STATE ESTIMATION APPLIED TO ACTIVE DISTRIBUTION NETWORKS WITH MINIMAL MEASUREMENTS

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Abstract – Traditionally, state estimation is applied to transmission networks to improve security and redundancy of the measurement system. This paper describes the application of state estimation to distribution networks in order to extend observability of the network. Key features of this application are that the network is active, minimal real measurements are available and there is minimal communications infrastructure. This paper presents results from a field trial which manages voltage levels in an 11 kV distribution network with distributed generation. The paper highlights the problems associated with this application and presents some solutions.

Keywords: State Estimation, Distributed Generation, Active Networks, Minimal Measurements

1 INTRODUCTION

1.1 Background

As part of their commitments to meeting the requirements of the Kyoto agreement on climate change, governments and power utilities around the world are attempting to increase the proportion of electrical power generated from renewable sources. The size of this generation is such that it is usually more economic to connect to the power system at lower distribution voltages. The type of the generation (e.g. landfill gas, wind, small scale hydro) often leads to it being distributed away from primary substations, for practical and economic reasons.

The penetration of distributed generation is currently limited by the passive operating methods of distribution networks. In the UK, at 11 kV, the size of connected generation is often restricted by the potential to exceed voltage limits. When connections are granted, output may be curtailed at times of low network load. At 33 kV, limitations are often due to line capacity limits. In both of these circumstances, increased distributed generation output may be possible under favourable network conditions, however the lack of measurement and control infrastructure precludes this.

1.2 Aims

The work reported here is part of a project with aims to increase the amount of renewable generation connected in the distribution network through the use of active control techniques. It must meet the technical and commercial requirements of independent generators and distribution network operators (DNOs). In the UK, an independent generator requesting a connection may pay some or all of the cost of upgrading the network infrastructure.

The resulting requirements can be summarised as:

- Ensure that all regulatory requirements of the DNO are met.
- Ensure that all plant limits are respected.
- Work with existing measurement and communications infrastructure.
- Minimise changes to existing operating methods.

1.3 Paper Organisation

This paper concerns the application of state estimation in the scenario described above. Theory appropriate to this application is stated in section 2. Section 3 describes the trial application and presents initial results. In section 4 these results are analysed and solutions are proposed.

2 STATE ESTIMATION

2.1 Normal Form of Basic Equations

Weighted least squares state estimation as presented in [1] aims to minimise the cost function

$$\min_{\mathbf{x}} J(\mathbf{x}) = \tfrac{1}{2}\Delta\mathbf{z}^{\mathrm{T}}.\Delta\mathbf{z}\,. \tag{1}$$

where $\Delta\mathbf{z} = \mathbf{z} - \mathbf{h}(\mathbf{x})$, $\mathbf{z}$ is a vector of measurements normalised by their respective standard deviations, $\mathbf{h}(\mathbf{x})$ is the equations defining the normalised measurements in terms of the state and $\mathbf{x}$ is the state vector for the system. $\Delta\mathbf{z}$ is the vector of measurement errors and has a standard Gaussian distribution n(0,1). The solution occurs when

$$\nabla J(\mathbf{x}) = \mathbf{H}^{\mathrm{T}}(\mathbf{x}).\Delta\mathbf{z} = 0 \tag{2}$$

where $\mathbf{H}(\mathbf{x})$ is the Jacobian matrix of $\mathbf{h}(\mathbf{x})$. The overdetermined non-linear system (2) can be linearised and solved iteratively leading to a state estimate $\hat{\mathbf{x}}$.

$$\mathbf{G}.\Delta\mathbf{x} \cong \mathbf{H}^{\mathrm{T}}.\Delta\mathbf{z} \tag{3}$$
$$\mathbf{x}_k := \mathbf{x}_k + \Delta\mathbf{x} \tag{4}$$

where

$$\mathbf{G} = \mathbf{H}^{\mathrm{T}}.\mathbf{H} \tag{5}$$

When applied to transmission systems, real and imaginary parts can often be decoupled leading to a linear system that can be solved analytically. This option is not appropriate on distribution systems due to lower line impedance angles.

2.2 Solution Methods

Equation (3) can be solved using Gaussian elimination with pivoting. However, loss of significance due to the wide magnitudes of values in the Jacobian matrix **H**, which is squared in (5), can cause errors and poor convergence of the estimator.

