad (7)$$

where  $RSRV_t$  is the required spinning reserve at hour  $t$ .

For the purposes of the probabilistic reserve assessment, the required reserve is assessed according to the desired level of reliability. Therefore, constraint (7) is replaced by the following equation:

$$\sum_{i=1}^{NG} ST_{i,t} P \max_i \geq LOAD_t, \quad t \in [1, T]. \quad (8)$$

The minimum capacity constraint is represented by:

$$\sum_{i=1}^{NG} ST_{i,t} P \min_i \leq LOAD_t, \quad t \in [1, T]. \quad (9)$$

The minimum up/down time constraints are given by:

$$Ton_{i,t} \geq Tup_i, \quad i \in NG; t \in [1, T] \quad (10)$$

$$-Toff_{i,t} \geq Tdown_i, \quad i \in NG; t \in [1, T] \quad (11)$$

where  $Ton_{i,t}$  is the consecutive time that unit  $i$  has been on at hour  $t$ , while  $Tup_i$  and  $Tdown_i$  are the minimum up-time and minimum down-time respectively of unit  $i$ .

Finally, the initial status (on/off) of each unit constitutes an additional constraint of the UC problem.

## 3 SIMULATED ANNEALING ALGORITHM

### 3.1 The Framework of Simulated Annealing

The SA method simulates the procedure of gradually cooling a metal, until the energy of the system acquires the globally minimal value. The concept of SA is based on the manner in which metals re-crystallize in the process of annealing. Beginning with a high temperature, a metal is slowly cooled, so that the system is in thermal equilibrium at every stage. At high temperatures the metal is in liquid phase and the atoms of the system are disordered and randomly arranged. By gradually cooling the metal, the system becomes more ordered, until it finally reaches a "frozen" ground state (zero temperature), where the energy of the system has acquired the globally minimal value [7].

Metropolis et al [8] proposed an iterative method to simulate the evolution to thermal equilibrium of a metal for a fixed value of the temperature. In each step of the algorithm, an atom is displaced through a random perturbation of its current state and the consequential change  $\Delta E$  in the energy of the system is calculated. If  $\Delta E \leq 0$ , the perturbation results in a lower energy for the metal. In this case, the change is accepted and the new

configuration of the system constitutes the starting point for the next step. If  $\Delta E > 0$ , the proposed change is accepted with a probability given by Boltzmann distribution:

$$P(\Delta E) = \exp(-\Delta E / k_B \text{Temp}) \quad (12)$$

where  $k_B$  is Boltzmann's constant and  $\text{Temp}$  corresponds to the current temperature. This acceptance rule for new states is referred to as the Metropolis criterion. The use of  $P(\Delta E)$  forces the system to evolve into thermal equilibrium, i.e. after a large number of perturbations, the probability distribution of the states approaches the Boltzmann distribution.

Consequently, if we start from an initial high temperature and gradually decrease it, using the above algorithm at each temperature, new lower energy states become reachable. At each temperature, the Metropolis criterion is applied for a sequence of trials, where the outcome of each trial depends only on the outcome of the previous one. This procedure is mathematically best described by means of a Markov chain, where the length of each chain is equal to a specific number of iterations performed at each temperature. As the temperature decreases, the Boltzmann distribution concentrates on the states with lower energy and finally, when the temperature approaches asymptotically to zero, only the minimum energy states have a non-zero probability of appearance. The above procedure is modeled through equation (12), due to which the probability of acceptance of higher energy states is large in high temperatures, whereas it becomes smaller as the temperature is lowered (convergence to "frozen" ground state).

By analogy, the SA algorithm is used for finding the optimal (or near optimal) solution of the combinatorial optimization problem of UC. Each configuration of the physical system corresponds to the current solution of the optimization problem, the energy of the atoms is analogous to the cost of the objective function and the final ground state is equivalent to the global minimum of the cost function. Moreover, the temperature plays the role of the control parameter of the optimization procedure.

### 3.2 Application of Simulated Annealing to UC

As mentioned above, SA is used for finding the optimal scheduling of the generating units during the dispatch period, while the economic dispatch sub-problem is solved separately for each hour through a quadratic programming routine. The basic steps of the SA algorithm are:

1. Find, randomly, an initial feasible solution, which is assigned as the current solution  $X_i$ , and perform the economic dispatch in order to calculate the operating cost of the system. Calculate the start-up cost by (2) or (3) and the total cost  $TC_i$  by (4).
2. Determine the initial value of the control parameter of temperature  $\text{Temp} = \text{Temp}_0$ .
3. Set iteration counter  $k=1$ .
4. Find a neighboring solution  $X_j$  through a random

perturbation of the current one and calculate the new total cost  $TC_j$ .

