

# NETWORK MODELLING FOR CONGESTION MANAGEMENT : ZONAL REPRESENTATION VERSUS NODAL REPRESENTATION

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**Abstract** – The introduction of competition in the European power systems has significantly increased the constraints on the cross-border lines. Besides, the electrical network in Europe is highly meshed, which makes it difficult to choose the technical and economic solution to organise coordinated access to the interconnections. The various possible options consist in gradually reducing the level of detail in the representation of network equations. In this way, nodes can be aggregated in zones. Considering the transmission lines, a first approach consists in retaining the limits of individual lines (which corresponds to the implicit flow based market coupling proposed by ETSO /EUROPEX). The lines may also be aggregated on interfaces. At last, the most important simplification consists in fixing import and export limits for each zone.

On a specific example, the global system costs due to the different allocation methods are compared with the global cost in the “nodal” reference case, where the influence of each node on each line is explicitly taken into account.

The most promising approach appears to be that proposed by ETSO provided that both the zones and some of the nodes close to the most congested links are described.

**Keywords:** *Market, modelling, congestion management, node groupings, PTDF, zonal, nodal*

## 1. INTRODUCTION

Historically, each European country has independently developed its own electricity network. For security reasons, these networks have been linked through interconnections – such that, when a problem occurred in a country, it could be helped by its neighbours.

The emergence of a European competitive electricity market has increased a lot the number of transactions and therefore the complexity of managing interconnection access. The cross-border lines are not well-dimensioned for commercial exchanges and are consequently often saturated.

Nowadays each user who wants to make an international transfer of energy has to make a demand on

each interconnection concerned by the transfer. Indeed, each interconnection is managed separately, which provides some inefficiency.

The European Union is willing to reduce this inefficiency by facilitating access to the cross-border lines [1]. Accordingly, to control exchanges between zones separated by structural congestion, the European Commission promotes the adoption of market splitting in the long term.

In order to check the feasibility of such an organization on such a huge electrical network, we must define the practical conditions for its setting up [2]. Particularly, the network representation has to be simplified for two main reasons : on the one hand, for the sake of liquidity, market participants favour zonal aggregation of the electrical nodes; on the other hand, TSOs would prefer to have a simplified network representation to cope with dynamic constraints like ramping rate, start up time of thermal units and hydro plants storage constraints.

This paper focuses on the different ways of grouping the nodes of the network and computing transfer limit. The consequences of such approximations are evaluated on some examples by comparing the global cost of the production system to the cost with a completely detailed representation of the system (i.e. when all the lines and all the nodes are taken into account) on a typical winter peak-hour.

## 2. NETWORK MODELLING

### 2.1 Electricity markets

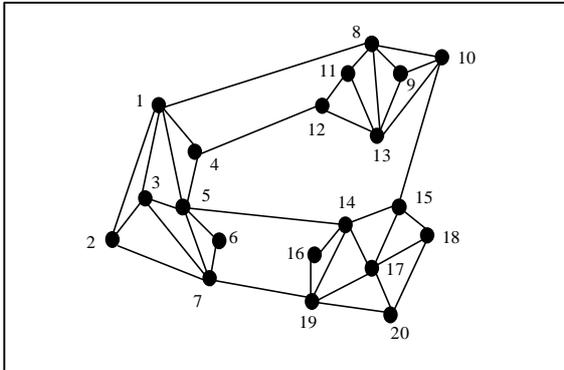
Approximately, an electrical network is composed of two kinds of components :

- the nodes, to which consumers and producers are connected.
- the lines that link the different nodes of the network so as to make possible the power exchanges between consumers and producers. Due to physical reasons, the power transfer on electrical lines is limited.

In this paper, the demand is supposed to be constant at each node. Thus, on the electricity market, the optimisation consists in minimizing the cost of production under the capacity constraints in order to meet this fixed demand.