The gain matrix **G** can be factorised to improve numerical stability. Since **G** is symmetric positive definite, Cholesky factorisation can be used to factorise $\mathbf{G} = \mathbf{U}^T.\mathbf{U}$ where **U** is upper triangular. Equation (3) is then solved in two direct substitution steps.

Other decomposition techniques can improve the solution method. In orthogonal transformation [2] the Jacobian **H** is factorised as $\mathbf{H} = \mathbf{Q}^T.\mathbf{U}$, where **Q** is an orthogonal matrix and **U** is upper triangular. Equation (2) can then be solved directly by rewriting as

$$\mathbf{U}.\Delta\mathbf{x} \cong \mathbf{Q}.\Delta\mathbf{z} \tag{6}$$

and since **U** is upper triangular solved by back substitution.

2.3 Measurements

Measurements **z** used in state estimation can be of 3 types: actual measurements, pseudo-measurements and virtual measurements. Actual measurements are metered on the system; pseudo-measurements of load are models of loads, usually defined as Gaussian distributions with their mean at half the transformer rating; virtual measurements are artificial values inserted when the value is known, for instance zero load injections at busses known to have no load connected. Virtual measurements are usually modelled with very low standard deviation, hence their elements in the error vector $\Delta\mathbf{z}$ will be weighted highly with respect to the other measurement types.

2.4 Distribution State Estimation

There are issues described in literature that have particular relevance when applying state estimation to distribution networks, and these are given below. They revolve around ill-conditioning of the gain matrix **G**, which can lead to poor convergence, or non-convergence, of the iterative equations (3) and (4).

It is normal in distribution state estimation to find a large number of load pseudo-measurements, which may cause problems of instability due to increased poor conditioning of the gain matrix. This is discussed in [3] however the cause of the poor condition is not explained.

Adjacent long and short lines (i.e. large and small impedances) are also a source of ill-conditioning [2]. This feature of state estimation can be expected to have an effect where there are very low impedance lines coupling busses. The authors of that paper proposed an orthogonal decomposition method as a general approach to reduce the effects of ill-conditioning.

The combination of measurements with very large and very small weighting factors leads to ill-conditioning of the gain matrix **G** [4]. The use of virtual measurements with load pseudo-measurements leads to this situation. A reformulation of the problem as a minimisation with equality constraints can improve numerical stability.

$$\begin{pmatrix} \mathbf{H}^T.\mathbf{H} & \mathbf{C}^T \\ \mathbf{C} & 0 \end{pmatrix} . \begin{pmatrix} \Delta\mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} \mathbf{H}^T.\Delta\mathbf{z} \\ \Delta\mathbf{c} \end{pmatrix} \tag{7}$$

where $\mathbf{c}(\mathbf{x}) = \mathbf{0}$ are the equality constraints, $\mathbf{C}(\mathbf{x})$ is its Jacobian matrix and $\Delta\mathbf{c} = -\mathbf{c}(\mathbf{x})$.

Equality constraints are used to fix known values, e.g. zero injection at a non-load bus. This removes the small weighting factors from the **H** matrix, which is squared.

Alternative approaches to distribution state estimation are presented in [5, 6].

3 IMPLEMENTATION

3.1 General Infrastructure Provision

The most prevalent form of control within networks is automatic voltage control (AVC) applied to primary substation transformers. Occasionally, this is supplemented by reactive power control equipment and inline regulators. Remote operation of line switches is sometimes possible for network reconfiguration, but use of these for dynamic response to network conditions is not common.

Available measurements within the network decrease substantially as the voltage level falls. In the UK, at 11 kV primary substations busbar voltage and primary transformer current is usually metered, and is typically reported back to the control centre using analogue SCADA systems. Feeder currents are normally measured only for protection purposes.

At 11 kV, remote from the substation there are usually no measurements made and there is no communications infrastructure available. Distributed generation has tariff metering but under the UK power system commercial structure this is often not available to the network operator in telemetered form.

3.2 Field Trial

The diagram for the field trial network presented in this paper is given in **Figure 1**. The original network contained over 400 nodes and was simplified to 86 nodes. The network includes one distributed generation site in the form of a 6x 660 kW windfarm.