5. If  $TC_j \leq TC_i$  accept the trial solution, set  $X_i = X_j$  and  $TC_i = TC_j$  and go to Step 6. If  $TC_j > TC_i$  calculate the deviation of cost  $\Delta C = TC_j - TC_i$  and generate a random number from the uniform distribution  $U(0,1)$ . If  $\exp(-\Delta C / \text{Temp}) \geq U(0,1)$  accept the new solution  $X_j$  to replace  $X_i$ , else preserve  $X_i$  as the current solution.
6. If  $k$  is less than the length of the Markov chain, increase the iteration counter  $k=k+1$  and go to Step 4, else go to Step 7.
7. Perform local optimization in the neighborhood of the last accepted solution.
8. If the stopping criterion is not satisfied decrease the temperature  $\text{Temp}$  and go to Step 3.

### 3.3 Implementation Details

The proposed SA algorithm adopts three different mechanisms for the generation of every candidate solution taking into consideration the minimum up/down time constraints of the generating units. The first mechanism changes the status of a random number of units for only one hour, while the other two change the status of a randomly selected unit for a random number of consecutive hours. The combination of these mechanisms results in a 2-dimensional exploration of the solution space, accelerating the convergence of the SA algorithm. In addition, these mechanisms guarantee that only feasible solutions are examined during the execution of the program.

The initial solution, which is required at the first step of the algorithm, is located through an effective iterative procedure, which is based on the alternative use of the above mechanisms. Furthermore, the initial value of the control parameter of the temperature is determined in such way that, in the initial stage of the algorithm, a big percentage (approximately 85%) of the trial solutions is accepted.

The length of each Markov chain of the SA algorithm is determined as the minimum value between an upper bound and the number of variables (product of the number of units with the number of hours). As soon as a Markov chain has been completed, the control parameter of temperature is gradually decremented by a relatively slow cooling schedule:

$$\text{Temp}_{k+1} = a \cdot \text{Temp}_k \quad (13)$$

where  $\text{Temp}_k$  is the temperature at stage  $k$  and  $a$  is a constant smaller than but close to 1.

In order to accelerate the convergence of the SA algorithm, a local optimization method is implemented at each temperature. This method locates the best solution in the neighborhood of the last accepted one by using iteratively the mechanisms developed for generating new random solutions.

The SA algorithm is terminated when one of the following criteria is satisfied: a maximum allowed number of iterations or a pre-selected final value of the temperature.

## 4 PROBABILISTIC RESERVE ASSESSMENT

### 4.1 Overview of Proposed Method

The objective of the probabilistic spinning reserve assessment is the determination of the required reserve capacity for each hour of the dispatch period taking into consideration the availability of the generating units. Several reliability indices are found in the literature [9], the most commonly used, though, are the loss of load expectation (LOLE) and the expected unserved energy (EUE). The proposed method adopts the EUE reliability index, which expresses the expected energy that will not be served by the generation system. The incorporation of the above index in the formulation of the UC problem is accomplished by implementing a maximum allowed limit of the EUE. The determination of this limit is based on the desired level of reliability.

For each feasible trial solution provided by the SA algorithm, the EUE of the dispatch period is calculated by constructing the capacity outage probability table (COPT), as in the standard literature [2], and computing the numerical convolution of the COPT with the given load curve. The construction of the COPT is based on the units committed at each hour of the dispatch period according to the current solution of the SA algorithm. The above procedure is further described in section 4.2.

In case the calculated EUE exceeds the given maximum allowed limit, a quadratic penalty term, related to the amount of violation, is included in the operating cost of the current solution. The weighting factor of the penalty term is dynamically adjusted, so that it is low at the early stages of the algorithm and gradually grows, until it reaches a relatively high final value. This procedure allows the solutions which violate the above reliability constraint to evolve into feasible solutions.

Finally, the algorithm converges to a near optimal solution of the UC problem, while the spinning reserve capacity at each hour of the dispatch period is given by:

$$RSRV_t = \sum_{i=1}^{NG} FST_{i,t} P \max_i - LOAD_t, \quad t \in [1, T] \quad (14)$$

where  $FST_{i,t}$  represents the status of unit  $i$  at hour  $t$  for the final solution of the problem.

The above method ensures that the availability of the generating units is taken into consideration in the solution of the daily UC problem and the reliability limit of the EUE is not violated in the final solution.