## 2.2 Nodal representation and introduction of PTDF

The figure below displays an example of network where all the lines and all the nodes are represented :



**Figure 1** : Example of a detailed network

This network is composed of 20 nodes linked by nearly 40 lines. We consider that it is made up of three “groups” of nodes: the nodes 1 to 7 that are closely linked to each other, the nodes 8 to 13 and the nodes 14 to 20. This example has not been aimlessly chosen : indeed, it aims at representing the principle characteristics of the European network where the “groups” of nodes could be seen as the countries and lines 1-8, 4-12, 5-14, 7-19 and 10-15 as the interconnections.

In the nodal representation, all the nodes and all the lines are taken into account : it means that the influence of all the injections of power and all the withdrawals of power on each line are taken into account for the computation of each flow. Under the DC load flow approximation, this influence is linear and comes from the Kirchhoff’s laws. It is mathematically represented by a coefficient called **nodal PTDF** (**P**ower **T**ransfer **D**istribution **F**actor).

So as to briefly present the notion of nodal PTDF [3], we have to introduce a slack node that can be any of the 20 nodes of the network but has to be the same node during all the computation : this slack node will be referred to as node “s”. The  $PTDF^{nodal}(L,i,s)$  is the oriented power flow calculated according to Kirchhoff’s laws when the node “i” produces (or consumes) 1 MW and the node “s” consumes (or produces) 1 MW. The PTDF are often expressed in percentage. For example, the fact that 1 MW injected at the node “5” and consumed at the node “13” increases the power flow on line “4-12” of 0.2 MW, means that the  $PTDF^{nodal}(“4-12”,“5”,“13”)$  has a value of 20 %.

On a nodal representation, the market operators would have to get (among other things) the following pieces of information in order to optimise the systems:

- the demands at each node.
- the details offers of production (quantity and price) at each node.
- the dynamic constraints of each power plants.
- the PTDF of each node on each line.
- the limits of each line.

As a consequence, this type of optimisation in Europe would require too many data to make the computation easily realised. What’s more, as we have specified it in the introduction, for sake of liquidity, the markets participants prefer zonal aggregation.

It seems doubtful that the nodal representation would be used in continental Europe. Nevertheless, it is a first-best solution and can be used as a reference [4]. That is why in the results presented at the end of the paper, the consequences of the simplifications in terms of exchanges and generation costs are compared with the values obtained in the nodal representation.

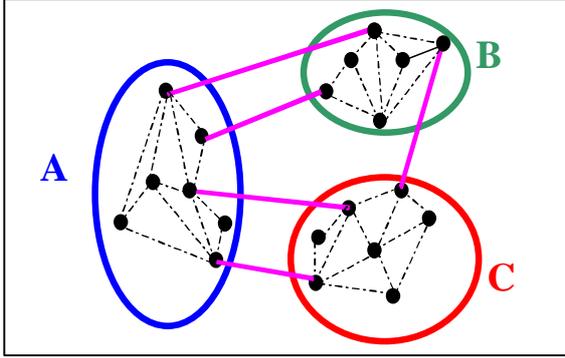
## 2.3 “Zonal - Line” representation

Compared to the nodal representation, the “zonal line” representation simplifies two aspects of the network :

- the nodes are aggregated in several groups, corresponding in our example to A, B, C (cf. Figure 2). Hence, it means for instance that a 1 MW injection in node 3 will have the same impact on the cross-border lines that a 1 MW injection in node 4 because the nodes 3 and 4 are grouped in the same zone : the zone A. On the European network, these groups stand for the countries.
- the only lines on which the power flow is computed are those subject to constraints. Those lines are called the **supervised lines**. In the European network, those supervised lines are mainly the interconnections. The limit could refer to N-1 security analysis.

This representation corresponds to the implicit flow based market coupling proposed by ETSO/EUROPEX [5].

The figure below shows our example of network where the nodes are grouped by “country” and the only supervised lines are the “interconnections” :



**Figure 2** : Zonal-line representation of the network

To compute the flows on the supervised lines, the injections-withdrawals in any node of a given zone are supposed to have the same consequences on the supervised lines : as a consequence, we have to introduce the notion of **zonal PTDF**. They are calculated on the basis of the average distribution of consumption in the different zones.