The equipment installed in the field trial adjusts the target voltage setpoints of transformer AVC relays so is sited in the principal primary substation at node 6. Existing metering and communication infrastructure is limited. Packet radio (GPRS) communications are used between the windfarm and the principal primary substa-

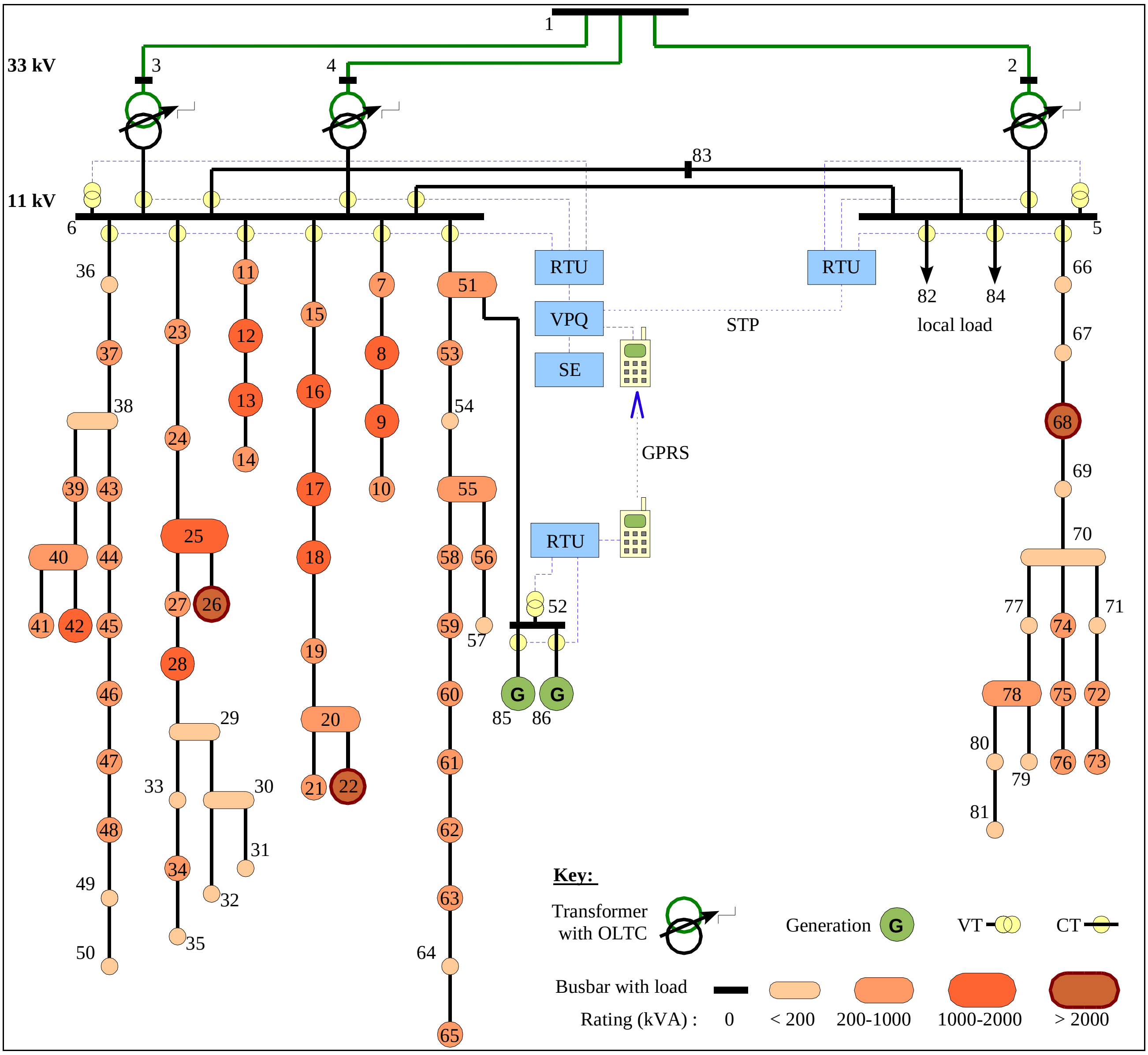

Figure 1: Diagram of trial network

tion. A shielded twisted pair (STP) pilot cable carrying Ethernet is used between the two primary substations at nodes 5 and 6.