### 4.2 Calculation of Expected Unserved Energy

In order to take into consideration the availability of the generating units, each unit is represented by a two-state model according to which a unit is either available or unavailable for generation. In view of this model, the unavailability of the generating unit  $i$  during a short time interval  $LT$  (known as the system lead time) is given by [1]:

$$Pdown_i(LT) = \frac{\lambda_i}{\lambda_i + \mu_i} (1 - e^{-(\lambda_i + \mu_i)LT}) \quad (15)$$

where  $\lambda_i$  and  $\mu_i$  are the failure and repair rates of unit  $i$ . The probability  $Pdown_i$  is known as the outage replacement rate (ORR) of the unit.

It is assumed that the lead time is much shorter than the repair times of the units. So, the repair process can be neglected, which results in a more simplified expression of the ORR of each unit:

$$Pdown_i(LT) = 1 - e^{-\lambda_i LT} \quad (16)$$

The calculation of the EUE for a given solution of the UC problem is based on the construction of successive capacity outage probability tables using the ORR of each unit. For each hour  $t$  of the dispatch period a COPT is constructed, where each row  $i=1..n$  represents a generation level that may be outaged, the total capacity  $CAPR_i$  that remains in service and the corresponding probability  $PR_i$  [3]. A numerical example of the COPT can be found in [2].

At this point, the expected unserved energy  $EUE_t$  for each hour  $t$  can be calculated by:

$$EUE_t = \sum_{i=1}^n PR_i E_i, \quad t \in [1, T] \quad (17)$$

where:

$$E_i = \begin{cases} LOAD_t - CAPR_i, & \text{if } CAPR_i \leq LOAD_t \\ 0, & \text{otherwise} \end{cases} \quad (18)$$

Afterwards, the total EUE of the dispatch period can be calculated by:

$$EUE = \sum_{t=1}^T EUE_t \quad (19)$$

The execution time required for the construction of each COPT can be considerably reduced by rounding the outage levels to a fixed increment [3]. However, the increment must be carefully chosen in order to retain the precision of the obtained results [9].

### 4.3 Dynamic Penalty Function

The inclusion of the EUE index in the formulation of the UC problem is accomplished by using a dynamic penalty function. For each feasible solution provided by the SA algorithm, the corresponding EUE of the dispatch period is calculated by the procedure described in the previous section. Then, the calculated EUE is compared to a given maximum allowed limit  $UNLM$  of the expected unserved energy:

- If  $EUE \leq UNLM$ , the trial solution provides an acceptable level of reliability.
- If  $EUE > UNLM$ , a quadratic penalty term  $UNSVL$  is added in the operating cost of the corresponding solution.

The penalty term is a quadratic function of the amount of violation of the above reliability constraint:

$$UNSVL = \begin{cases} (EUE - UNLM)^2, & \text{if } EUE > UNLM \\ 0, & \text{otherwise} \end{cases} \quad (20)$$

The above penalty term is added in the total cost  $TC$

of the corresponding solution, which results in an augmented objective function  $TC_{aug}$  of the UC problem:

$$TC_{aug} = TC + PM \cdot UNSVL \quad (21)$$

where  $TC$  is given by (4) and  $PM$  is a dynamic penalty multiplier.

The value of  $PM$  is low at the early stages of the algorithm and gradually grows, until it reaches a very high final value  $PM_{max}$ . Each time a solution violates the implemented reliability constraint, the penalty multiplier is increased by the following rule:

$$PM_k = MIN(PM_{max}, inc^k \cdot PM_0) \quad (22)$$

where  $PM_k$  is the value of  $PM$  after  $k$  violations of the reliability constraint,  $PM_0$  is the initial value of the penalty multiplier,  $inc$  is a constant greater than but very close to 1 and  $MIN(X,Y)$  is the minimum value between  $X$  and  $Y$ .

The initial value of the penalty multiplier is chosen small enough (close to 0) to encourage the acceptance of solutions which violate the reliability constraint during the first stages of the algorithm. However, as the algorithm proceeds, the  $PM$  constantly grows, until it reaches a very high final value. Afterwards, the probability of acceptance of any solution which violates the reliability constraint is practically insignificant (due to the great increase of the total cost of the corresponding solution).

The implementation of the above dynamic penalty function allows the solutions which violate the reliability constraint to evolve into feasible solutions. In addition, this procedure guarantees that the final solution of the UC problem is always feasible and provides the desired level of reliability.