The flows on the supervised lines are calculated on the basis of Zonal PTDF, and take into account the initial distribution of observable flows (which contributes to define  $T_L^{noTrans}$ ) in a reference situation for the network. In this reference situation, we use the average start up plan due to nodal optimisation.

Let  $\{P_i\}$  be the production of the available plants, and  $C_i$  the consumption on each node  $i$ .

Mathematically, flow  $T_L$  on a given line “L” is calculated by the Zonal PTDF according to the following formula :

$$T_L = \sum_{z \in Z} \left[ \text{PTDF}^{zonal}(L, z_0, z) \sum_{i \in \Omega_z} (P_i - C_i) \right] + T_L^{noTrans} \quad (1)$$

where :

- $Z$  is the set of all the zones “z”.
- $z_0$  is the slack zone chosen arbitrarily.
- The  $\text{PTDF}^{zonal}(L, z_0, z)$  is the fraction of the power flow on L calculated, according to Kirchhoff’s laws, only when the consumption of the zone  $z$  is +1MW and the consumption of the zone  $z_0$  is -1 MW. For the calculation of PTDF, the consumptions are pro rata distributed on all nodes of the zones.
- the  $\Omega_z$  is the set of the nodes belonging to the zone “z”.

- $T_L^{noTrans}$  is the “no Transaction” flow on the line L : when each zone is balanced ( $\sum_{i \in \Omega_z} P_i = \sum_{i \in \Omega_z} C_i$  for each zone “z” in  $Z$ ), there are power flows on the lines, that is the reason why it is necessary to introduce  $T_L^{noTrans}$ . This term is computed according to the following formula :

$$T_L^{noTrans} = T_L^{Ref} - \sum_{z \in Z} \left[ \text{PTDF}^{zonal}(L, z_0, z) \sum_{i \in \Omega_z} (P_i^{Ref} - C_i) \right] \quad (2)$$

where :

- $P_i^{Ref}$  is the reference start up : it corresponds to the average start up of each generating unit “i” computed with a “pure” nodal representation. The average is based on 500 scenarios of unavailability of generating units.
- $T_L^{Ref}$  is the average transit (based on the same 500 scenarios as the  $P_i^{Ref}$ ) on the line L computed with the reference start up in a nodal representation.

The formula (2) proves that  $T_L^{noTrans}$  has been chosen so that the reference start up gives the same flows in the nodal and in the zonal representations.

There is no single way for the determination of the reference situation used to calculate the no-transaction flow. For this study, the values  $T_L^{noTrans}$  are deducted from the average of the optimal start up plans observed during nodal optimisation of the network. Accordingly, it is said that aggregate models will give results close to the optimum average nodal reference situation.

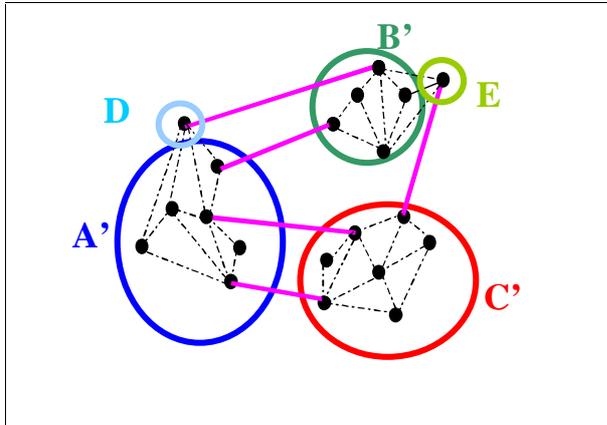
#### 2.4 “Zonal + Particular Nodes – Line” representation

In some cases, a few electrical nodes may display very different marginal costs from the costs of their neighbours. This happens for example when a node is connected to a very saturated line. On the basis of the study of marginal nodal costs, it is possible to individualise these particular nodes.