Measurements are taken using existing VTs and CTs. Therefore, for current measurement this is generally from class 5P CTs. The RTUs calculate local vector quantities of voltage and current, which are communicated to the principal primary substation. The VPQ block converts these into magnitudes of voltage and real and reactive power for use by the state estimator (SE block). The results of the state estimation are used by the control algorithms, described in [7, 8].

3.3 *Initial Implementation*

The state estimation algorithm has been coded and compiled in MatLab (double precision floating point mathematics). The power flow equations used to define $\mathbf{z} = \mathbf{h}(\mathbf{x})$ have been implemented in complex form. Equation (3) can be restated as

$$\Delta \mathbf{x} = \mathbf{G}^{-1}.\mathbf{H}^{T}.\Delta \mathbf{z} \qquad (8)$$

and this is implemented in MatLab directly without calculation of the matrix inverse $\mathbf{G}^{-1}$ by

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Dx = Gain \ (H' * Rinv * Dz) ;                (9)
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MatLab operators which may require explanation are:

\ used to solve the matrix equation. Since the gain matrix is symmetric positive definite, Cholesky decomposition with partial pivoting is used.

' matrix transpose

* matrix multiplication

In the implementation, $\mathbf{z}$ and $\mathbf{h}(\mathbf{x})$ are not normalised, hence the inclusion of the diagonal matrix Rinv, with diagonal elements being the inverse of the measurement variances.