## 5 NUMERICAL RESULTS

The proposed model has been implemented in Fortran 77 and executed on a Pentium IV PC. The effectiveness of the algorithm has been tested on a system of ten generating units, considering a dispatch period of 24 hours. The system data are taken from [10]. The data referring to the failure rates ( $\lambda_i$ ) of each unit (for the purposes of the probabilistic spinning reserve assessment) were added by the authors of this paper, as shown in Table 1. In order to show the efficiency of the proposed model in solving realistic UC problems, the data of the test system were appropriately scaled in an attempt to obtain three new systems with 20, 40 and 80 generating units.

Unit Number	Failure Rate (f/hr)	Unit Number	Failure Rate (f/hr)
1	0.00025	6	0.00030
2	0.00025	7	0.00030
3	0.00040	8	0.00035
4	0.00040	9	0.00035
5	0.00050	10	0.00035

**Table 1:** Failure rates of generating units.

Due to the random nature of the SA algorithm, the program was executed 10 times for each case study, in order to validate the stability of the algorithm. The initial temperature for each case study was determined using the procedure described in the relevant section. In all studies, the simple cooling schedule described in (13) was used, with  $\alpha=0.95$ . Two stopping criteria were adopted for all case studies: a final temperature of 0.001 or 100 iterations of the algorithm. The length of each Markov chain was determined as the minimum value between 800 and the number of the variables. Finally, 50 iterations were used for the local search at each temperature.

In order to show the significance of incorporating the probabilistic spinning reserve assessment in the solution of the UC problem, a base case study with deterministic assessment of spinning reserve was examined. For the purposes of the above case study, the spinning reserve requirements of the system were considered to be at least equal to 10% of the hourly load. The above deterministic criterion is commonly used in the literature.

The simulation results of the base case study are shown in Table 2, which gives the best cost, the worst cost and the average cost obtained through ten executions of the proposed model. The small variation between the cost of the best and the worst solution of the program for each scaled system prove the robustness of the algorithm. Moreover, the average execution time of the SA algorithm for each system is presented in Table 2. It is obvious that the time requirements of the proposed model are reasonable, even for the large system of 80 generating units.

For the purposes of the probabilistic spinning reserve assessment, the method presented in this paper was applied on the same test system. In this case, the maximum allowed limit of the EUE was selected to be equal to 0.1% of the total energy demand of the dispatch period, while a lead time of 6 hours was considered.

The simulation results of the above case study are shown in Table 3. It can be seen that the proposed method leads to significant reduction of the total operating cost of each scaled system, in comparison to the results of the base case study. However, the procedure used for the calculation of the EUE increased the average time requirements of the model. In spite of this, the execution time of the model remained in acceptable levels.

Units	Best Cost (\$)	Av. Cost (\$)	Worst Cost (\$)	Av. Time (sec)
10	565828	565988	566260	3.35
20	1126251	1127955	1129112	16.80
40	2250063	2252125	2254539	88.28
80	4498076	4501156	4503987	405.01

**Table 2:** Results for deterministic assessment of spinning reserve.

Units	Best Cost (\$)	Av. Cost (\$)	Worst Cost (\$)	Av. Time (sec)
10	558107	559084	561094	8.23
20	1103845	1105075	1105984	58.84
40	2202729	2203798	2206047	405.59
80	4400271	4403316	4405477	1713.28

**Table 3:** Results for a specific EUE limit of 0.1% and a lead time of 6 hours.

Table 4 presents the units which are committed (denoted with 'x') at each hour of the dispatch period, referring to the best solution of the 10-unit system for a fixed EUE limit of 0.1% and a lead time of 6 hours.

Hour	Unit Number								
	1	2	3	4	5	6	7	8	9
1	x	x							
2	x	x							
3	x	x							
4	x	x	x		x				
5	x	x	x		x				
6	x	x	x	x	x				
7	x	x	x	x	x				
8	x	x	x	x	x				
9	x	x	x	x	x	x			
10	x	x	x	x	x	x	x		
11	x	x	x	x	x	x	x		
12	x	x	x	x	x	x	x	x	
13	x	x	x	x	x	x	x		
14	x	x	x	x	x	x			
15	x	x	x	x	x	x			
16	x	x	x	x	x				
17	x	x	x	x	x				
18	x	x	x	x	x				
19	x	x	x	x	x				
20	x	x	x	x	x			x	x
21	x	x	x	x	x				
22	x	x	x	x	x				
23	x	x							
24	x	x							

**Table 4:** Unit commitment schedule for the best solution of the 10-unit system for a fixed EUE limit of 0.1% and a lead time of 6 hours (unit 10 is off during the entire period).

The results presented in Tables 3 and 4 are based on fixed values of the maximum allowed limit of the EUE and the lead time. However, the selection of the above parameters has a significant effect on the solution of the UC problem and the associated operating costs. This effect is examined by the following sensitivity analysis.