Singling out these particular nodes could lead to a more refined optimisation on highly congested locations.

The “Zonal + Particular nodes – Line” representation is only a special case of the “Zonal – Line” representation in which some zones are composed of only one node.

The figure below shows our example of network where the nodes are mostly grouped by “country” (A’, B’ and C’) but where some particular nodes have to be isolated (D and E). The only supervised lines are the “interconnections” :



**Figure 3 :** Zonal + Particular Nodes - Line representation of the network

Note that the union of the zones A’ and D is equal to the zone A of the Figure 2. Same remark for the union of B’ and E which is equal to the zone B.

### 2.5 “Zonal - Interface” representation

Compared to the “Zonal-line” representation, the “zonal-interface” representation does not consider the limit of each potentially saturated line but only the transfer possibilities on the interfaces between the zones. As a consequence, in this representation, the constraint is such that the sum of the power flows on the lines belonging to each interface must not exceed the limits set either by an expert user (GRTs, TSOs, ..) or on the basis of the historical flows.

Let us take an example:

Let  $I_{FD}$  be the set of all lines on the border between France and Germany (i.e. the France-Germany interface).

For this type of optimisation with this zonal-interface representation, we have the following constraint:

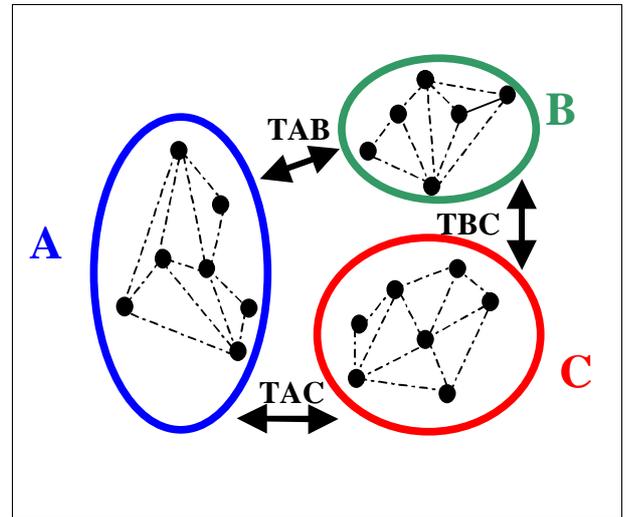
$$T_{I_{FD}}^{\min} < \sum_{k \in I_{FD}} T_k < T_{I_{FD}}^{\max} \quad (3)$$

Where :

- $T_k$  is the flow on a line belonging to the French-German border. This flow is calculated by the Zonal PTDF (the formula was reviewed above in paragraph 2.3).
- $T_{I_{FD}}^{\max}$  is the maximum export from France to Germany.
- $T_{I_{FD}}^{\min}$  is the minimum import from France to Germany.

$T_{I_{FD}}^{\max}$  and  $T_{I_{FD}}^{\min}$  are, in all cases, defined on the basis of observations of historical flows or according to

expert opinion. In this optimisation, we therefore group the lines between two zones. In a zonal-interface representation, our 20-node network would be represented as below :



**Figure 4 :** Zonal-Interface representation of the network

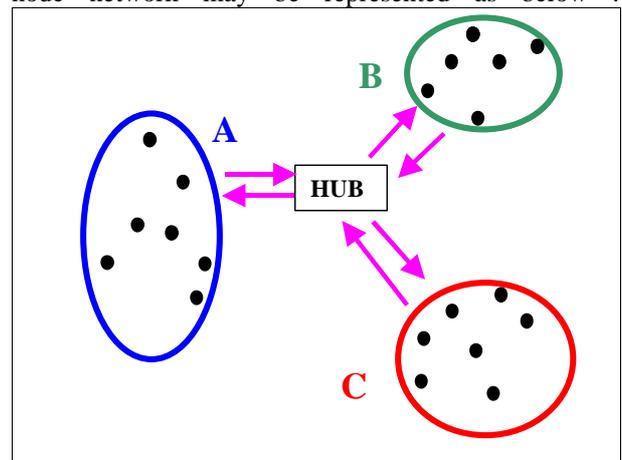
In this figure,  $T_{ij}$  is the interface flow between zone  $i$  and zone  $j$  and must not exceed the maximum and the minimum flows chosen.