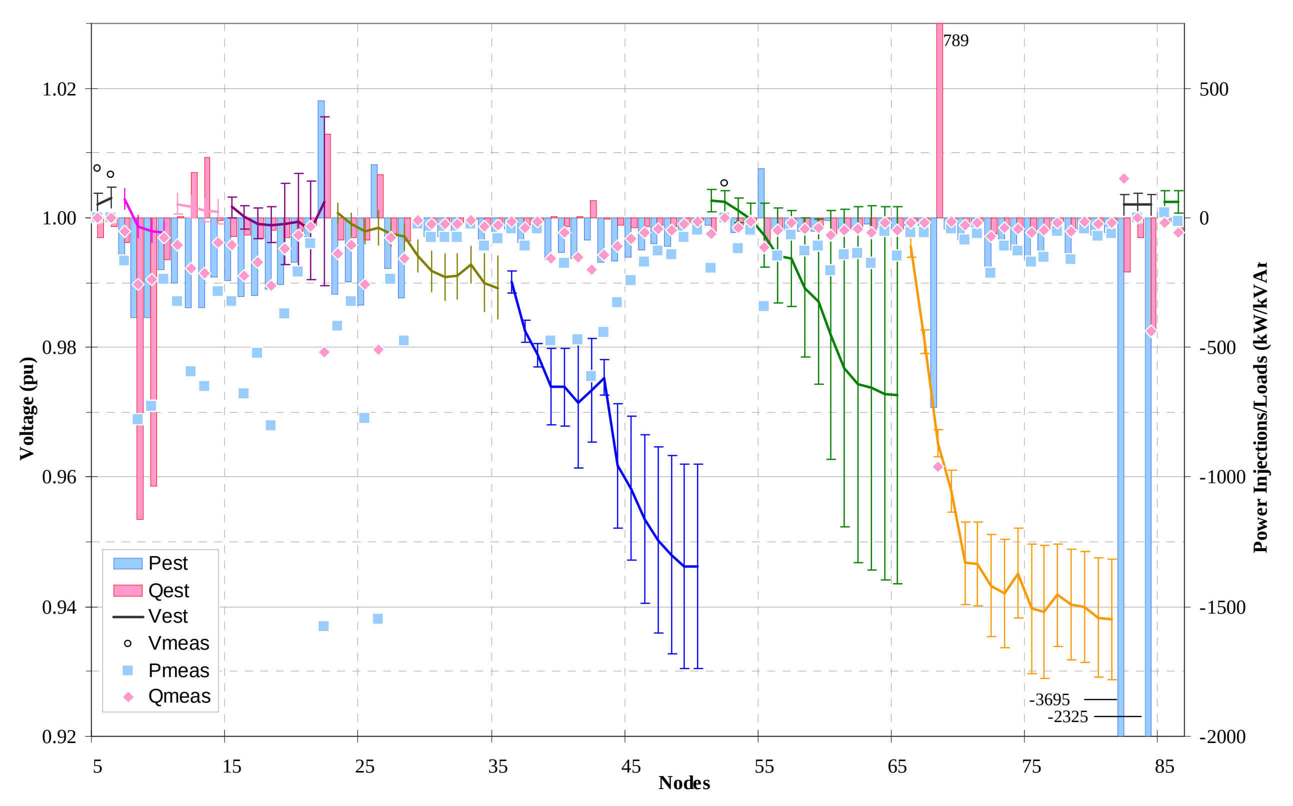

Figure 2: Results from trial using data for 10th November, 2004 at 0846

All nodes that have loads (as indicated in **Figure 1**) and no measurement of power injection have pseudo-measurements of real and reactive load allocated. The measurement value (mean) is set to half the rated load of the node and 3 standard deviations are also equated to half the load. This is equivalent to a probability of 99.7% for the load being between zero and the rating.

3.4 Initial Results

When executed on a sequence of data the state estimator often failed to converge. A typical example of the results when convergence did occur is shown in **Figure 2**. This shows voltage and power injection/load measurements and estimates for each node.

Voltage is on the left scale in per units with the line graph indicating the estimate and the error bars indicating the extent of 3 standard deviations of the estimate. The circle plots indicate the three voltage measurements provided to the state estimator (nodes 5, 6 and 52). The real and reactive load estimates are shown as bars (using the right scale) and the pseudo-measurements used to form these are indicated using square and diamond plots respectively.

Features of the results that are unsatisfactory include

- allocation of positive injection of real and reactive power to load nodes,
- poor correlation of voltage estimates to actual voltage measurements on those nodes where they are present, and
- frequent failure of the state estimator to converge.

These are discussed in the following section.

4 ANALYSIS OF RESULTS

4.1 Allocation of Generation to Load Nodes

In **Figure 2**, the state estimator has allocated positive injection of real and reactive power to a number of nodes that are loads. It can be seen from **Figure 1** that this tends to be at nodes with larger values for its pseudo-measurements (in comparison with those on the same feeder). The following is proposed as explanation for this.

Consider the example of **Figure 3**, with actual real power flow measurements P_1 and P_2 and pseudo load real power injection measurements P_A, P_B and P_C. η is the unnormalised measurement error, i.e. $\Delta z = \eta/\sigma$. For a lightly loaded network $P_1 + P_2$ will be substantially less than $P_A + P_B + P_C$. The network model will enforce the measurement estimates $\overline{P}_1 + \overline{P}_2 = \overline{P}_A + \overline{P}_B + \overline{P}_C$. Therefore, if η_1 and η_2 are small, η_A, η_B and η_C will be large, and vice versa.

These errors add towards the cost J, which is to be minimised. There are multiple locally optimal solutions to these problems. One often found by the estimator is to keep the measurement error small for many of the smaller pseudo loads (η_A and η_C), resulting in low contribution to J. In order to keep η_1 and η_2 (with medium weights) to a minimum, a very low (or negative) value is allocated to one large load, which will have a small weighting value (η_B).