In order to demonstrate the effect of lead time on the UC problem, the desired limit of the EUE was fixed at 0.1% of the energy demand of the dispatch period and the system lead time was varied from 6 to 15 hours. The corresponding results for the 40-unit scaled system are presented in Table 5. It can be seen that the operating cost of the system increases as the lead time increases. The above conclusion is justified by the fact that the

increase of the lead time causes a proportional increase of the ORR of the units. Consequently, additional units must be committed in order to provide the same level of reliability. Moreover, the results presented in Table 5 show that the execution time of the model is practically unaffected by the value of the lead time.

LT (hr)	Best Cost (\$)	Av. Cost (\$)	Worst Cost (\$)	Av. Time (sec)
6	2202729	2203798	2206047	405.59
9	2208844	2210305	2213191	394.82
12	2213639	2216322	2219971	396.87
15	2215699	2218922	2224148	404.55

**Table 5:** Results for the 40-unit system for a fixed EUE limit of 0.1% and variable lead times (LT).

Table 6 presents the spinning reserve assessment for each hour of the dispatch period, referring to the best solution obtained in the above case study. The results demonstrate that, for several hours of the considered period, the spinning reserve capacity increases as the system lead time increases. However, this is not always the case, due to the fact that the implemented reliability index is calculated over the 24-hour period. Therefore, the desired level of reliability can be achieved by counterbalancing the shortage of spinning reserve at several hours of the period with the excess of the remaining hours.

Hour	Lead Time (hr)			
	6	9	12	15
1	840	840	840	840
2	640	640	640	640
3	240	402	402	402
4	100	2	424	262
5	62	224	354	192
6	52	214	246	376
7	14	274	468	436
8	74	204	528	366
9	46	126	128	208
10	53	133	108	133
11	18	43	98	128
12	38	8	63	93
13	53	133	103	188
14	128	208	208	288
15	138	528	398	478
16	738	868	868	998
17	938	1068	1068	1198
18	538	668	668	798
19	298	428	348	558
20	3	8	3	78
21	238	188	238	368
22	132	536	406	456
23	137	364	494	461
24	245	147	570	439

**Table 6:** Spinning reserve assessment (MW) for the best solution of the 40-unit system for a fixed EUE limit of 0.1% and variable lead times.

The sensitivity of the results for the 40-unit scaled system to the desired reliability level is shown in Table 7. For the purposes of the sensitivity analysis, the lead time was fixed at 6 hours and the desired EUE limit was varied in the range of 0.03% to 0.1% of the total energy demand of the dispatch period. It is obvious that, as it was expected, the operating cost of the system increases as the EUE limit decreases. In any case, even if a rather high reliability level is desired, the total operating cost of the system is considerably lower than the cost obtained using deterministic criteria for the assessment of the system spinning reserve. Hence, the proposed method can be effectively used for assessing the spinning reserve requirements of realistic power systems, providing significant cost reductions.

EUE Limit (%)	Best Cost (\$)	Av. Cost (\$)	Worst Cost (\$)	Av. Time (sec)
0.10	2202729	2203798	2206047	405.59
0.05	2211716	2213714	2215121	428.02
0.03	2219668	2221513	2224167	471.54

**Table 7:** Results for the 40-unit system for a fixed lead time of 6 hours and variable EUE limits.

## 6 CONCLUSIONS

A new method for incorporating the probabilistic spinning reserve assessment in the solution of the unit commitment problem is presented in this paper. The unit commitment problem is solved using a new simulated annealing algorithm, while the evaluation of the required spinning reserve capacity is performed by using the expected unserved energy reliability index. For each solution provided by the simulated annealing algorithm, the expected unserved energy of the dispatch period is calculated by convolving the capacity outage probability table with the given load curve. The required spinning reserve capacity for each hour of the considered period is determined by adding a quadratic penalty term in the operating cost of any solution which leads to violation of a maximum allowed limit of the expected unserved energy. In this way, the required spinning reserve capacity is effectively scheduled, resulting to near optimal unit commitment solutions, which provide a reasonable level of reliability.

The application of the proposed method is demonstrated using a test system from the bibliography. The results obtained in several case studies show the effect of the lead time and the desired level of reliability (measured by the expected unserved energy index) on the final solution of the unit commitment problem. In addition, the various test results prove that the proposed method leads to significant reduction of the total operating cost of the system, in comparison to the

results obtained using deterministic criteria for the assessment of the system spinning reserve.

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This work is dedicated to the memory of Professor George Contaxis who supervised it until he suddenly passed away on November 1<sup>st</sup>, 2004.

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