### 2.6 “Zonal - Import/Export” representation

Compared to the “Zonal-Interface” representation, the “zonal-import/export” does not take the network into account.

In this representation, each zone must have its import-export balance in the range of two values (minimum and maximum) predefined by an expert user. Of course, the sum of the balances of all zones must be equal to zero.

In a “zonal-import/export” representation, our 20-node network may be represented as below :



**Figure 5 :** Zonal-Import/Export representation of the network

In the representation chosen in the figure above, each zone has to exchange with the “HUB” zone a quantity delimited by the values predefined by an expert.

## 2.7 Examples of computations

Computations have been made in EDF R&D on a stylised network representative of the European network so as to compare the efficiency of each representation.

In this stylised network, the demand in each node is supposed to be inelastic. As a consequence, the optimisation consists in a minimisation of the generating cost under network and exchange constraints.

Concerning these constraints, the different power flow representations detailed in the previous paragraphs have a direct influence on the use of the network and the economics of the system. The efficiency of each representation is measured by making a distinction between two stages :

- The market splitting stage (day-ahead) : this first stage is devoted to the determination of the program of exchanges. Under the networks constraints retained (which could be, depending on the kind of zonal optimisation chosen, the limits of some supervised lines, the limits of the exchanges between zones or the limits of the import/export balance for each zone) , the global cost (i.e. the generating cost) of the system is minimised. The results of this minimisation are, among other things, the exchanges between the different zones : these exchanges make it possible to compute the balance for each zone. This balance calculation can be considered as the program of exchanges of each zone.
- The re-dispatching stage (D-day) [6] : the second stage optimises the generating cost with a nodal representation of the system with the additional constraints of respecting the program of exchanges computed in the day-ahead market.

The overall system cost resulting from these two stages is compared with that achieved in the reference case (“pure” nodal representation), where all nodes are shown and where *de facto* the first allocation relieves every constraints.

For the “Zonal – line”, on the examples on which the computations have been made, the zones have not only always been equivalent to countries. As we explained in 2.4. (the “Zonal + Particular Nodes – Line” representation) , we have also individualised some particular nodes on the basis of the study of marginal nodal costs.

For the “Zonal – Interface” representation, we have made the computation with limits on the interface defined on the basis of observations of historical flows and limits on the interface defined according to expert opinion.

As a consequence, we have 5 computations to compare to the “pure” nodal representation :

- Zonal-Interface with limits on interface fixed by experts.

- Zonal-line with zones equivalent to countries + some individual nodes.
- Zone-line with zone equivalent to countries.
- Zonal interface with limits on interface computed using many nodal optimisations.
- Zonal-Import/Export.

The results on which the comparisons are done are average values obtained, in each representation, using the calculations on 500 different scenarios of availability of generating units on a typical winter peak-hour.

In the following table, there are three columns with values :

- the first one gives, in percentage, the global cost variation (compared to the global cost calculated in the nodal optimisation) realised day ahead, in the first stage:

$$\frac{\text{Cost}^{\text{Zonal}} - \text{Cost}^{\text{Nodal}}}{\text{Cost}^{\text{Nodal}}} \times 100 \quad (4)$$

Most of the time, the zonal representation leads to over optimisation, which results in a decrease of the global cost.