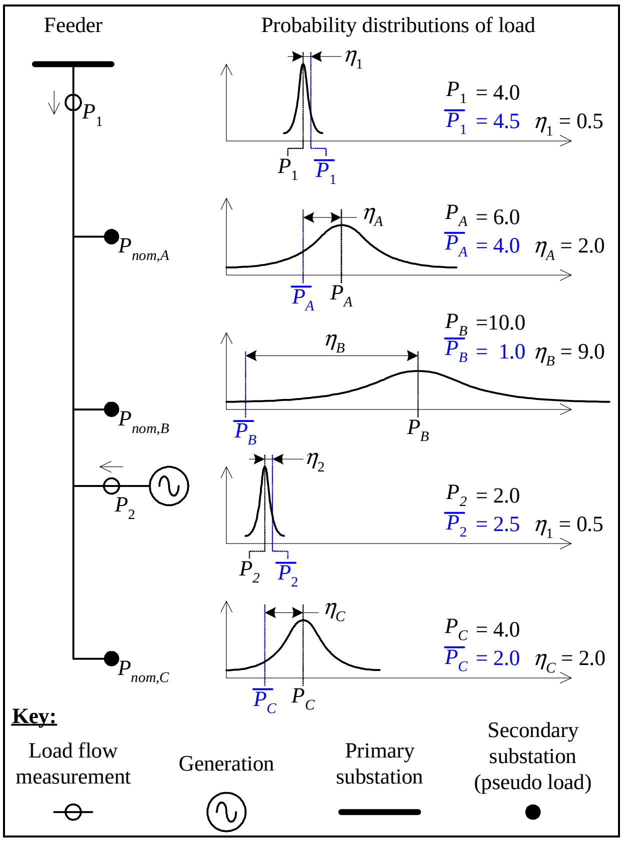

Figure 3: Example of the Effect of Pseudo Loads on State Estimation

As a solution it is proposed to take the power flowing into each load group and allocate it to the nodes of that group in proportion to the rating of the load at each node. In the example of **Figure 3** the real power flowing into the group is P_1 + P_2 = 6.0 and this would be allocated as P_A = 1.8, P_B = 3.0 and P_C = 1.2.

The actual measured values would not then be in opposition to the pseudo-measurements. A potential problem resulting from this is that a very small load on a feeder would result in some extremely large weights on the pseudo-measurements, which may result in further convergence problems. Additionally, it is necessary to correctly match actual load flow measurements in the network to load groups, including distributed generation.

It has previously been proposed to take the measured load for the group and distribute it to each load in the group in proportion to its rating, scaled according to load type, time of day, day of week and season [9]. In the UK, profiles are available for different load types. In order to implement this it would be necessary to estimate the proportion of each load type connected at each point.

A comparison has been carried out for the three types of model described above, termed the load invariant, load variant and scaled-load variant models respectively. The effect of these different load models on voltage profile was studied and results are presented in **Figure 4**.

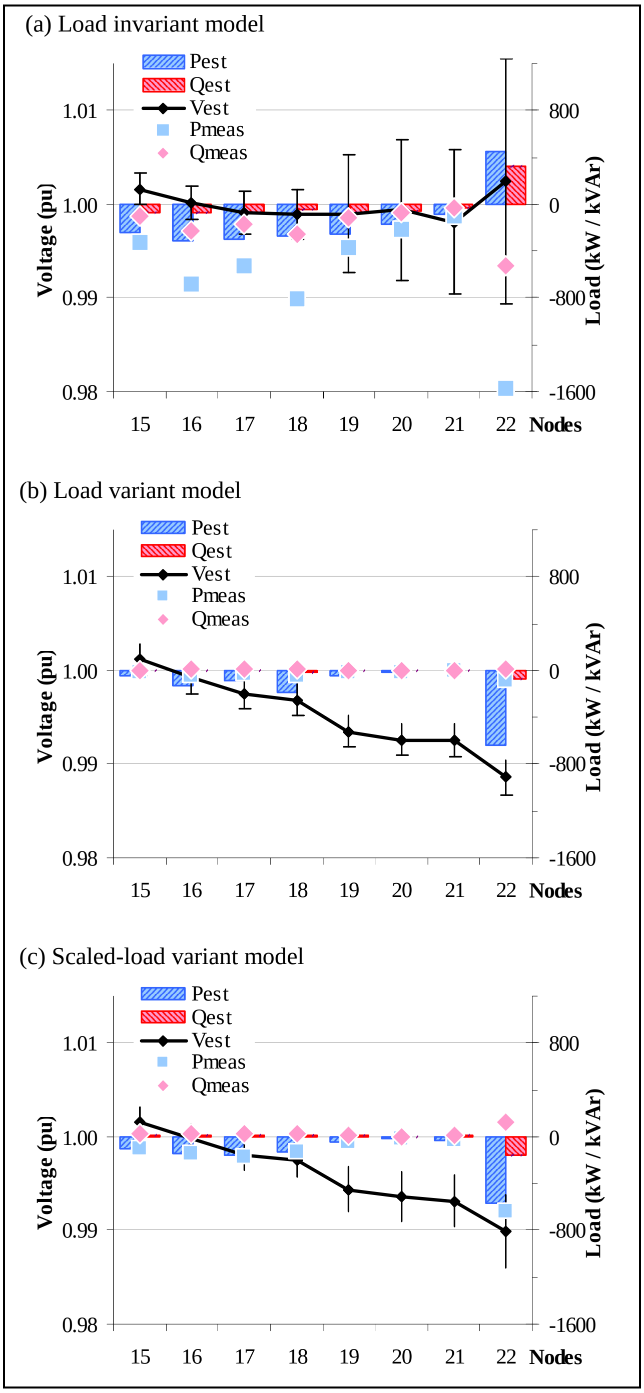

Figure 4: Comparison of different models for load pseudo-measurements

The figure shows one feeder (nodes 15 to 22) with measurements for 10th November 2004 at 0846. Valid data was not available for the lightly loaded period over the Summer, which could be expected to show a more marked improvement.