- The second column with values displays, in percentage of the nodal cost, the increase of the global cost (compared to the global cost calculated in the zonal optimisation) due to the D-Day re-dispatching needed to realise the exchanges found in the day ahead phase :

$$\frac{\text{Cost}_{\text{Capacity allocation constraints}}^{\text{Nodal}} - \text{Cost}^{\text{Zonal}}}{\text{Cost}^{\text{Nodal}}} \times 100 \quad (5)$$

- The last one gives the total cost premium due to the simplification in the representation of the network :

$$\frac{\text{Cost}_{\text{Capacity allocation constraints}}^{\text{Nodal}} - \text{Cost}^{\text{Nodal}}}{\text{Cost}^{\text{Nodal}}} \times 100 \quad (6)$$

It is interesting to note that in some scenarios of certain zonal representations, no possible re-dispatching has been found in the second phase to realise the exchanges calculated in the first phase : it is the case of the “zonal line where zones are equivalent to countries” (3 scenarios out of the 500 have led to unserved energy) and in the “zonal interface with limits on interface based on nodal averages ” (1 scenario out of the 500 has led to unserved energy).

### 3. CONCLUSION

The “zonal line + a few detailed nodes” and “zonal-interface” solutions lead to a global system cost of the same order. In both cases, the proximity with the optimum solution depends closely on the efficiency of the network operator in identifying nodes to be highlighted or when establishing suitable limits for the interfaces. Nevertheless, it may be assumed that the discretionary powers of the transmission system operator is more extensive for the calculation of interface limits, an extremely complex task with no clear-cut solution, than in the identification of nodes close to structural constraints.

This approach was adopted in a slightly simplified form for the market splitting system now being tested in Italy: whilst the nodes directly associated with the most congested lines are taken into account properly, the impedances of the electrical grid used bear no relation with reality.

Finally, we do not find any true superiority of one form of market organisation to another between the two best solutions, provided that the node description is sufficiently refined or the limits for each interface have been sufficiently reduced. Hence, the formulae could be mixed according to the requirements of TSOs, the technical characteristics of networks and confidence in the regulation of the transmission system operator.

On the other hand, the highly simplified case of limits concerning only import/export balances leads to severe de-optimisation of the system and cannot be compared with the other methods.

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Type of modelling		% cost premium due to market splitting in day ahead (1)	% cost premium due to the completion of the exchange programme by the TSO (2)	% total cost premium (1)+(2)
Zonal interface with limits on interface fixed by experts	<b>Average</b>	<b>-1.24 %</b>	<b>1.55 %</b>	<b>0.31 %</b>
	Min	-2.28 %	0.84 %	0.04 %
	Max	-0.27 %	4.83 %	2.72 %
Zonal line with zones = countries + some individual nodes	<b>Average</b>	<b>-0.98 %</b>	<b>1.34 %</b>	<b>0.36 %</b>
	Min	-1.98 %	0.63 %	0.08 %
	Max	-0.19 %	3.79 %	2.07 %
Zonal line (Implicit flow based market coupling) with zones = countries	<b>Average</b>	<b>-0.43 %</b>	<b>1.13 %</b>	<b>0.7 %</b>
	Min	-1.51 %	0.52 %	0.15 %
	Max	0.86 %	3.22 %	2.37 %
Zonal interface with limits on interface based on nodal averages	<b>Average</b>	<b>-1.49 %</b>	<b>2.46 %</b>	<b>0.97 %</b>
	Min	-2.48 %	1.03 %	0.11 %
	Max	-0.70 %	7.06 %	5.16 %
Simplified zonal, with import/export balance constraints and limits on interface according to expert opinion	<b>Average</b>	<b>-1.62 %</b>	<b>3.20 %</b>	<b>1.58 %</b>
	Min	-2.73 %	1.35 %	0.22 %
	Max	-0.65 %	8.20 %	5.83 %

**Table 1** : Cost of different zonal representations compared to the nodal representation

In the previous table, the solutions are classified in increasing order of average total cost premium.

The best one seems to be the zonal interface with limits on interface fixed by experts. The cost of the second solution is close to the cost of the best one.

Only the case of optimisation with import/export balance constraints stands out due to its relatively high sub-optimisation. The other methods may produce results of the same magnitude, provided that certain modelling parameters are set correctly. It is important to note that these computations have been made for only one hour and the a one percent difference could lead to hundreds millions euros of difference in one year.