Case (a) is an extract from **Figure 2**. Case (b) shows a significant difference in voltage profile to (a). The node that was estimated as a power injection is now estimated as load. The voltage at node 22 (the feeder end) shows a difference of 0.015 pu between the two traces. Also, the confidence limit of the estimate, as indicated by the error bars is hugely reduced. In this application the maximum voltage on the feeder as used by the control software when setting limits would be taken as 1.015 pu for case (a) and 1.003 pu for case (b), a difference of 0.012 pu.

The accuracy of the estimates will be verified by placing confirmatory measurements at key points within the system, however this is yet to occur.

Case (c) was created by applying the following factors to each node, simulating variation of demand during the day. The factors were created artificially, but recognised that the data was for a weekday in Autumn, that most nodes are domestic loads and node 22 is a single industrial user. The factors applied are given in **Table 1**. Comparing cases (b) and (c), the scaled-loads show minimal effect on estimated voltage profile, or the loads allocated to each node, despite a large difference in measured value.

4.2 Poor Voltage Estimates

In **Figure 2** nodes 5, 6 and 52 have voltage measurements. Results show that the estimates for nodes 5 and 6 often place their confidence limits such that the measurement values are not within them. (The assumption is made that the measurements are not bad since the purpose of using SE is to extend observability.)

One reason for this is that the many load pseudo-measurements tend to push the busbar voltages lower than is expected from the voltage measurements alone. The state estimation is attempting to resolve a number of opposing tendencies: voltage measurements, zero-load busses and load pseudo-measurements. However, the load flow equations are conditions which must be met. The solution found is a compromise, the exact nature of which is dependent on the relative weightings of each measurement by its standard deviation.

It was found through experimentation that decreasing the standard deviation applied to voltage measurements can significantly improve the estimated voltage – as can be expected. However, decreasing the voltage standard deviation has the effect of increasing the load allocated to nodes 5 and 6, which are zero-load busses. In order to decrease this to an acceptable level the standard deviation applied to the virtual measurements must be reduced, which increases the condition of the gain matrix **G** to a level that causes failure to converge.

The load modelling methods described in section 4.1 would also assist in solving this problem.

4.3 Failure to Converge

Ill-conditioning of the gain matrix **G** causing loss of significance during Gaussian elimination can lead to failure of the state estimator to converge. This is countered to some extent by pivoting the gain matrix during the solution process.

The factorisation and transformation methods reviewed in section 2.2 can allow a poorer condition to be tolerated while still converging. The other approach to improve convergence is to decrease the condition number.

Node	15	16	17	18	19	20	21	22
Factor	0.73	0.49	0.73	0.36	0.61	0.23	0.71	0.90

Table 1: Demand factor applied to nodes

	Case 1	**Case 2**	**Case 3**
Method	normal	constraints	
Base	1 MVA		1 MVA
SD V	0.015 pu		0.003 pu
SD P	0.003P + 0.0015		0.003P+ 0.0015
SD Q	0.003Q+ 0.015		0.003Q+ 0.015
SD P,Q virtual	0.001 pu		0.001 pu

Table 2: Parameters used to evaluate method of minimisation with equality constraints

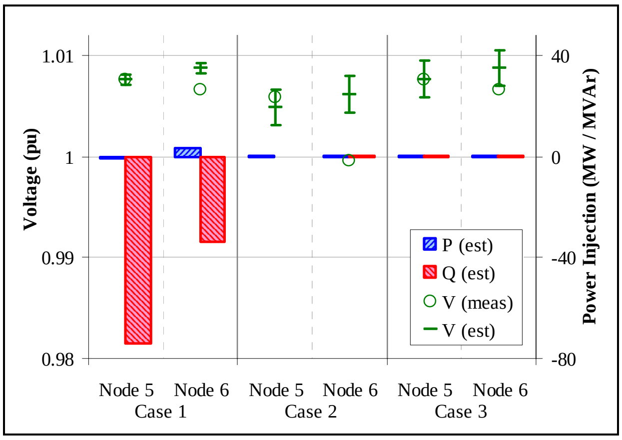

Figure 5: Evaluation of method of minimisation with equality constraints

The large number of pseudo-measurements is one source of ill-conditioning, due to the presence of many high values in the error vector. Poor scaling of measurement values has been identified as another cause [10]. It is usual for power system problems to be evaluated with a per unit base of 100 MVA. While this is satisfactory for transmission system state estimation, the above suggests that for distribution state estimation the base should be reduced.

Decreasing the standard deviation of virtual measurements to a level giving satisfactory estimates was found to be a source of poor condition. This was reviewed in section 2.4 and the proposed solution was to form the virtual measurements as equality constraints. This was applied to the trial network with parameters as given in **Table 2**, giving the results of **Figure 5**.

The base case (case 1) shows poor voltage estimates and allocation of large loads to nodes 5 and 6 (zero-load busses) when the normal equations ((3) and (5)) are used. If the method of equality constraints (equation (7)) is applied, the allocated load is correctly reduced to zero – case 2. Smaller standard deviations can now be applied to the voltage measurements producing voltage estimates which are acceptable while allowing convergence – case 3.

5 CONCLUSION

The proposition is that distribution state estimation can be used to extend the observability of networks and provide satisfactory estimate of voltage with a minimum of actual measurements from the system. Attempting to do this has revealed three main issues which can be traced to one root cause: a large number of load pseudo-measurements acting in contradiction to observed actual measurements, particularly in periods of light loading.

Although changes in solution technique can improve the convergence of the state estimator, improved load models have the biggest effect on convergence, load allocation and voltage estimates.

With respect to the accuracy of the voltage estimates further work is necessary to verify these estimates. To this end recording of measurements at key nodes within the network is in progress. This will allow the different solution methods and load models to be compared objectively against a relevant standard: the actual network voltage.

To date, the load model analysis has been completed on a subsection of the network data. Analysis of the full network using valid daily demand factor profiles for different load types, together with an estimate of the composition of each load could be undertaken. However, the initial analysis has indicated that the benefits of doing this may be small.

The application of satisfactory load models to distributed state estimation can be expected to cause two improvements: to the voltage estimate accuracy and to convergence. It is the latter of these that is more significant.

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REFERENCES

[1] F. C. Schweppe, "Power System Static State Estimation," *IEEE Transactions on Power Apparatus and Systems*, vol. 89, pp. 120-135, 1970.

[2] A. Monticelli, C. A. F. Murari, and F. F. Wu, "A hybrid state estimator: solving normal equations by orthogonal transformations [power systems]," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-104, pp. 3460-8, 1985.

[3] J. W. Gu, K. A. Clements, G. R. Krumpholz, and P. W. Davis, "The solution of ill-conditioned power system state estimation problems," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-102, pp. 3473-80, 1983.

[4] F. C. Aschmoneit, N. M. Peterson, and E. C. Adrian, "State estimation with equality constraints," presented at Power Industry Computer Applications Conference, Toronto, Ont., Canada, 1977.

[5] S. Naka, T. Genji, T. Yura, and Y. Fukuyama, "A hybrid particle swarm optimization for distribution state estimation," *IEEE Transactions on Power Systems*, vol. 18, pp. 60-68, 2003.

[6] J. Wan and K. N. Miu, "Weighted least squares methods for load estimation in distribution networks," *IEEE Transactions on Power Systems*, vol. 18, pp. 1338-45, 2003.

[7] H. Leite, *Modelling and Real-time Testing of an Automatic Local Voltage Controller to Increase the Amount of Distributed Generation*, PhD Thesis, University of Manchester Institute of Science and Technology, Manchester, 2004.

[8] C. M. Hird, H. Leite, N. Jenkins, and H. Li, "Network voltage controller for distributed generation," *IEE Proceedings - Generation, Transmission and Distribution*, vol. 151, pp. 150-6, 2004.

[9] A. K. Ghosh, D. L. Lubkeman, and R. H. Jones, "Load modeling for distribution circuit state estimation," *IEEE Transactions on Power Delivery*, vol. 12, pp. 999-1005, 1997.

[10] D. S. Watkins, *Fundamentals of Matrix Computations*, 2nd ed. New York: Wiley, 2002.